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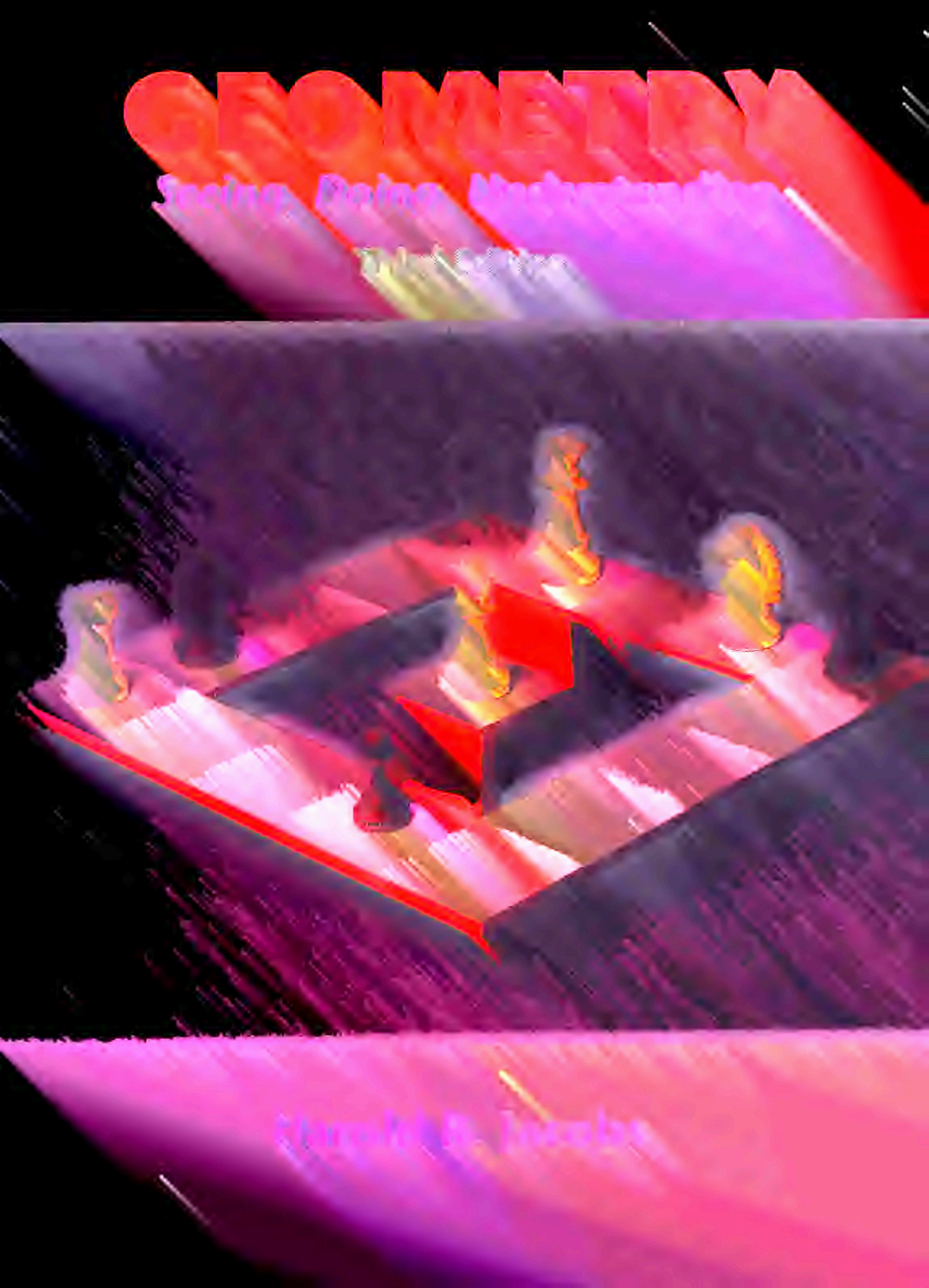

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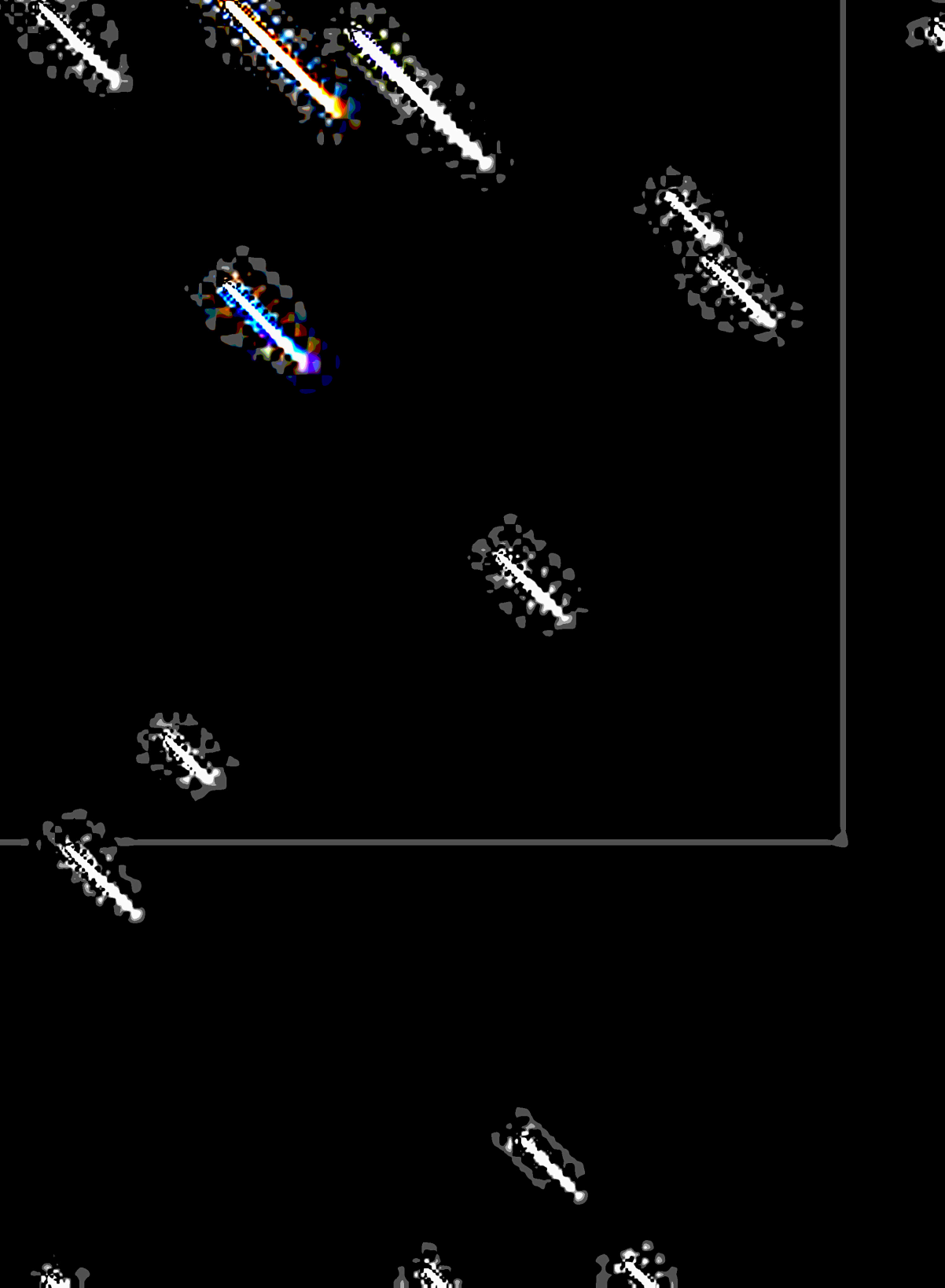

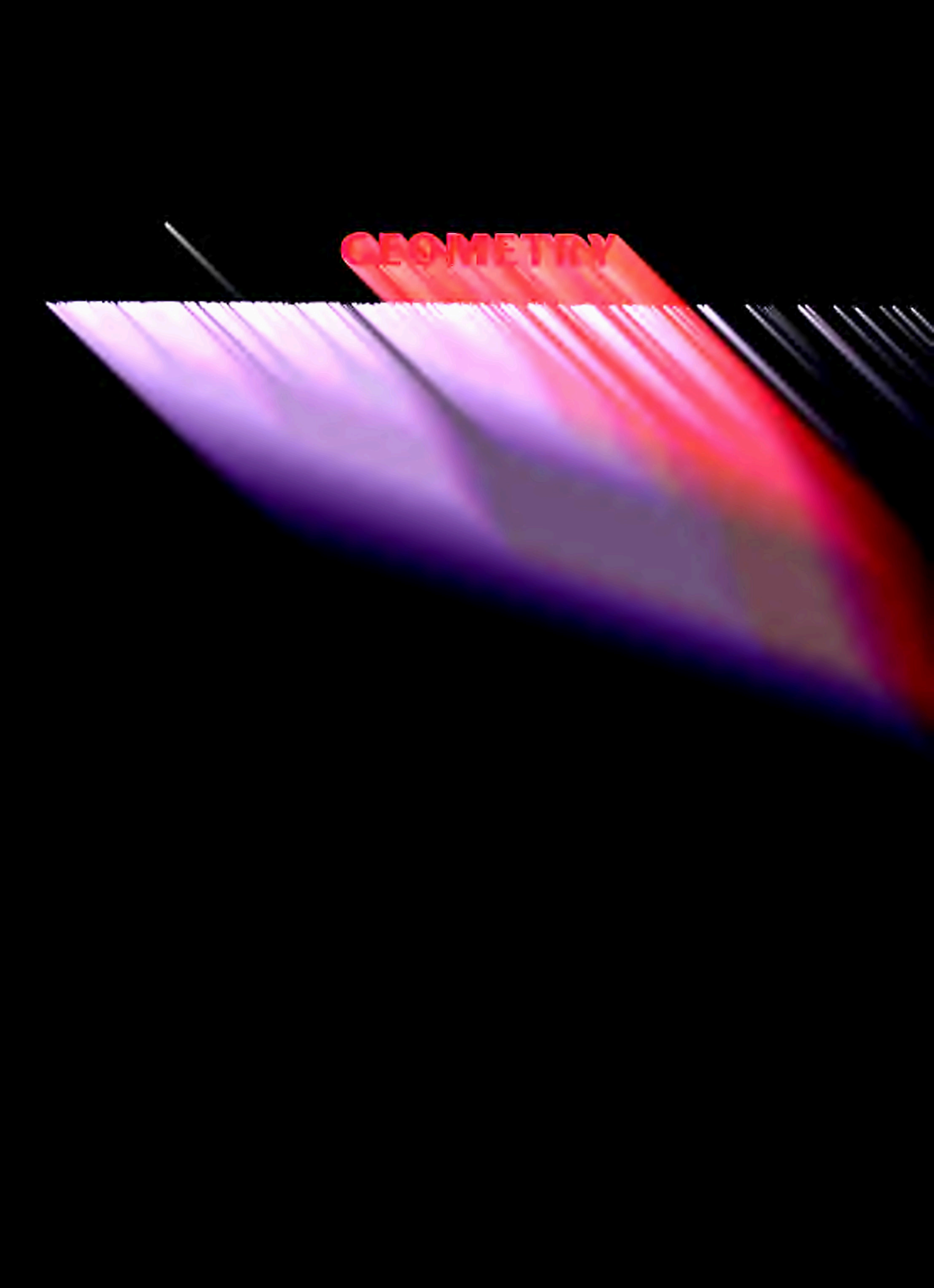
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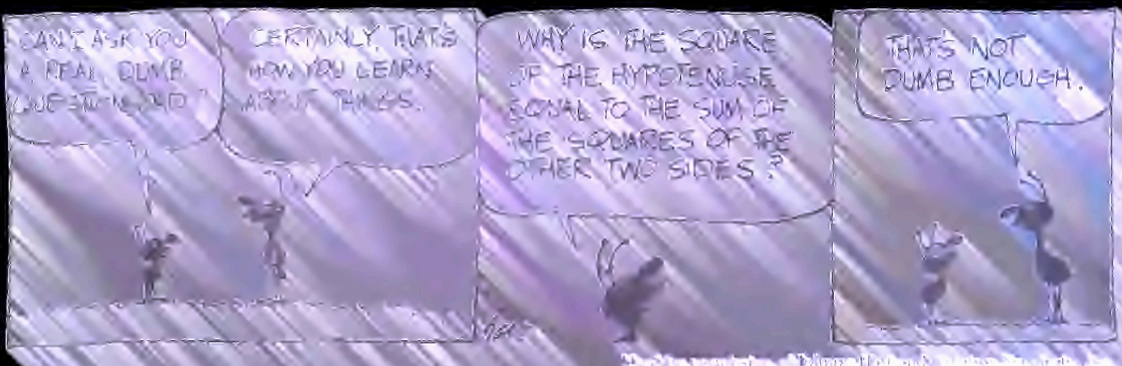
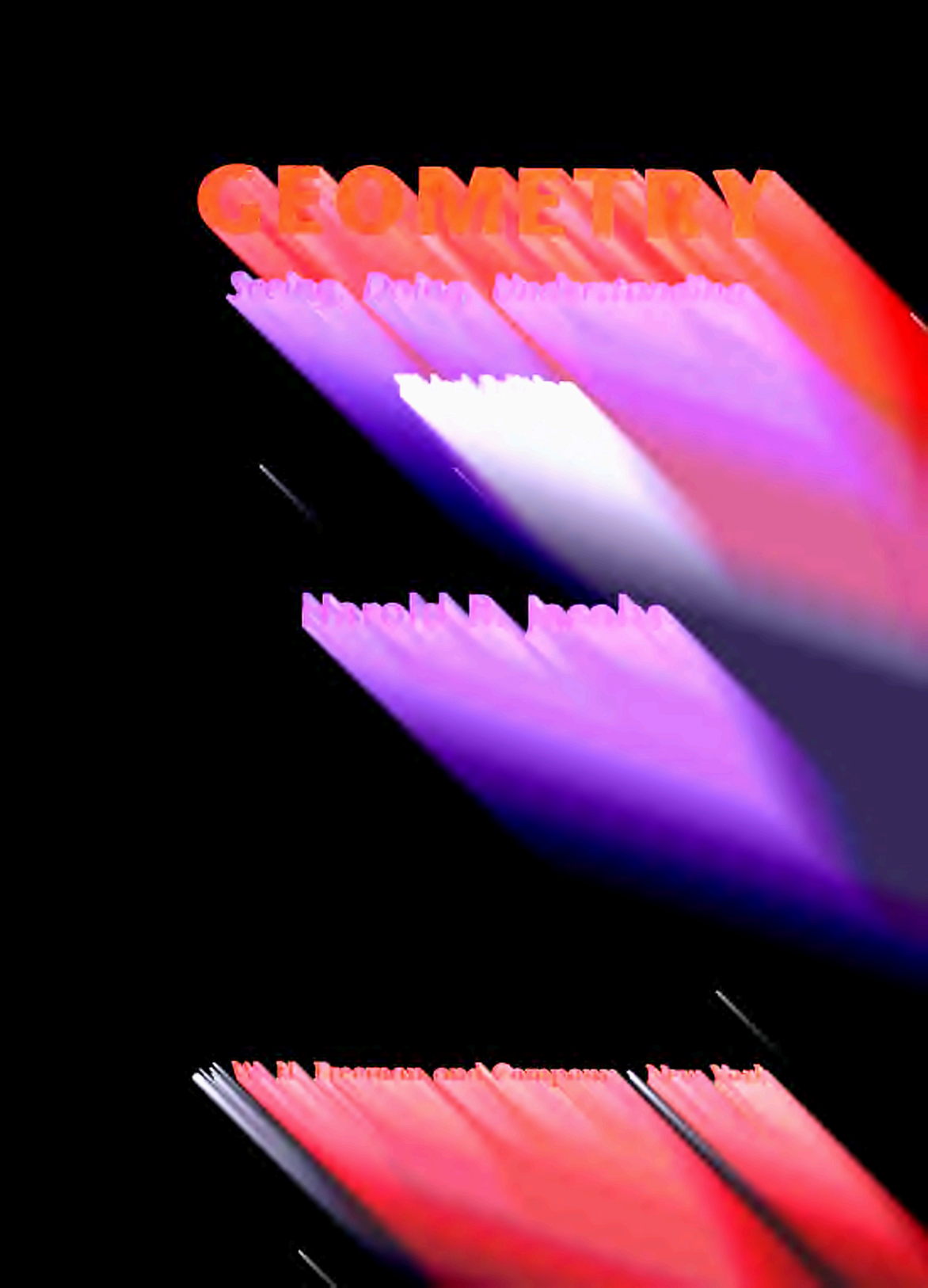

CAN I ASK YOU A REAL DUMB QUESTION, DAD?
CERTAINLY. THAT'S HOW YOU LEARN ABOUT THINGS.
WHY IS THE SQUARE OF THE HYPOTENUSE EQUAL TO THE SUM OF THE SQUARES OF THE OTHER TWO SIDES?
THAT'S NOT DUMB ENOUGH.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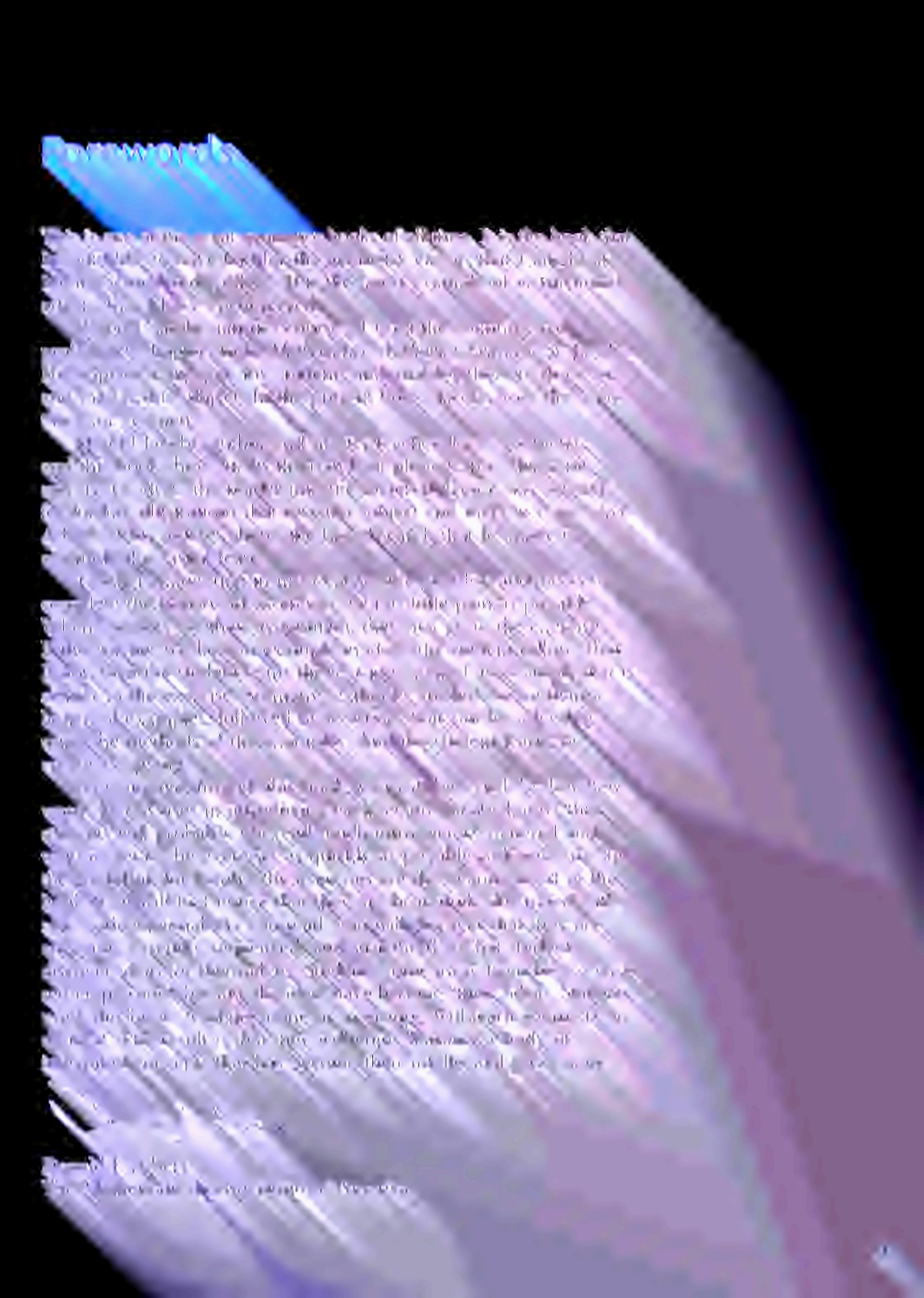
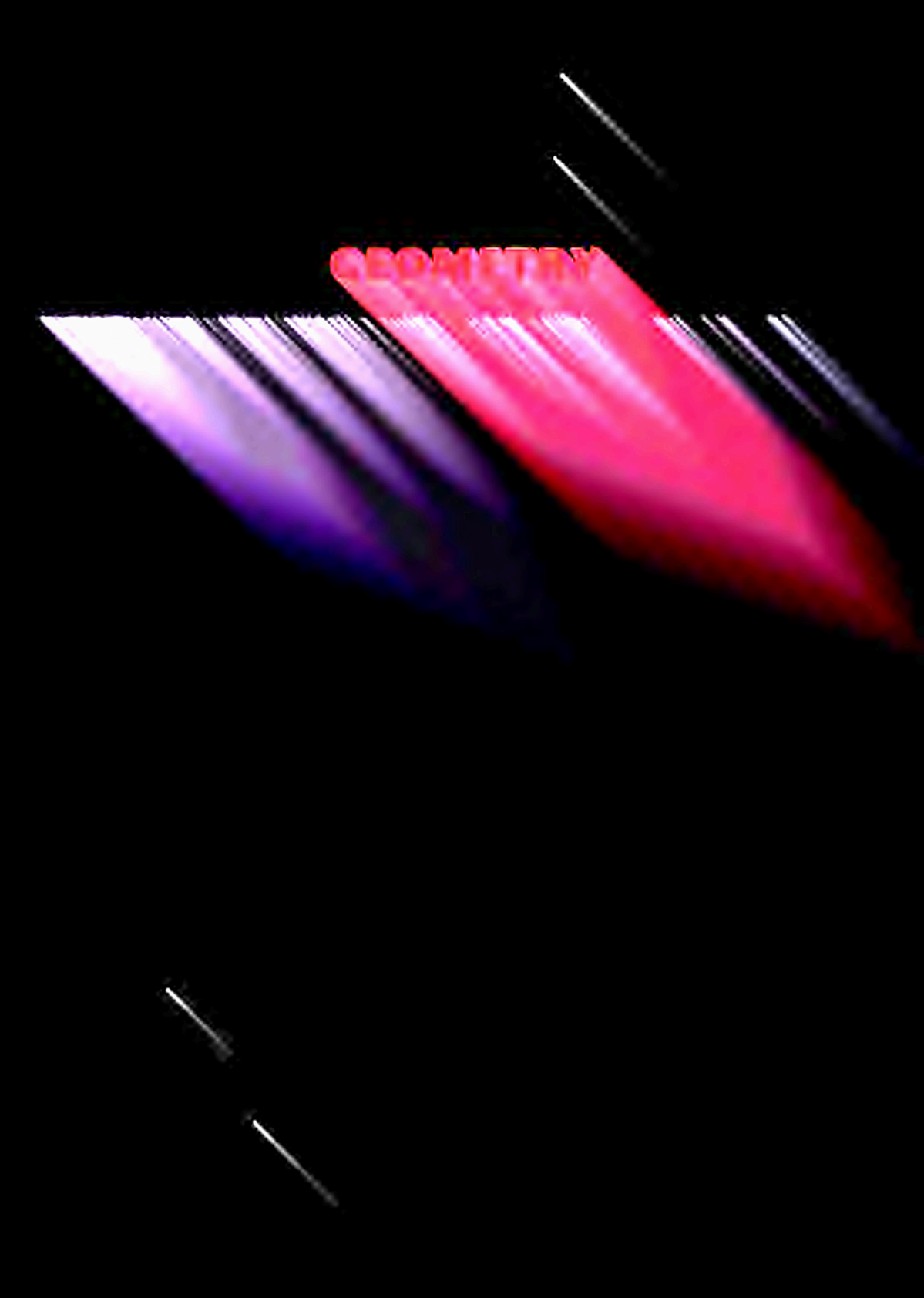


Sincerely,

Andrew McLean
Vice President

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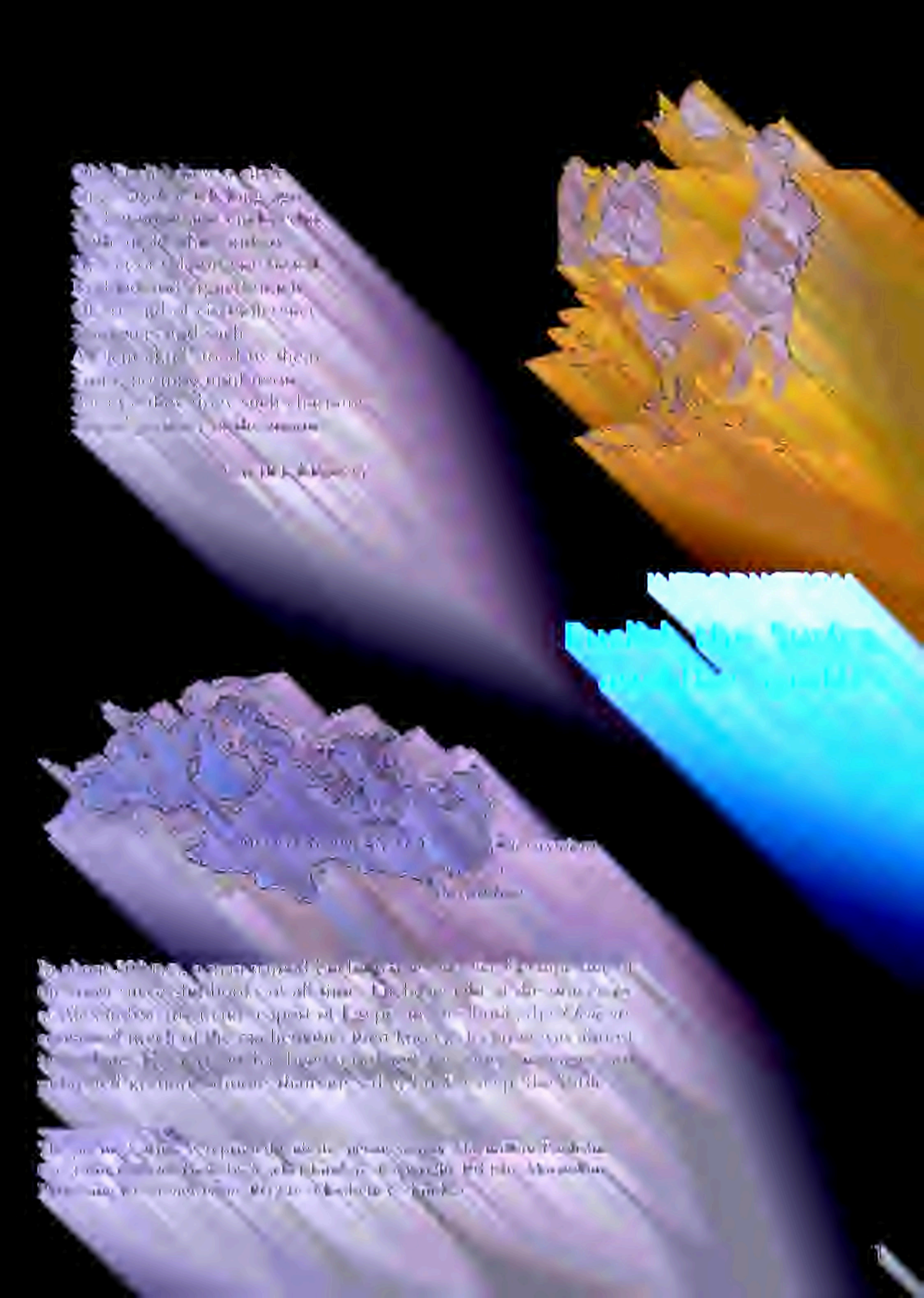

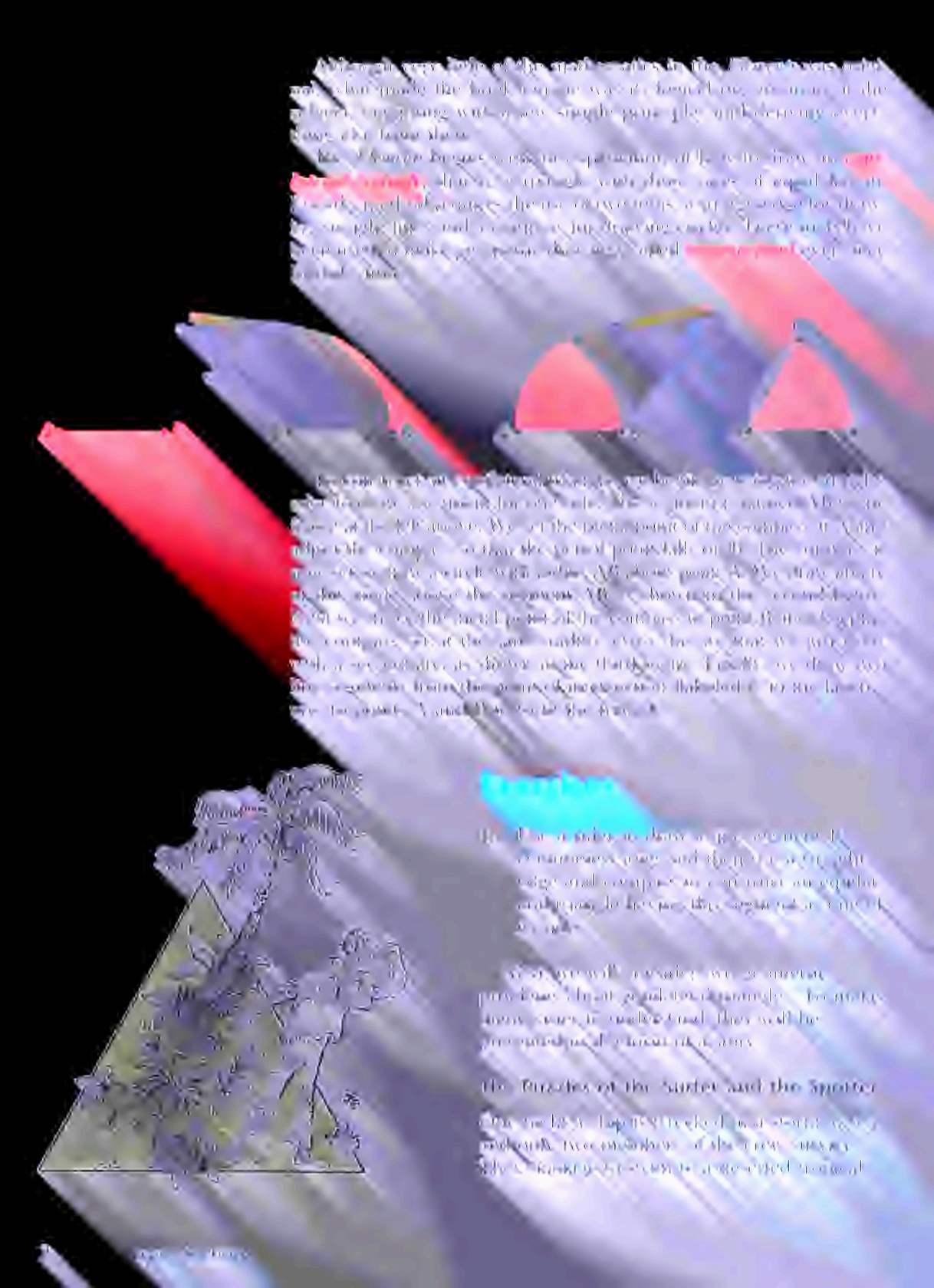

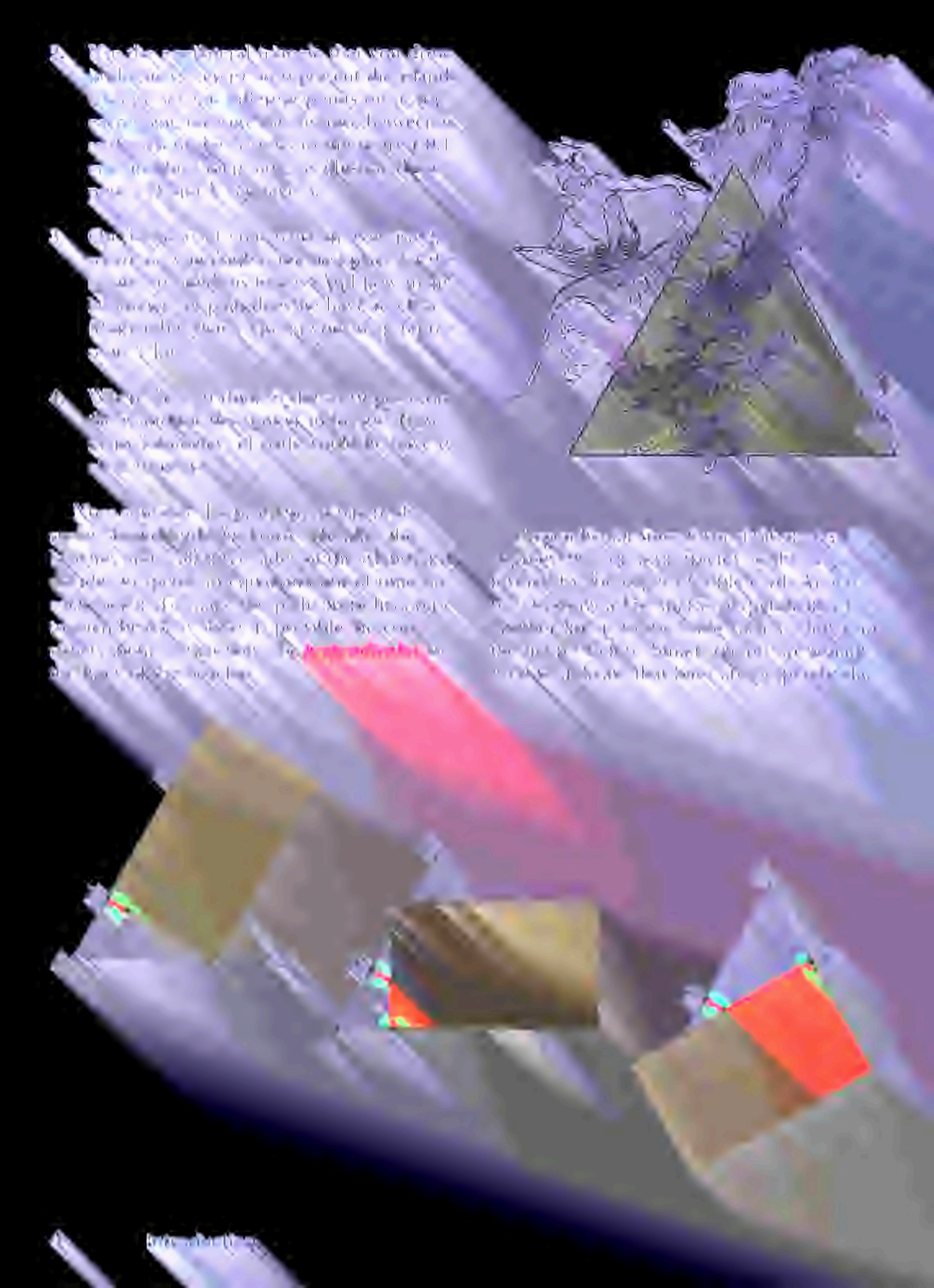





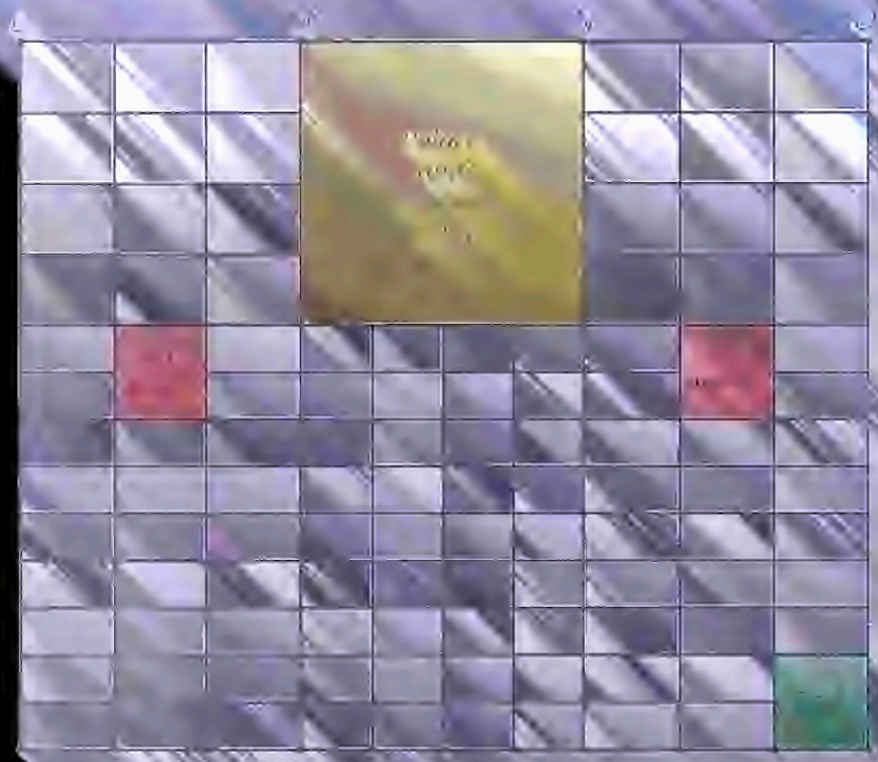

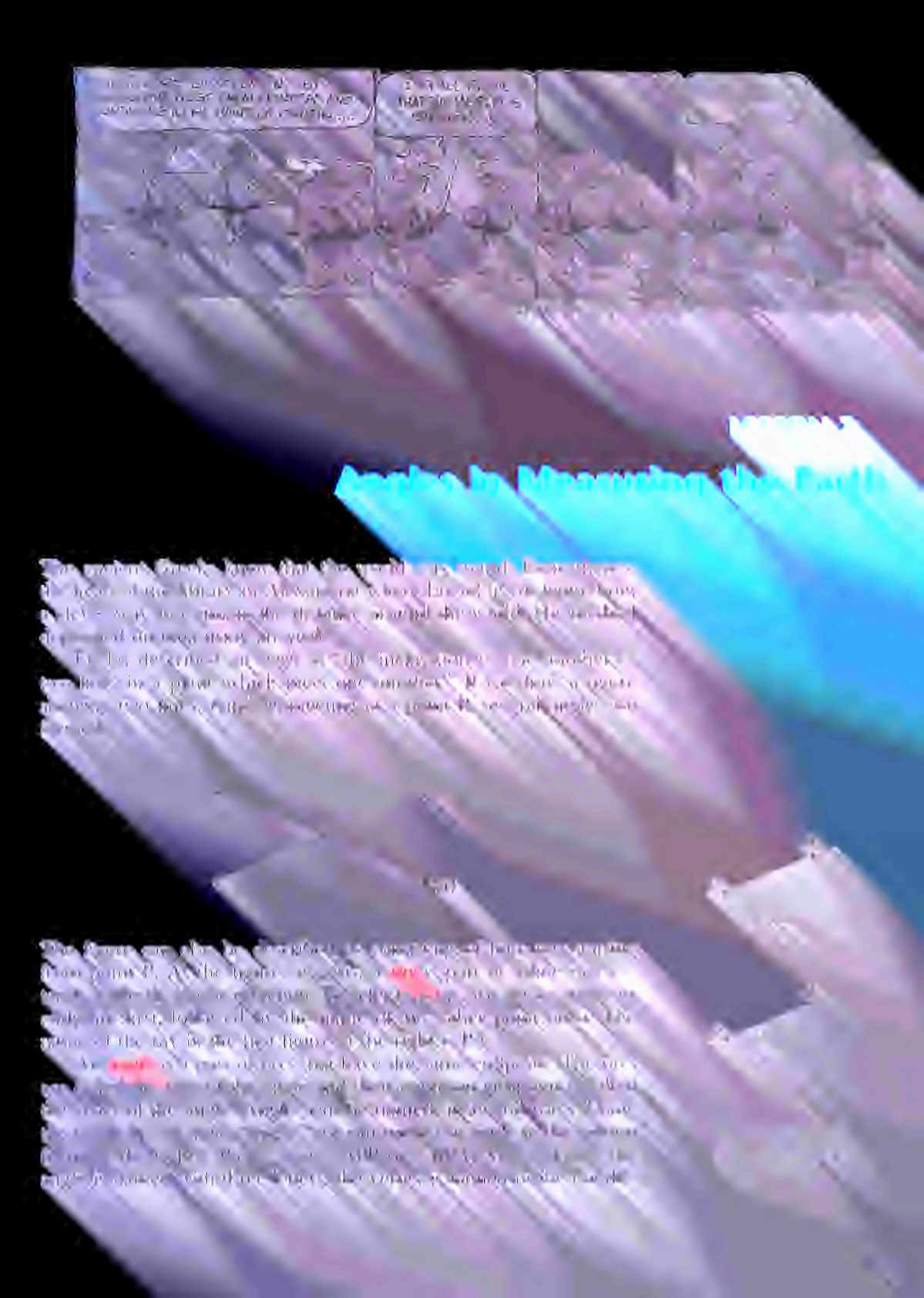















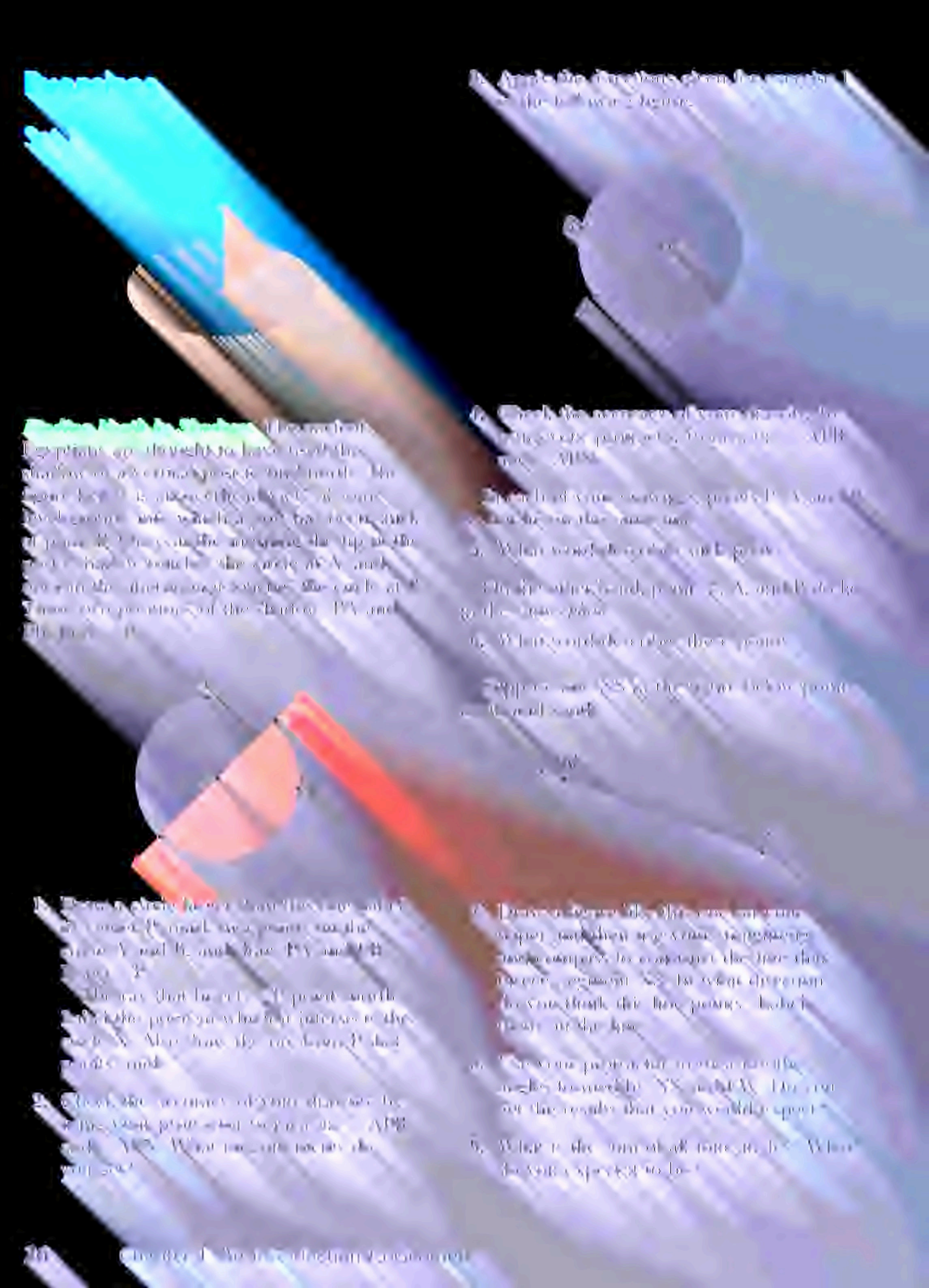


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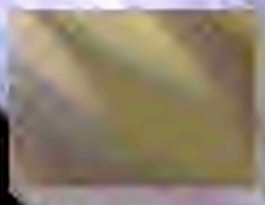





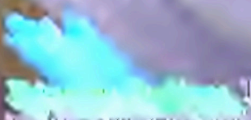







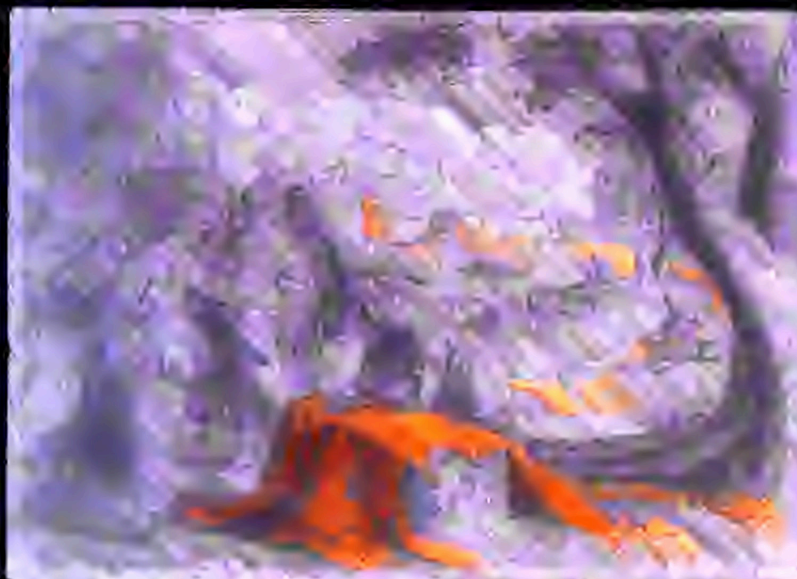





Addition

Multiplication







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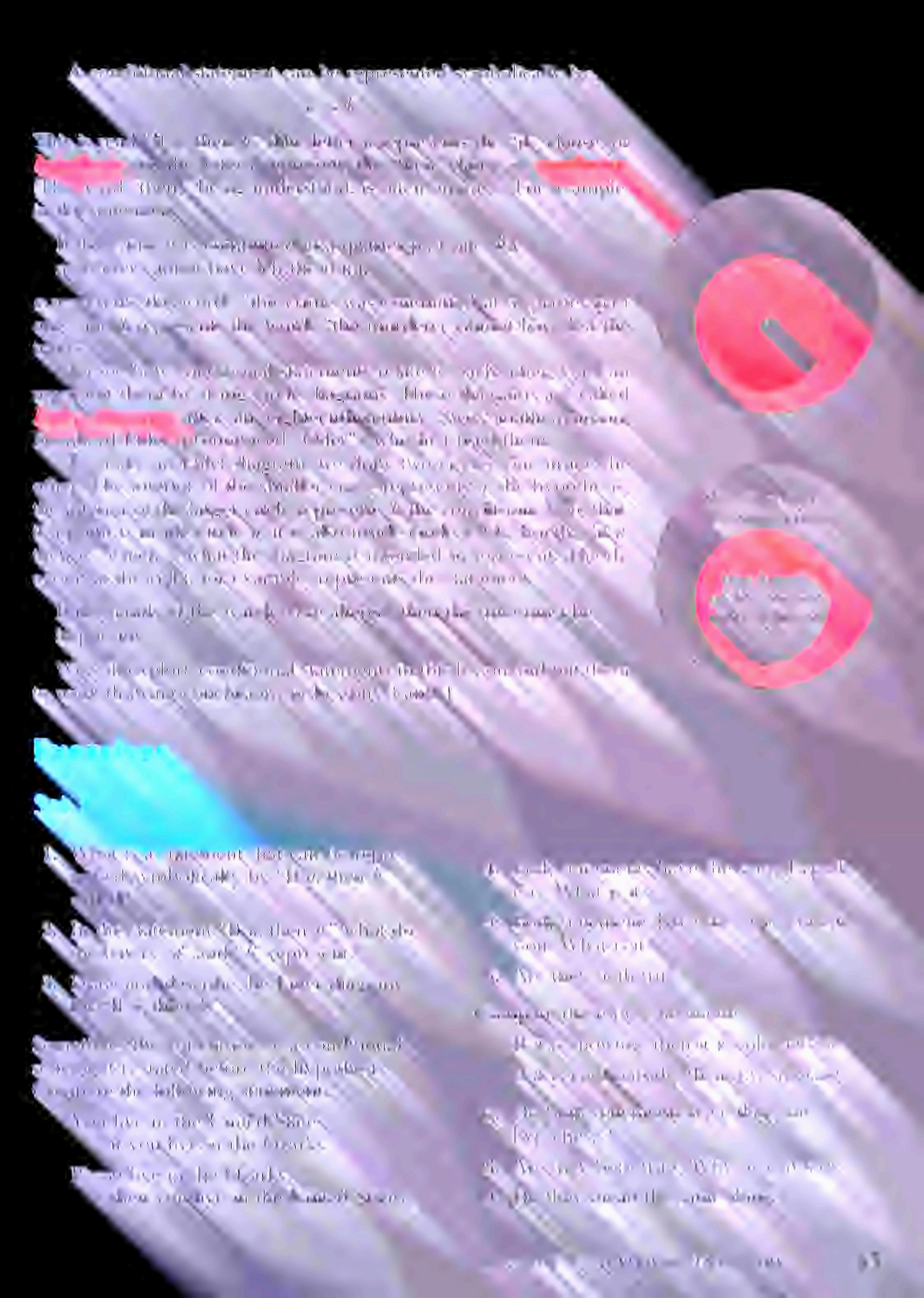
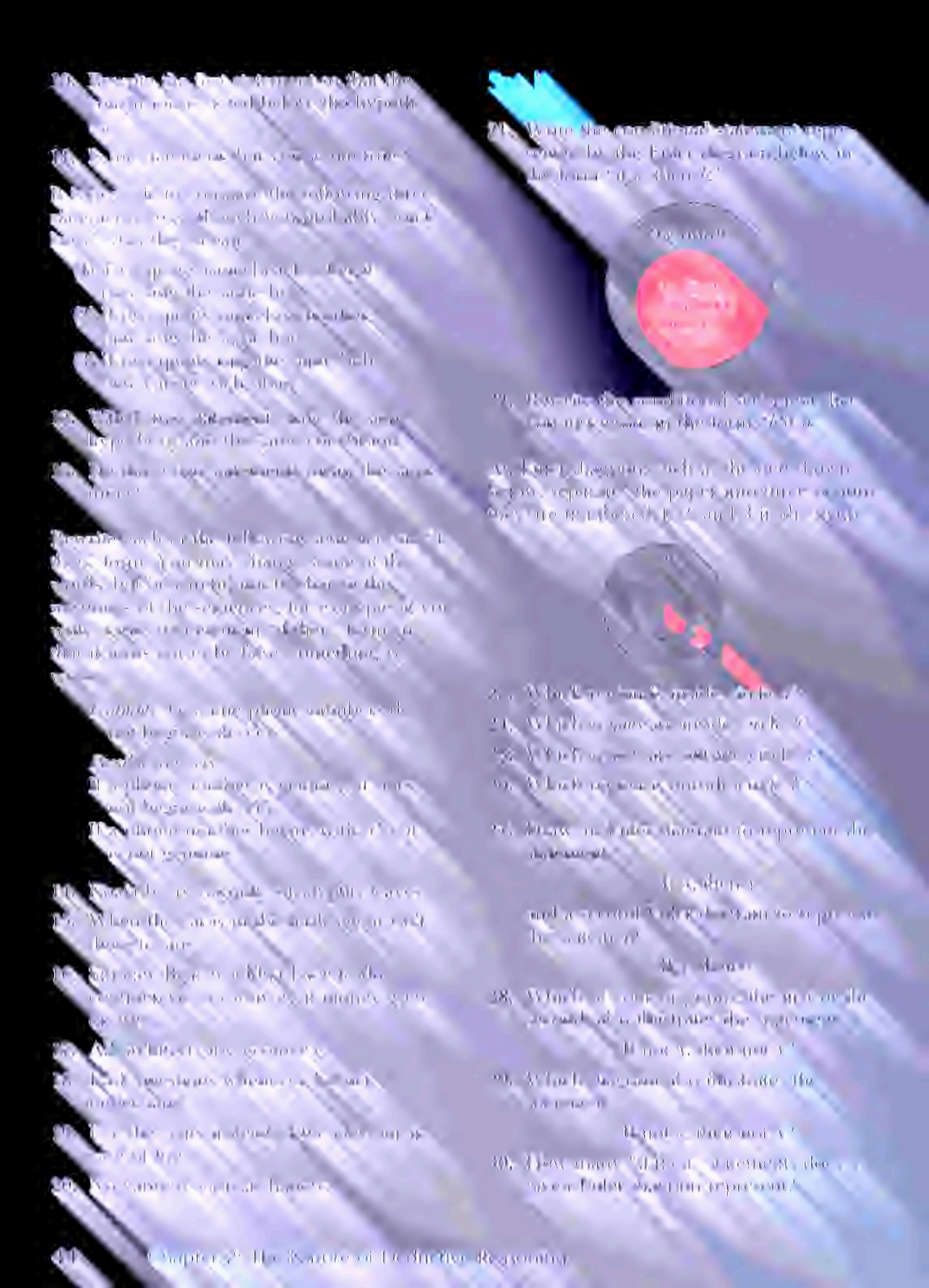




Horse by Any Other Name Is Still a Horse?

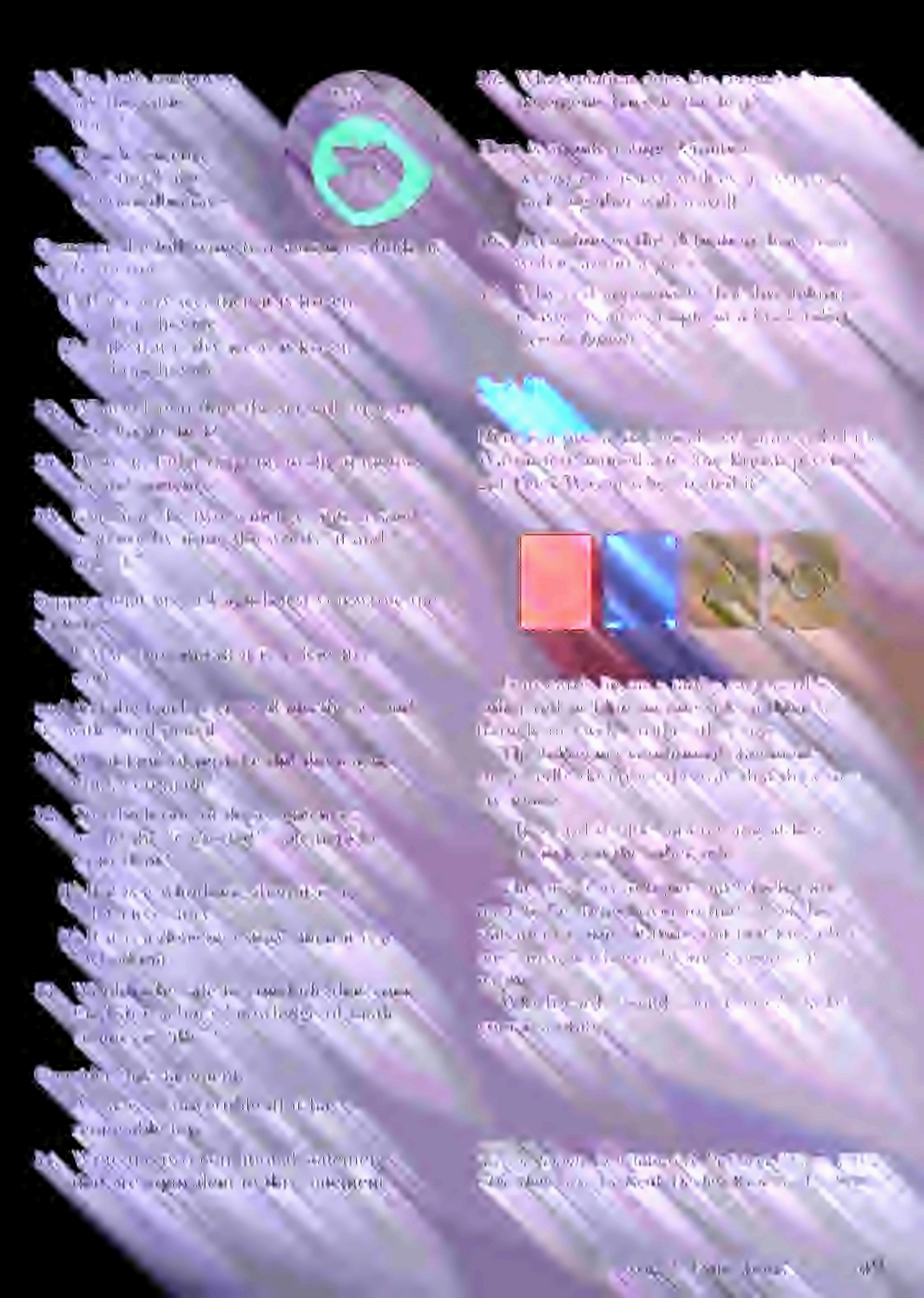
It's All in the Name



ACCORDING TO A HORSEMAN:

ACCORDING TO THE DICTIONARY:

Definitions



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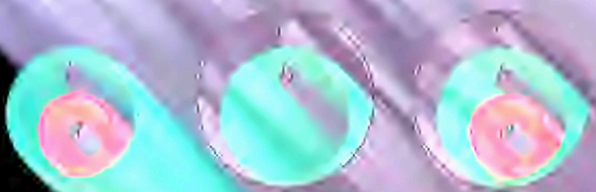
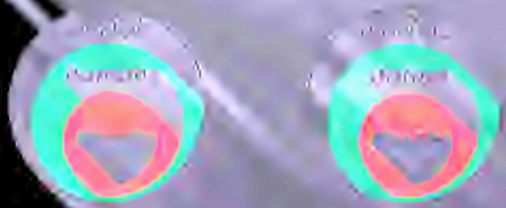
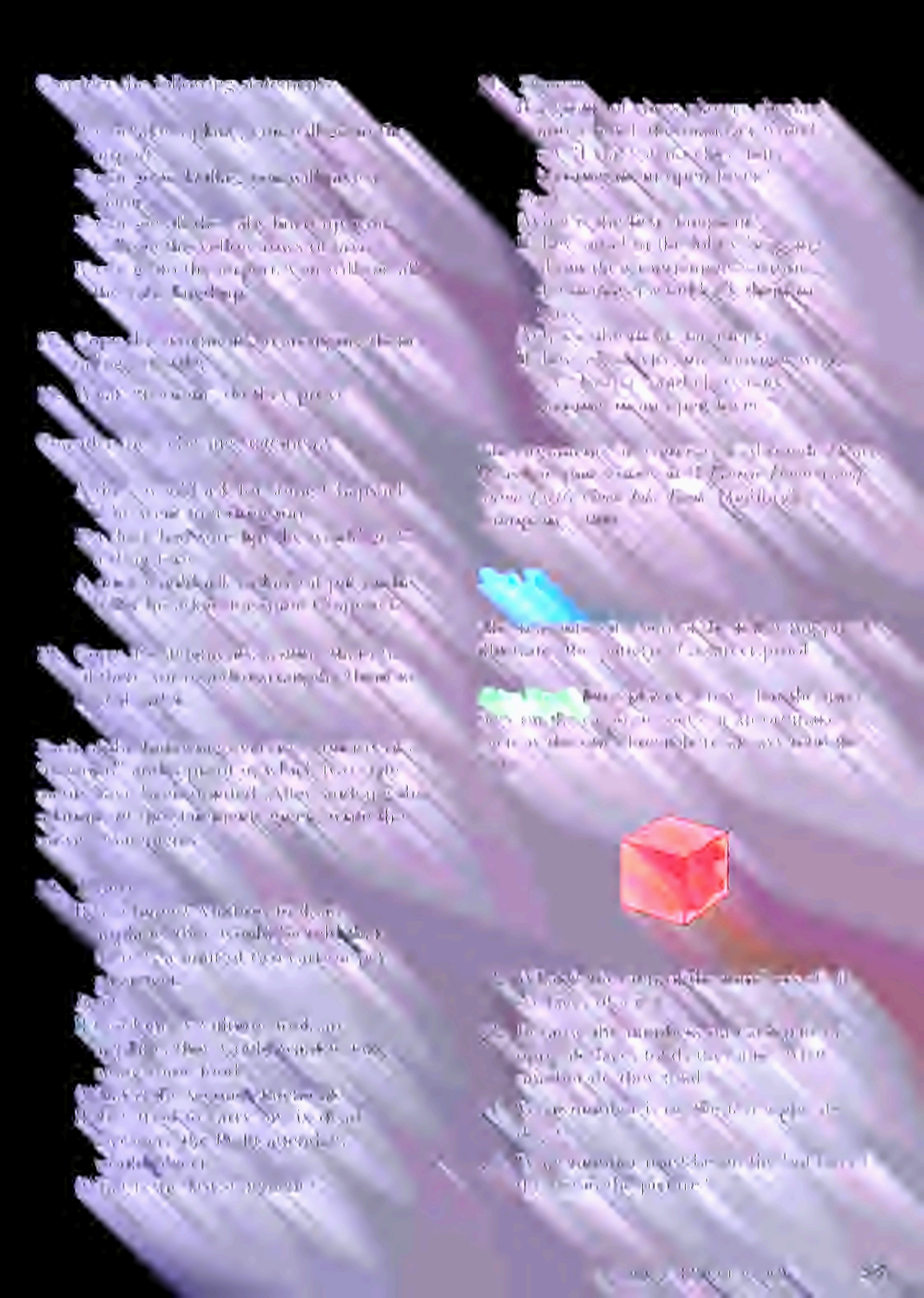








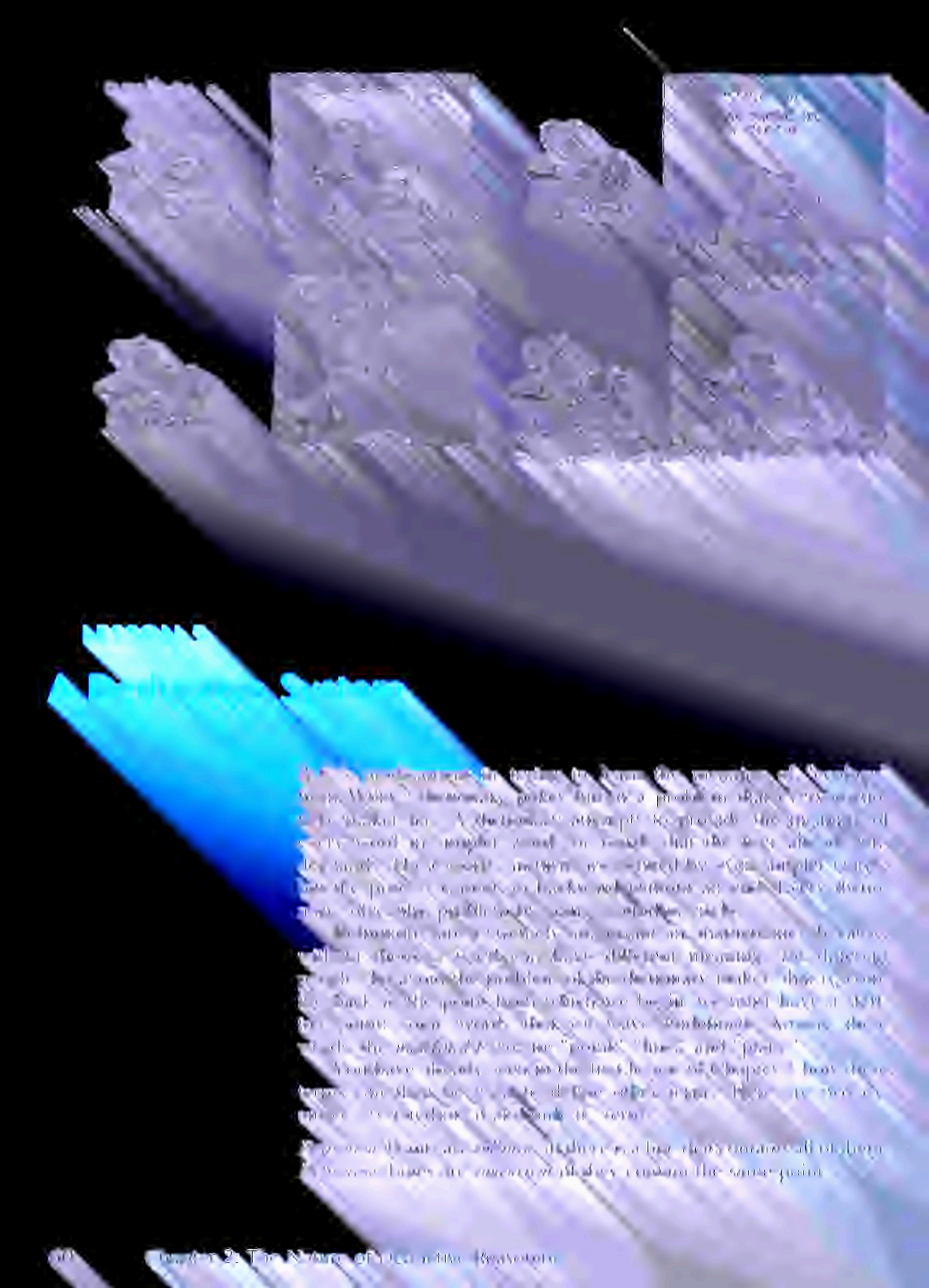


Indirect Proof

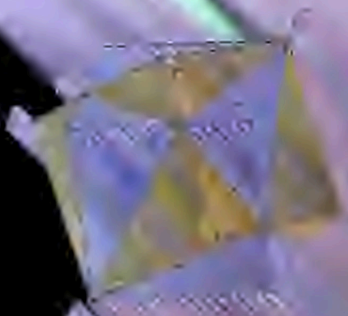






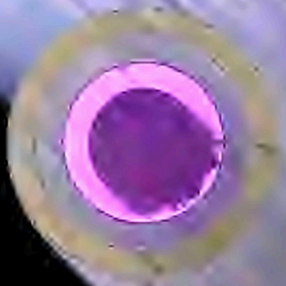


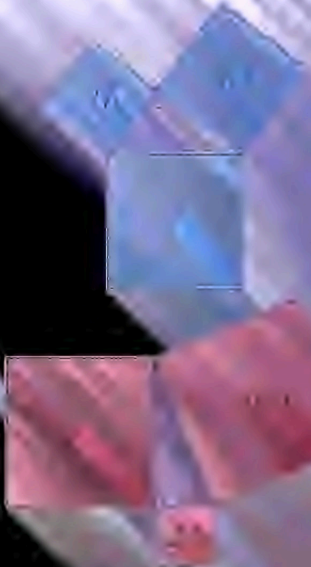
Chapter 2: The Nature of Decision Behaviors

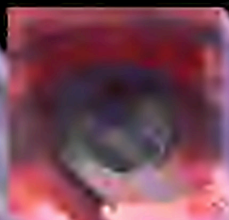


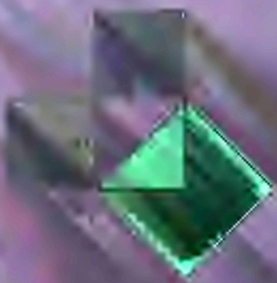




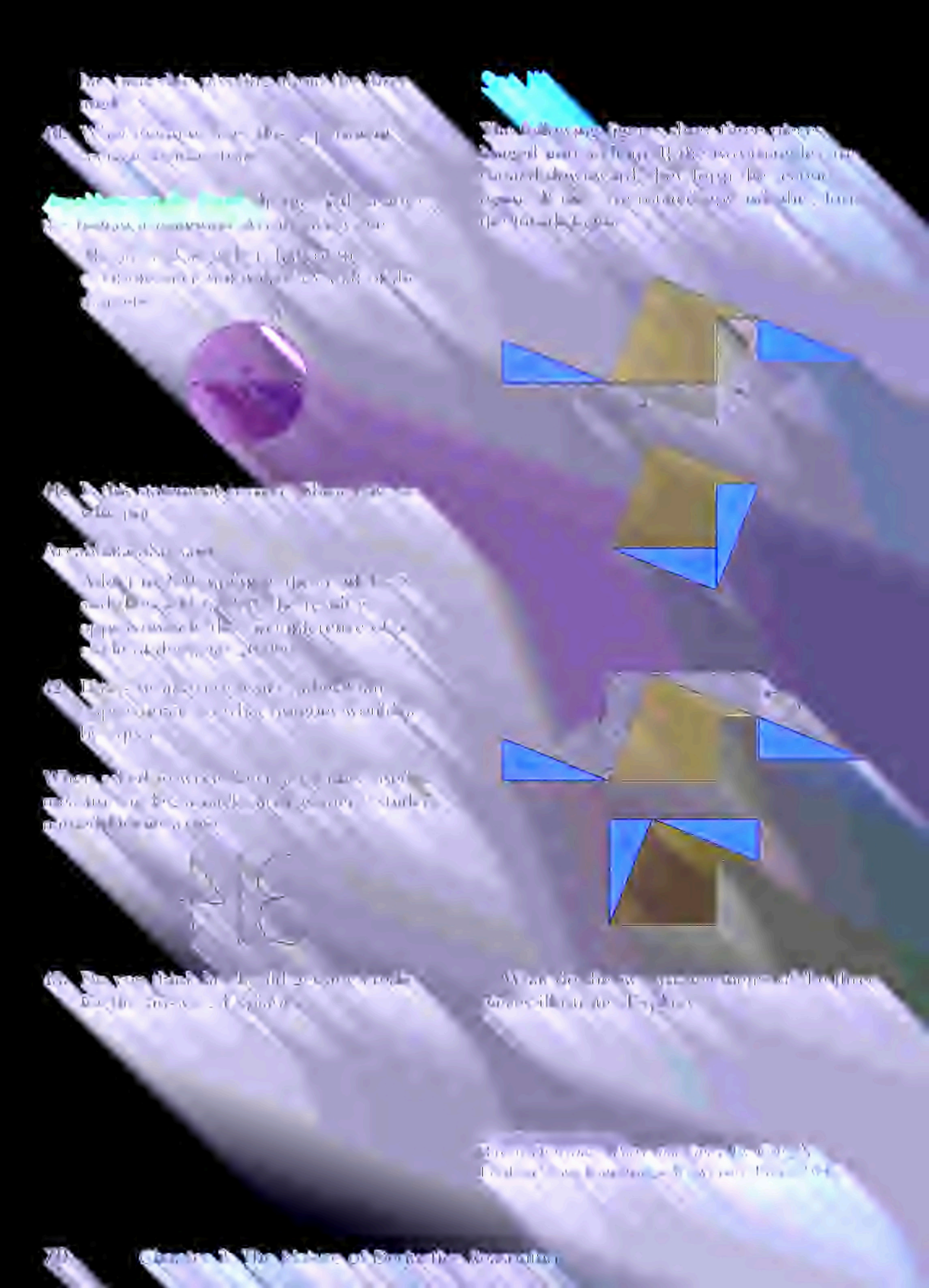




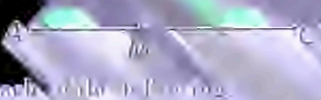




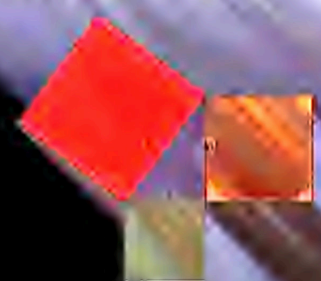






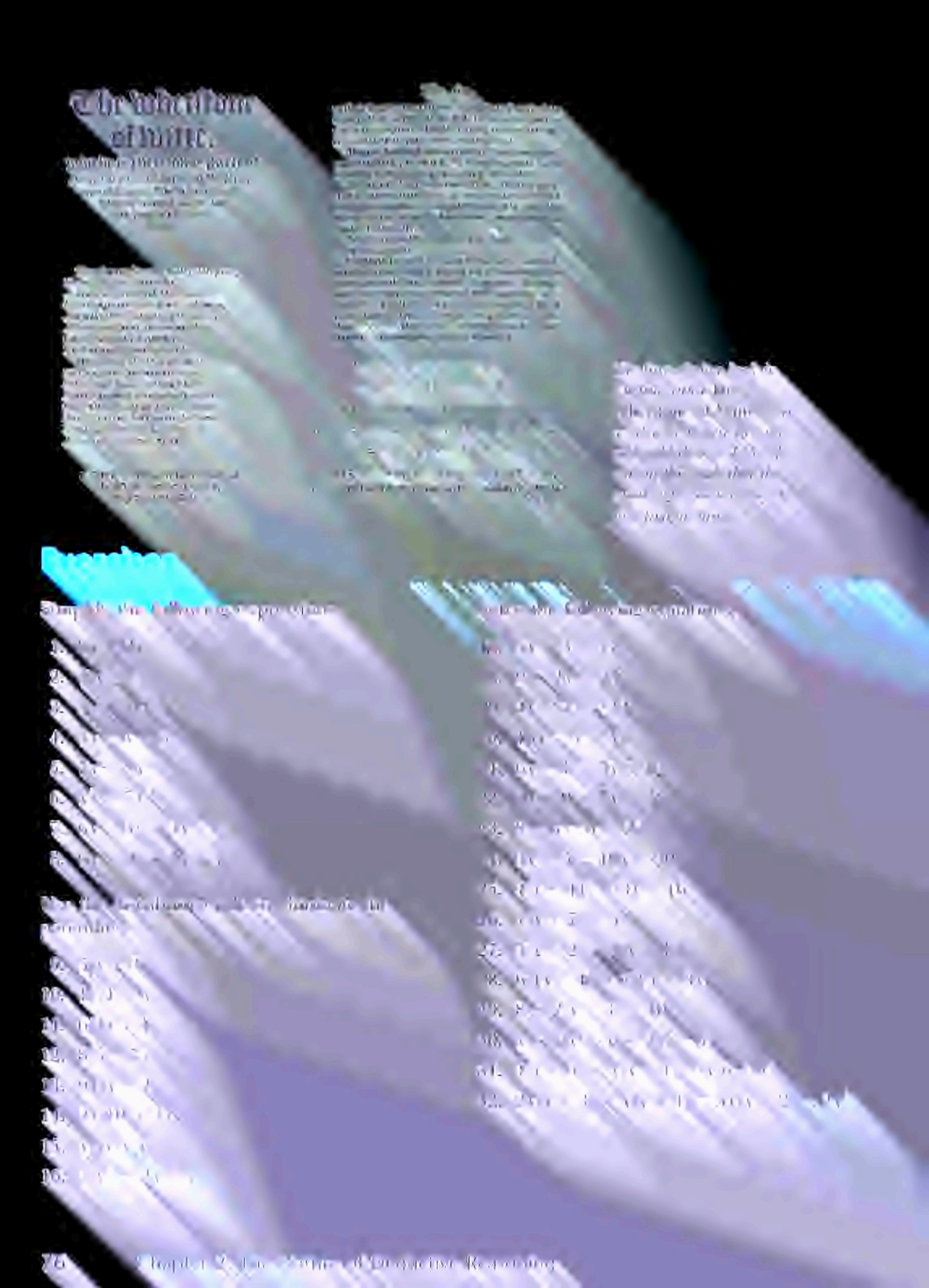








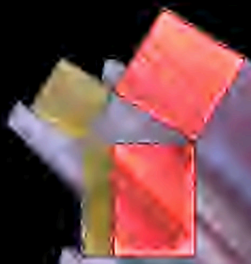


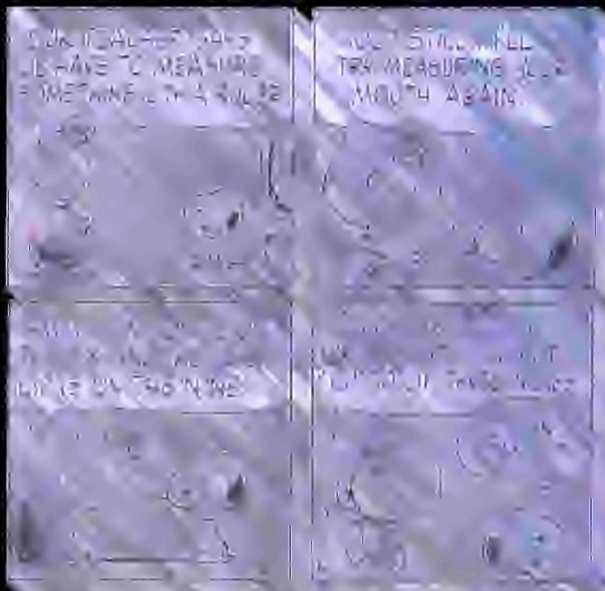


Write the proof in each of the following exercises.





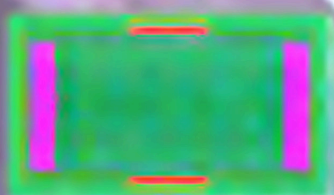












Suppose the base of a pyramid is a square with sides of length s units and the height of the pyramid is h units.

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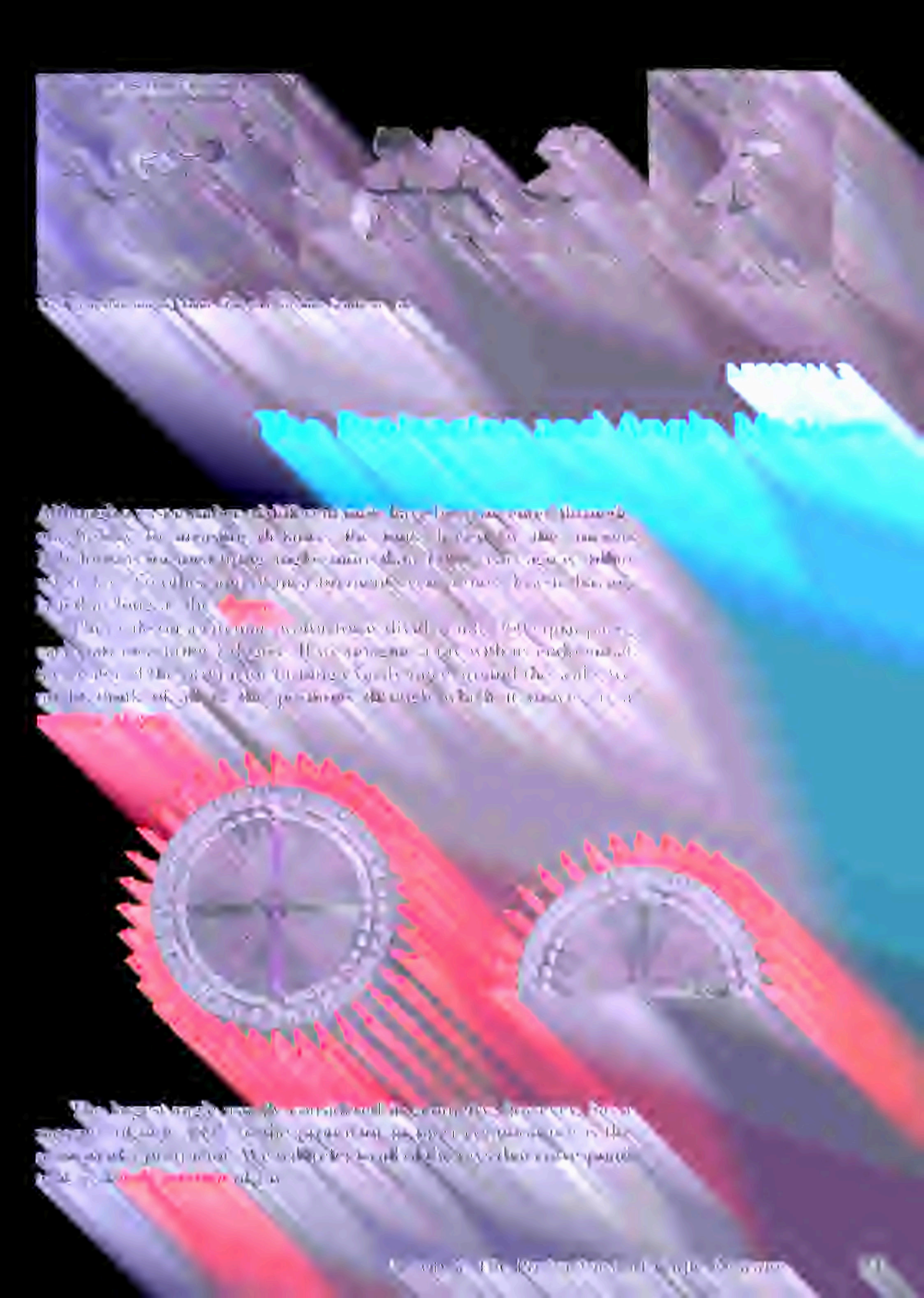

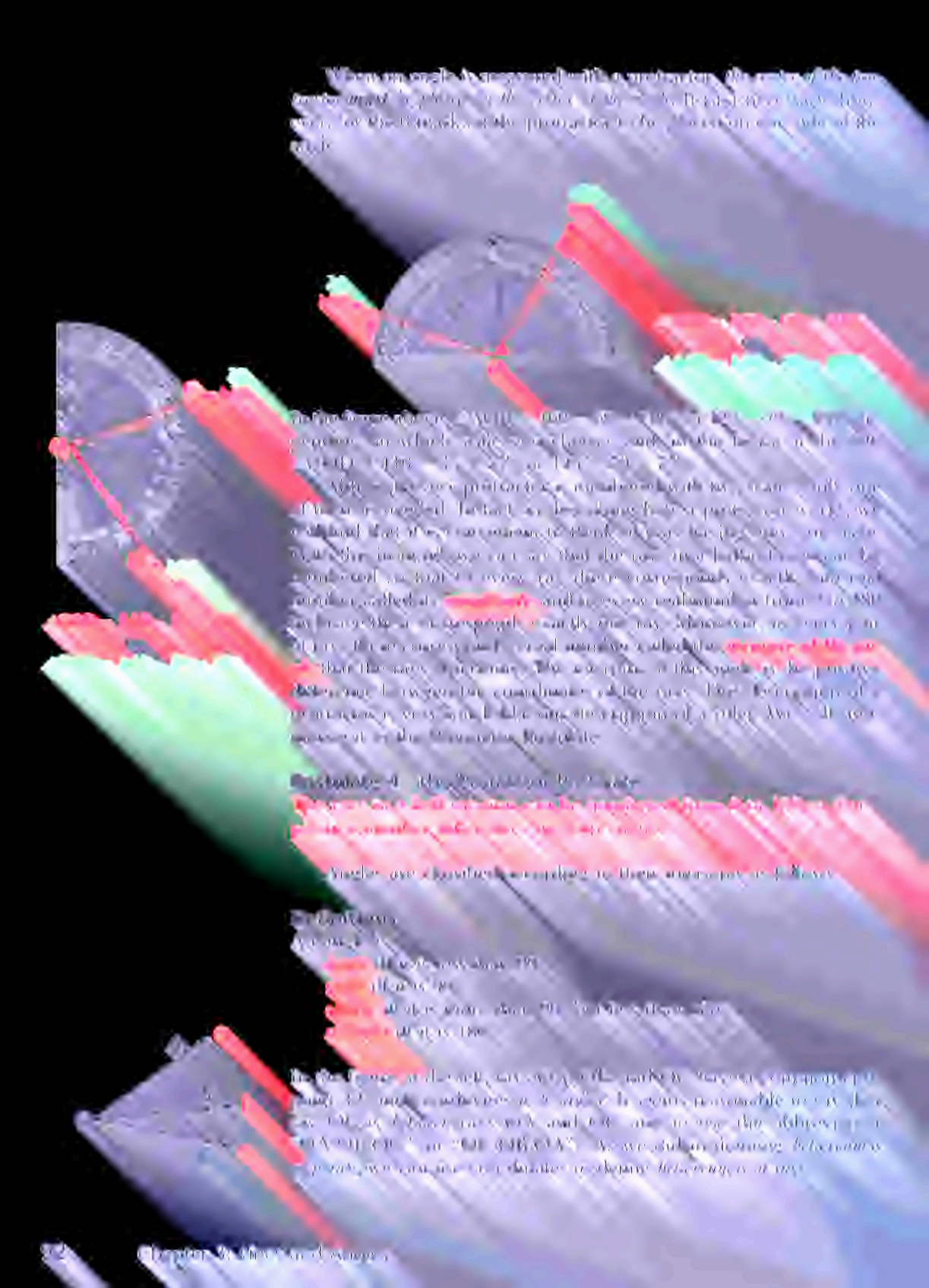

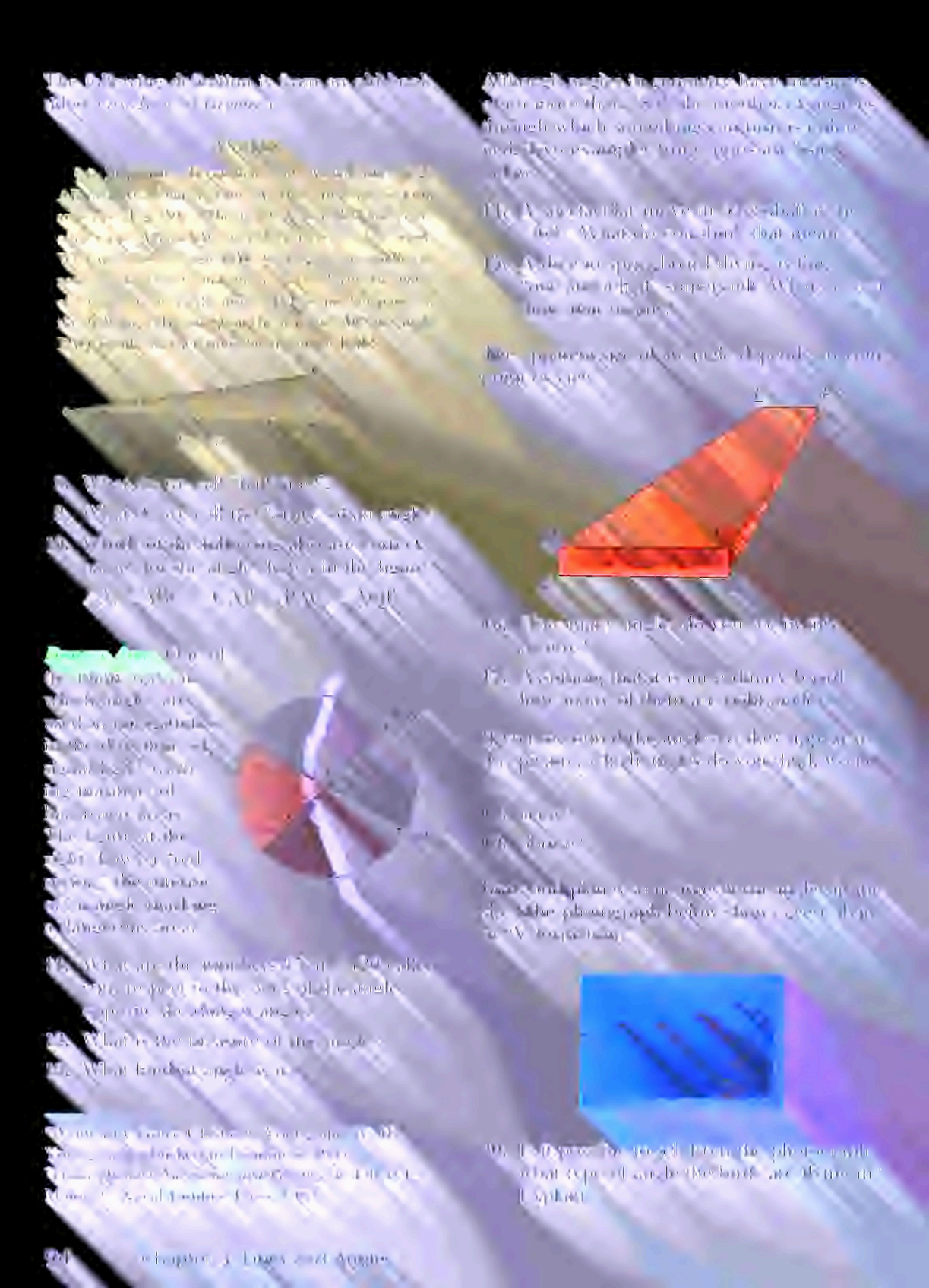

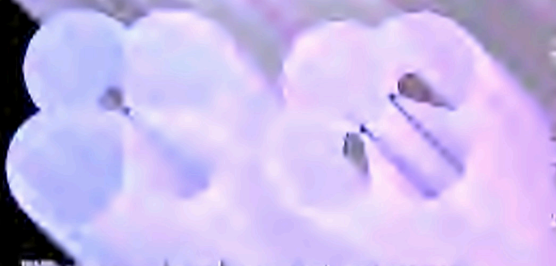



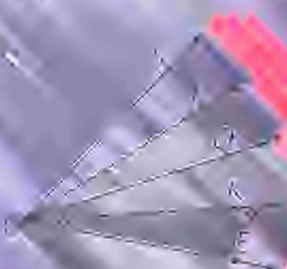













Direction

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Definition

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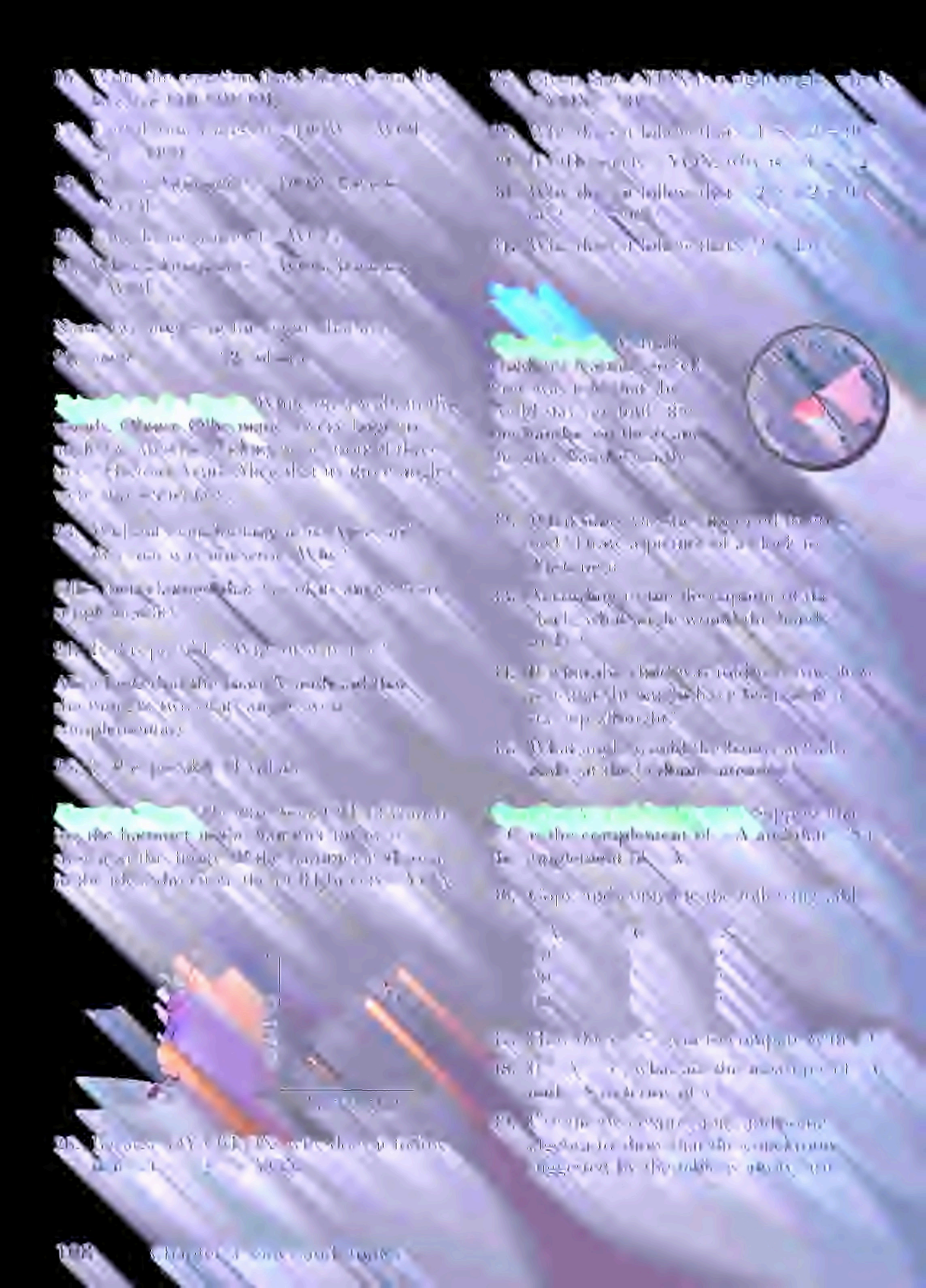
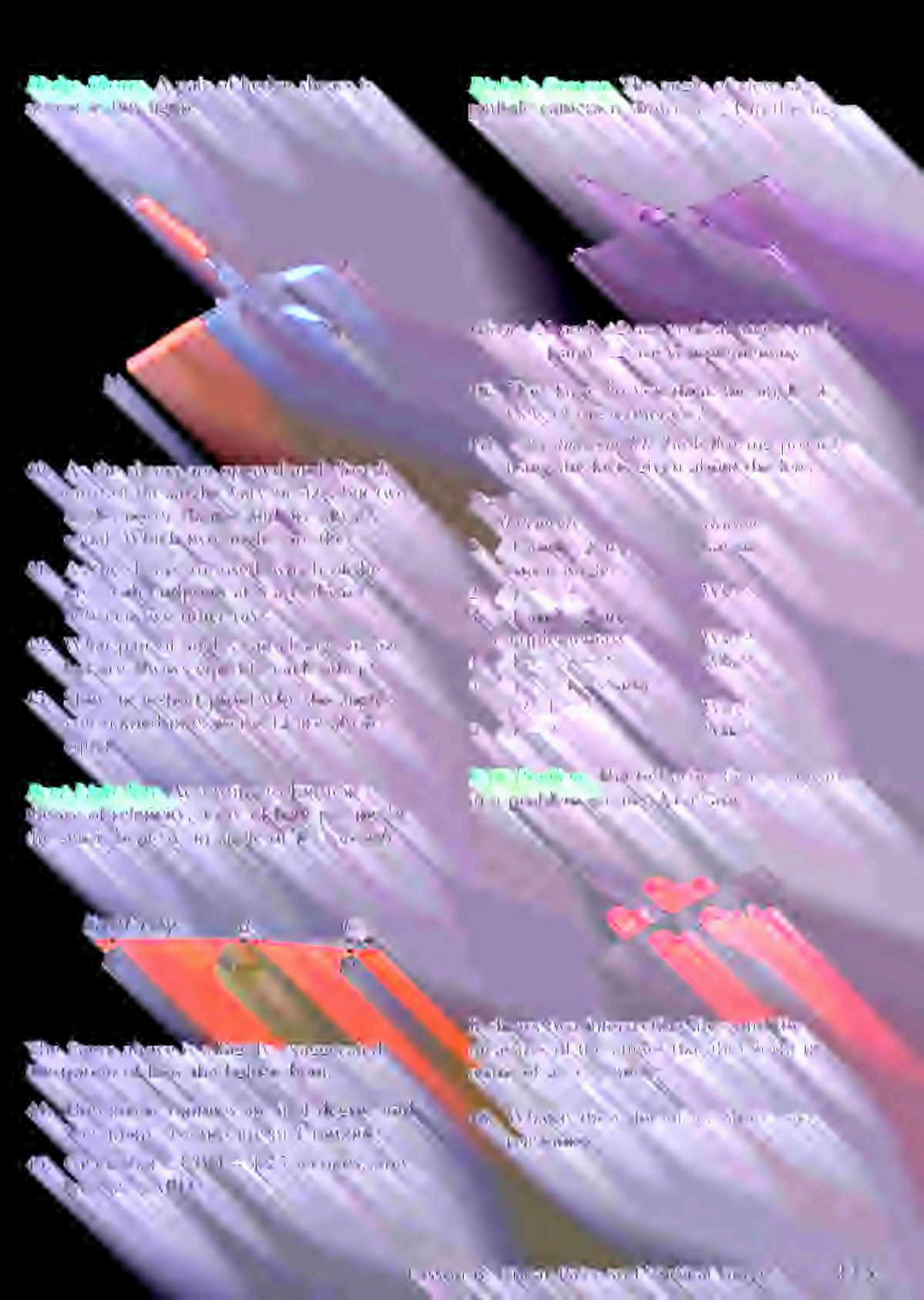
Complementary and Supplementary Angles







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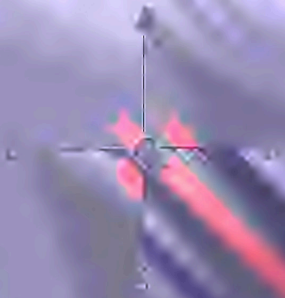







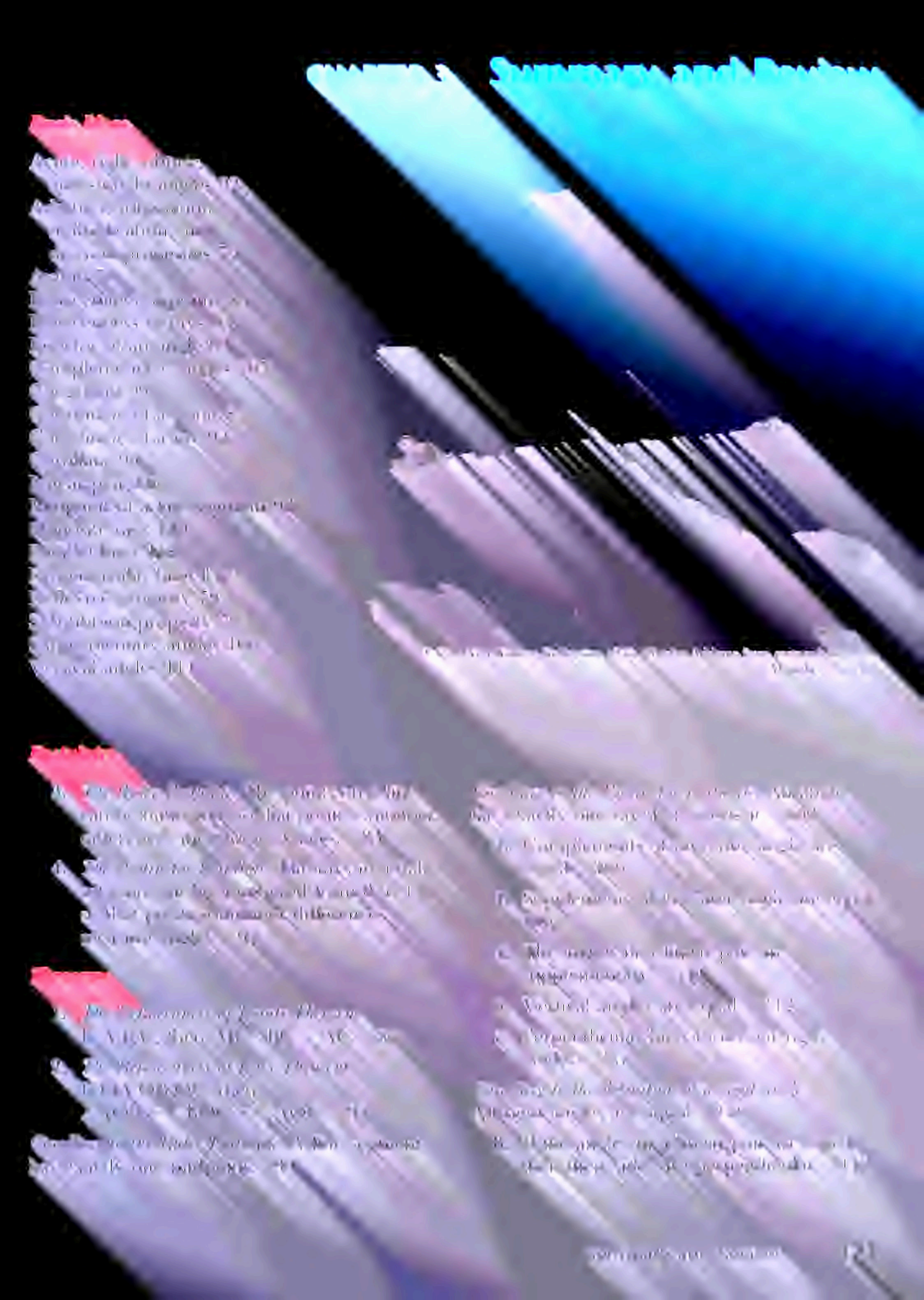





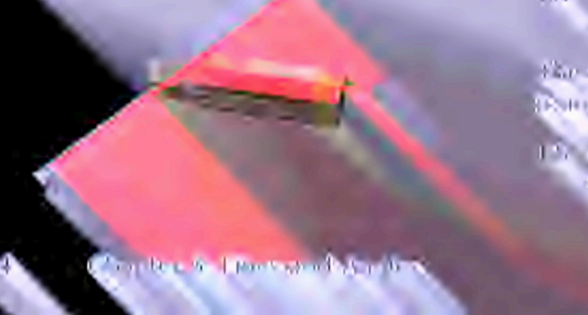

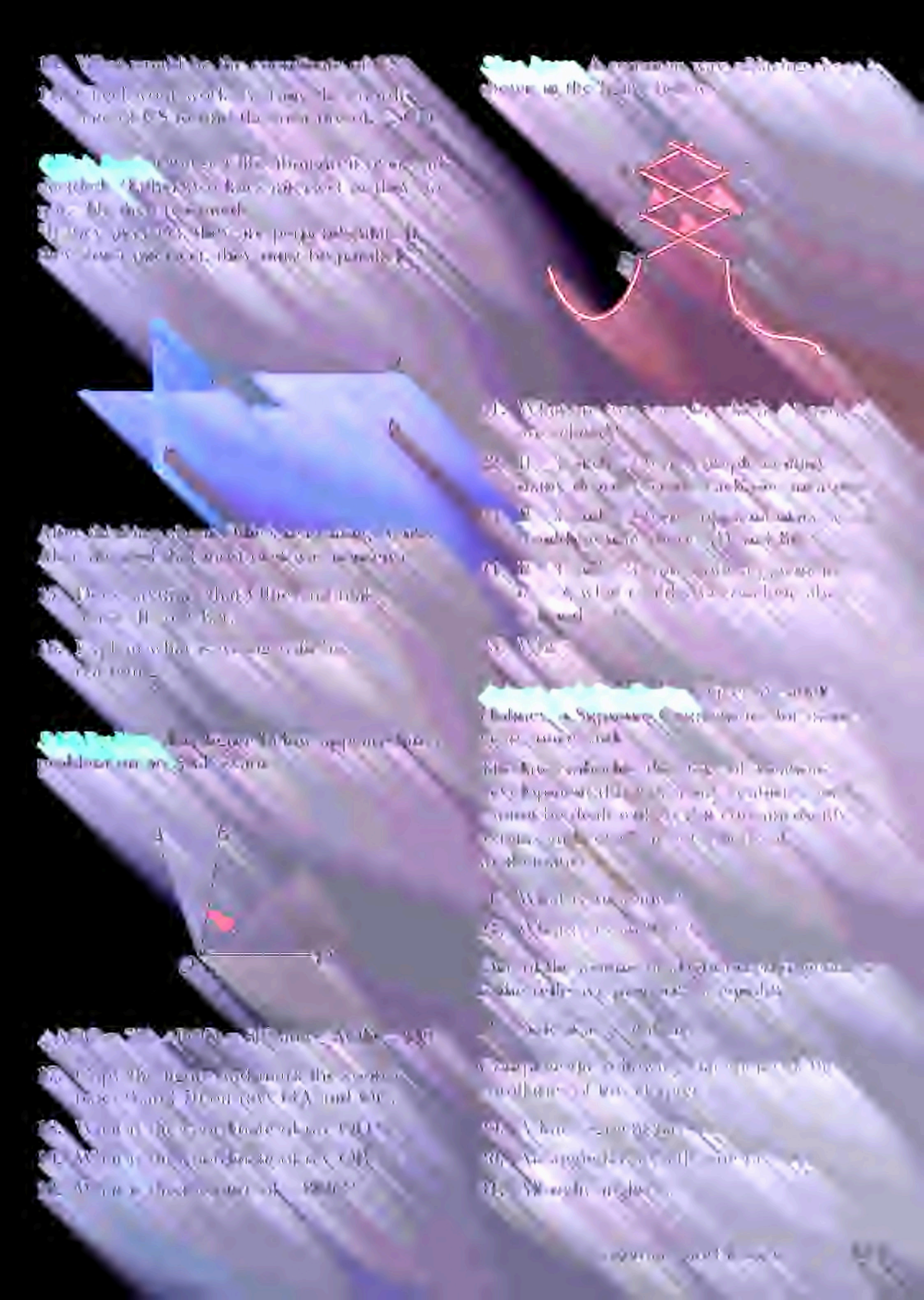





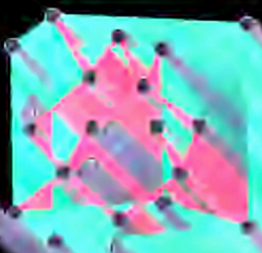

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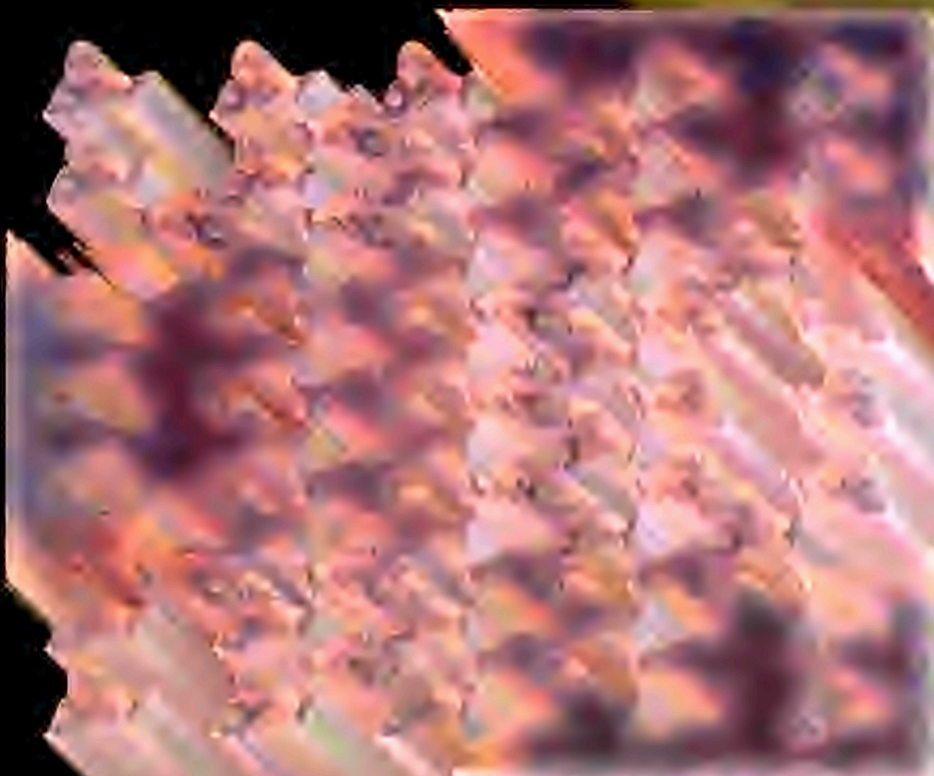
Chapter 4

Congruence



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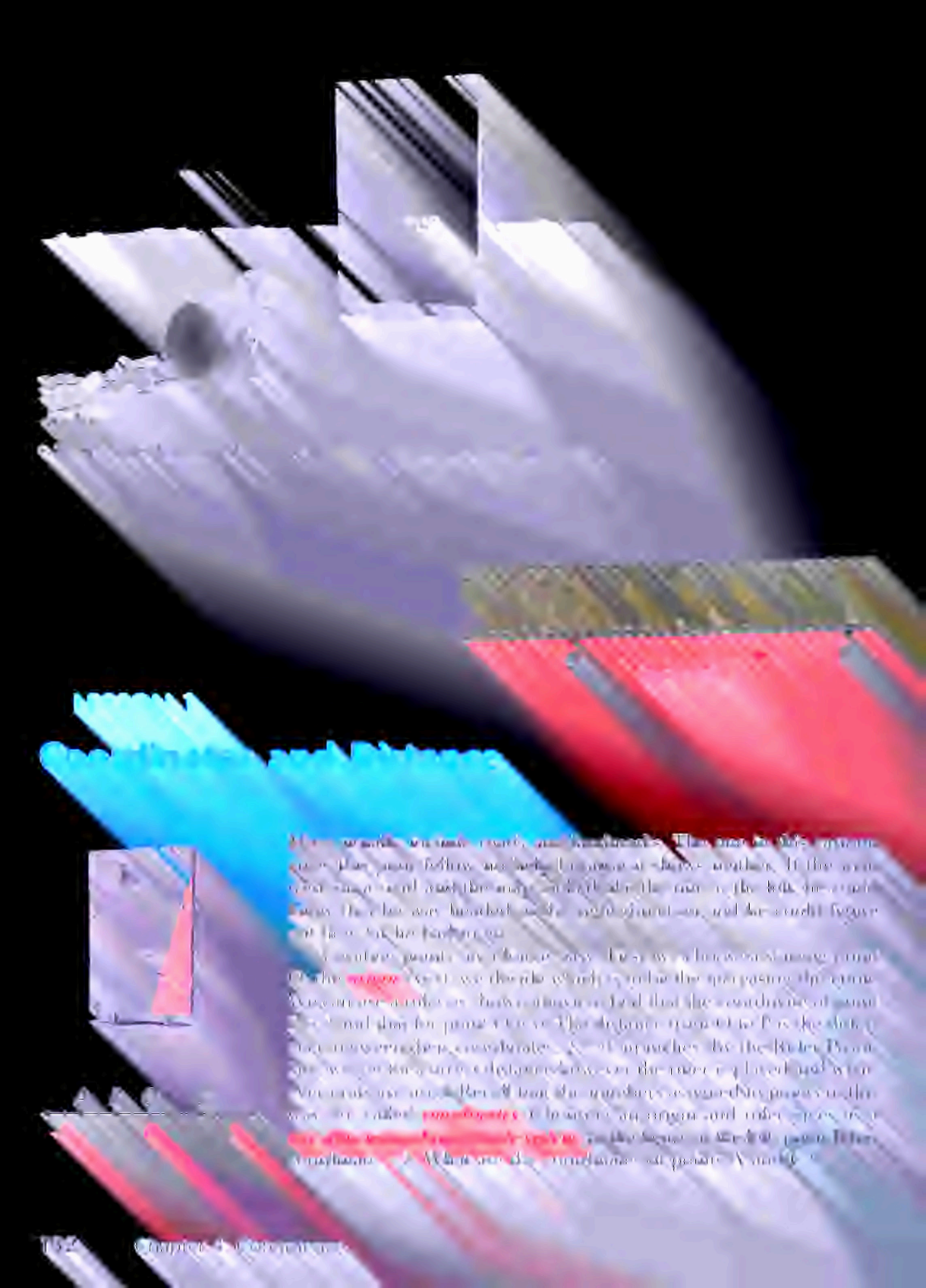

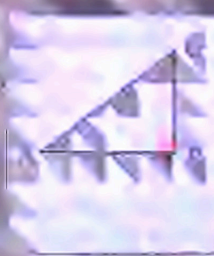











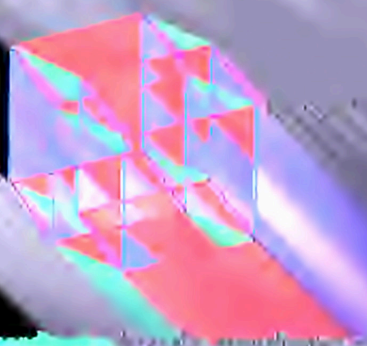

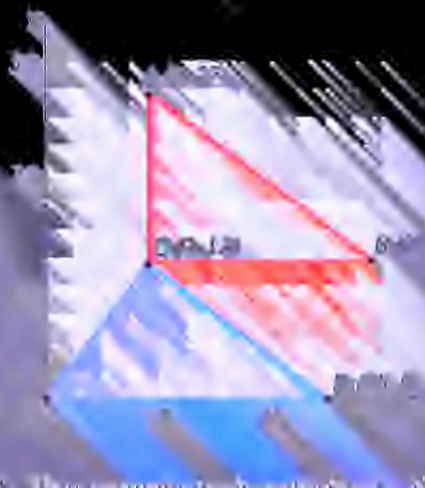



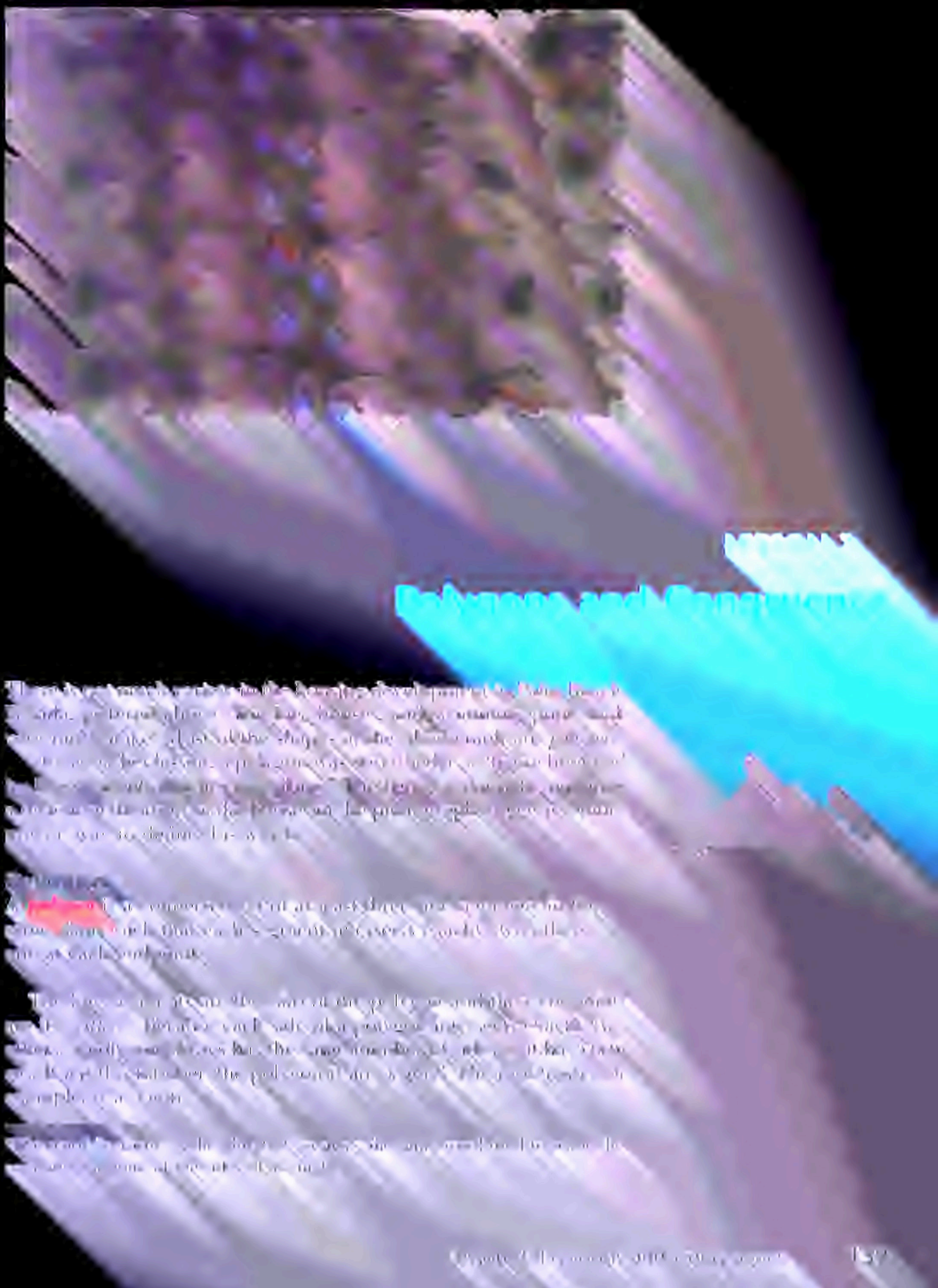

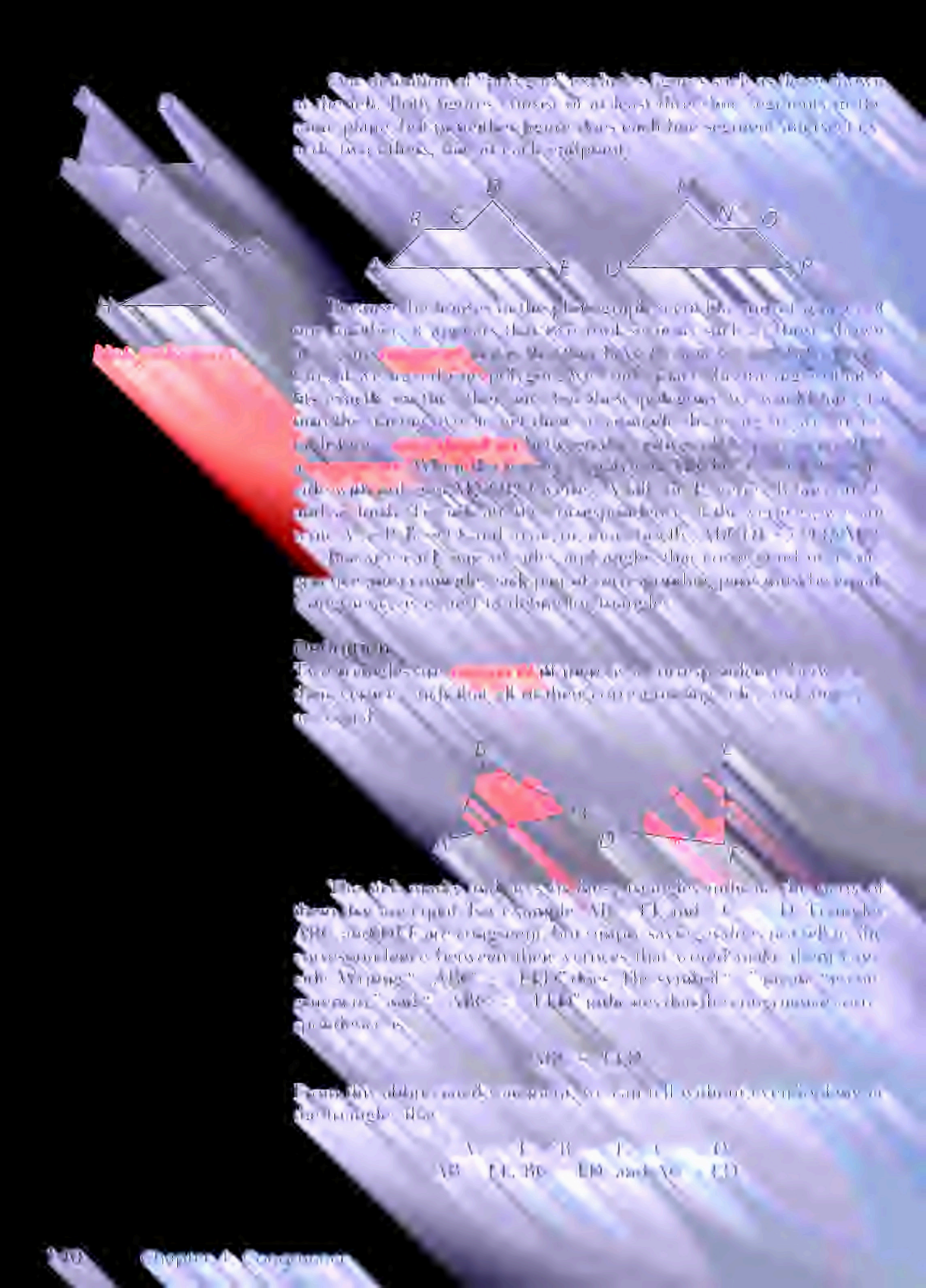



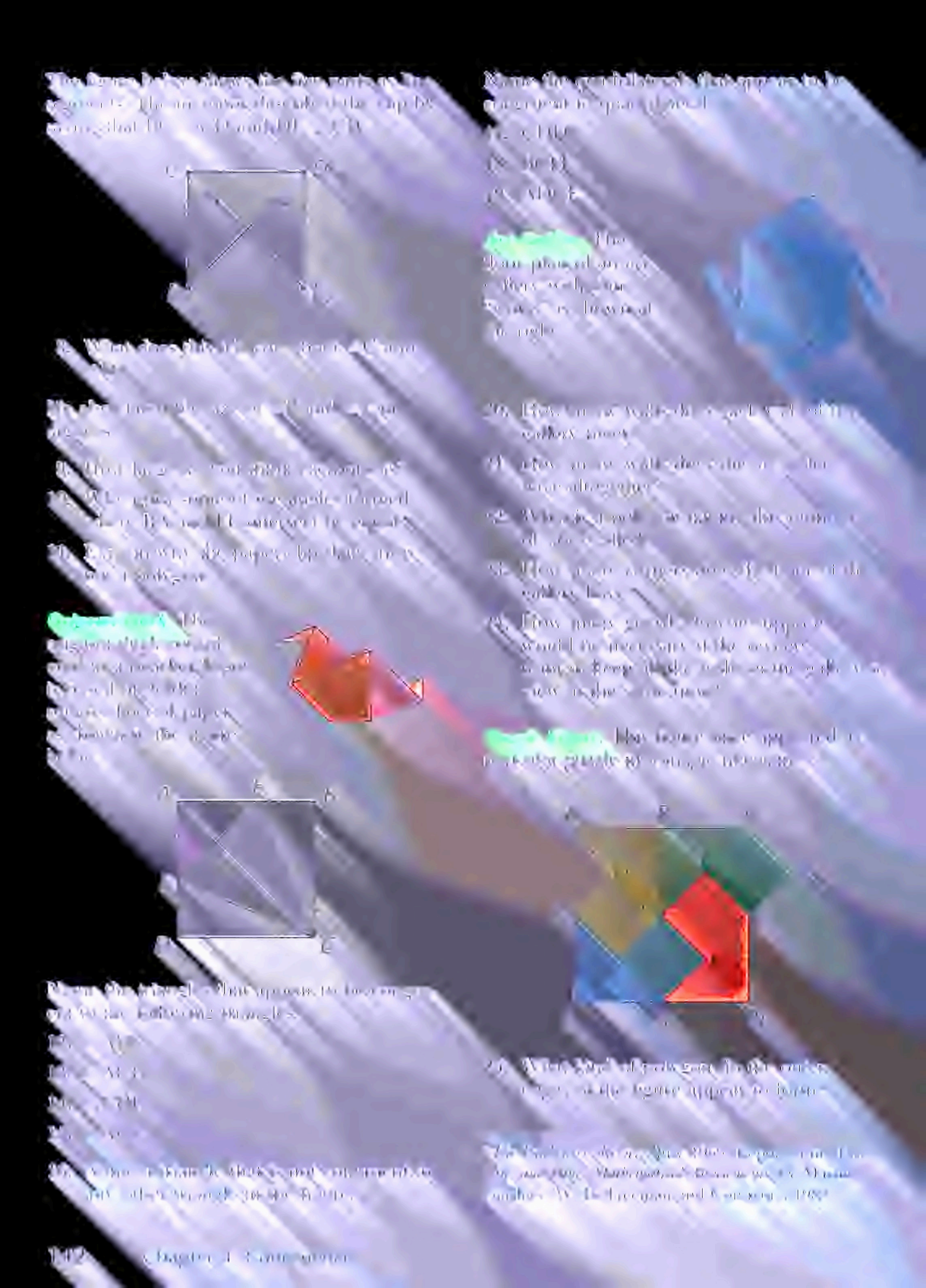

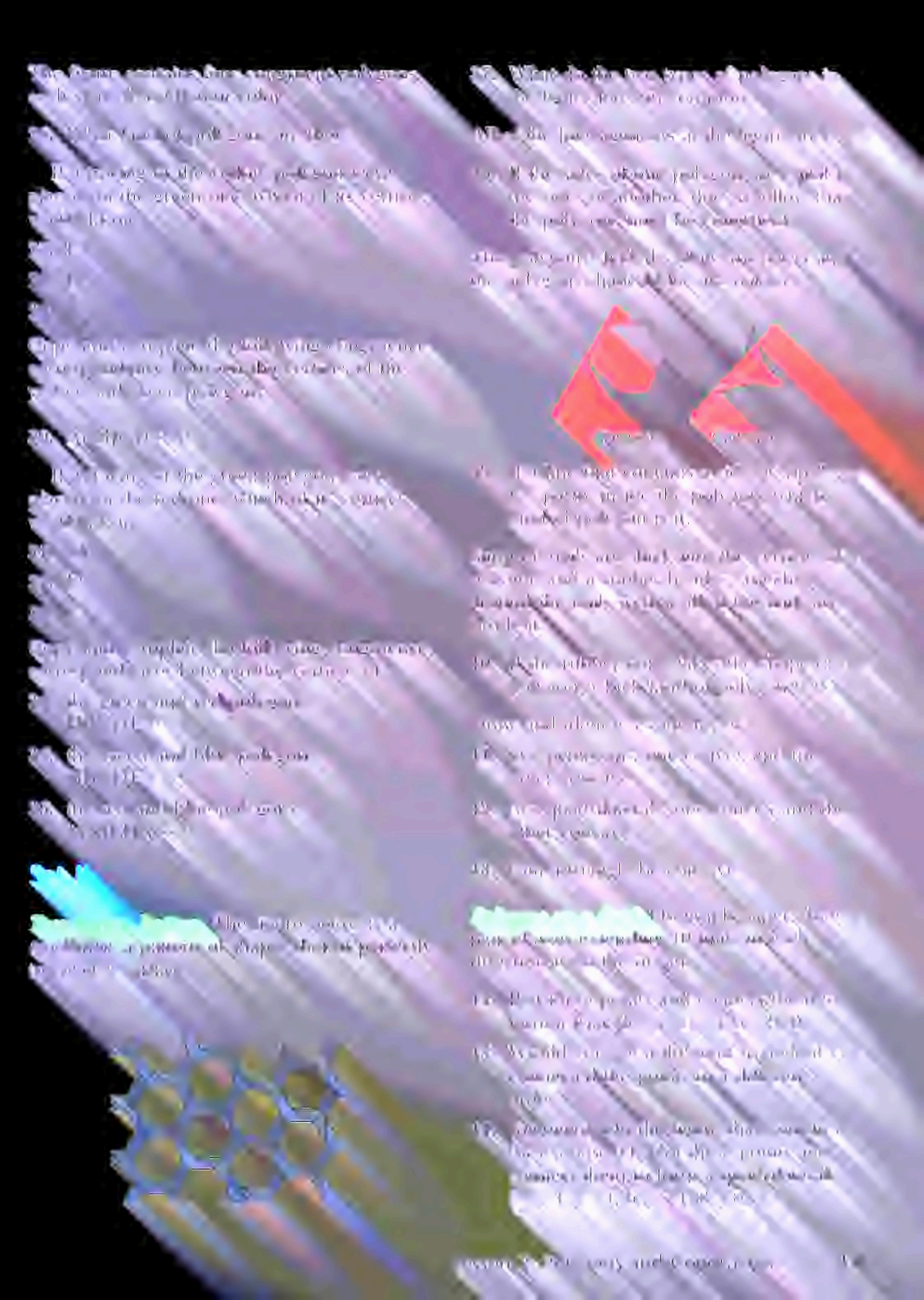

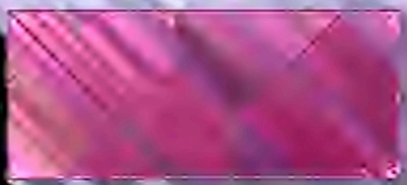







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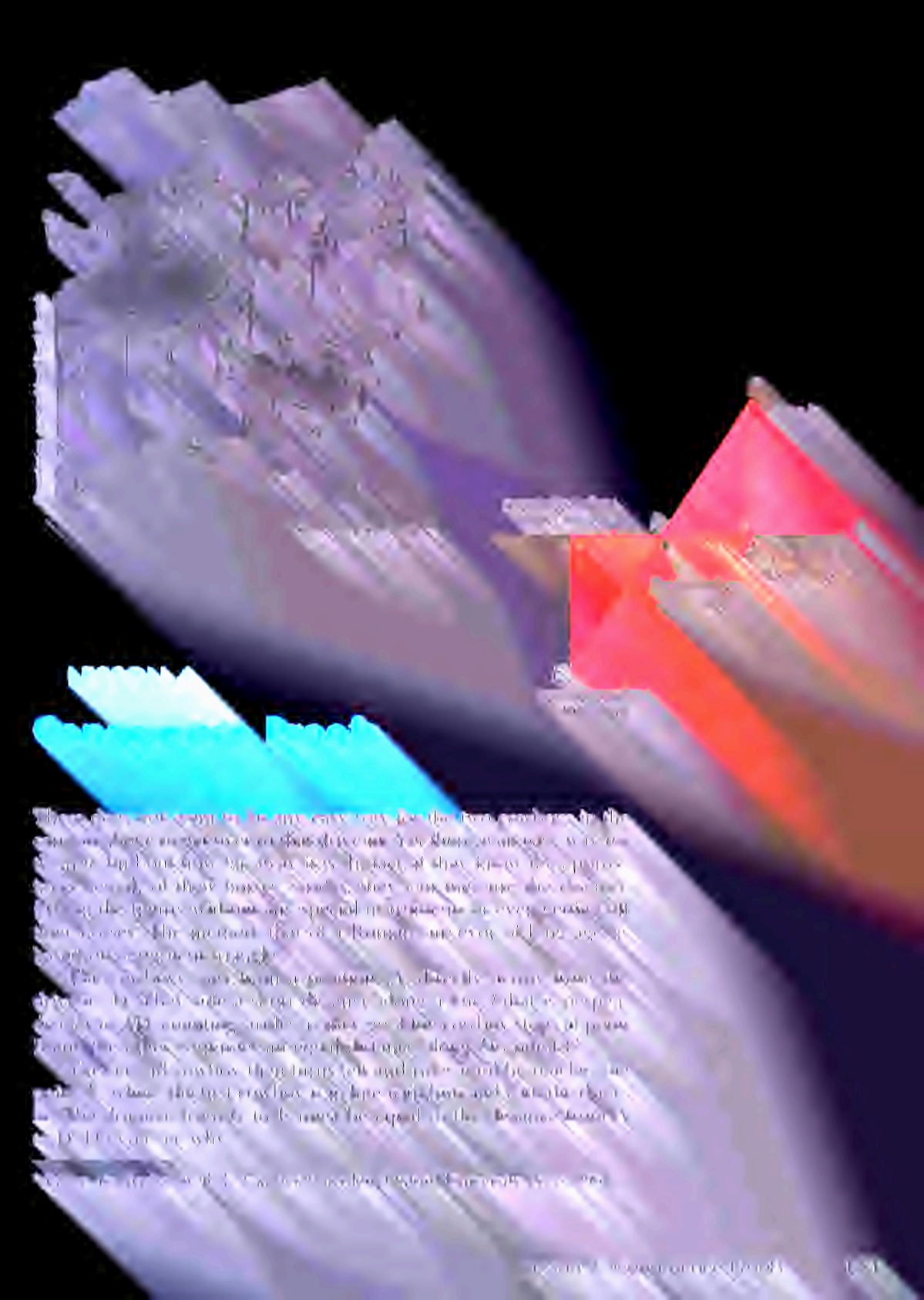

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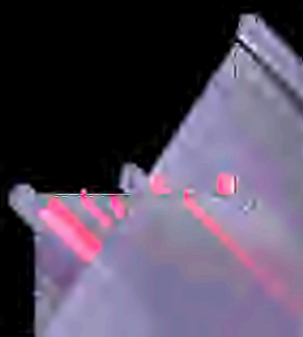

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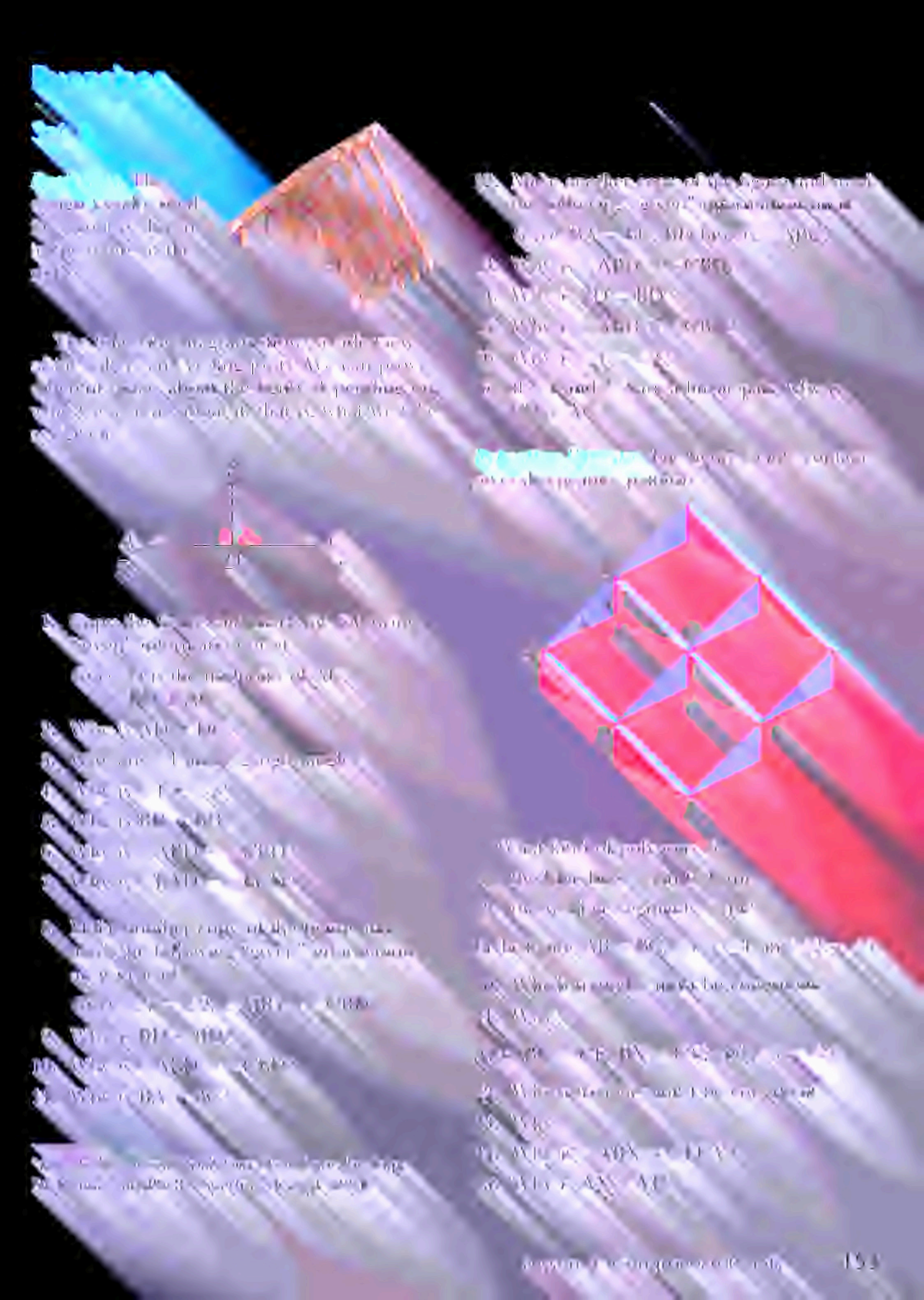
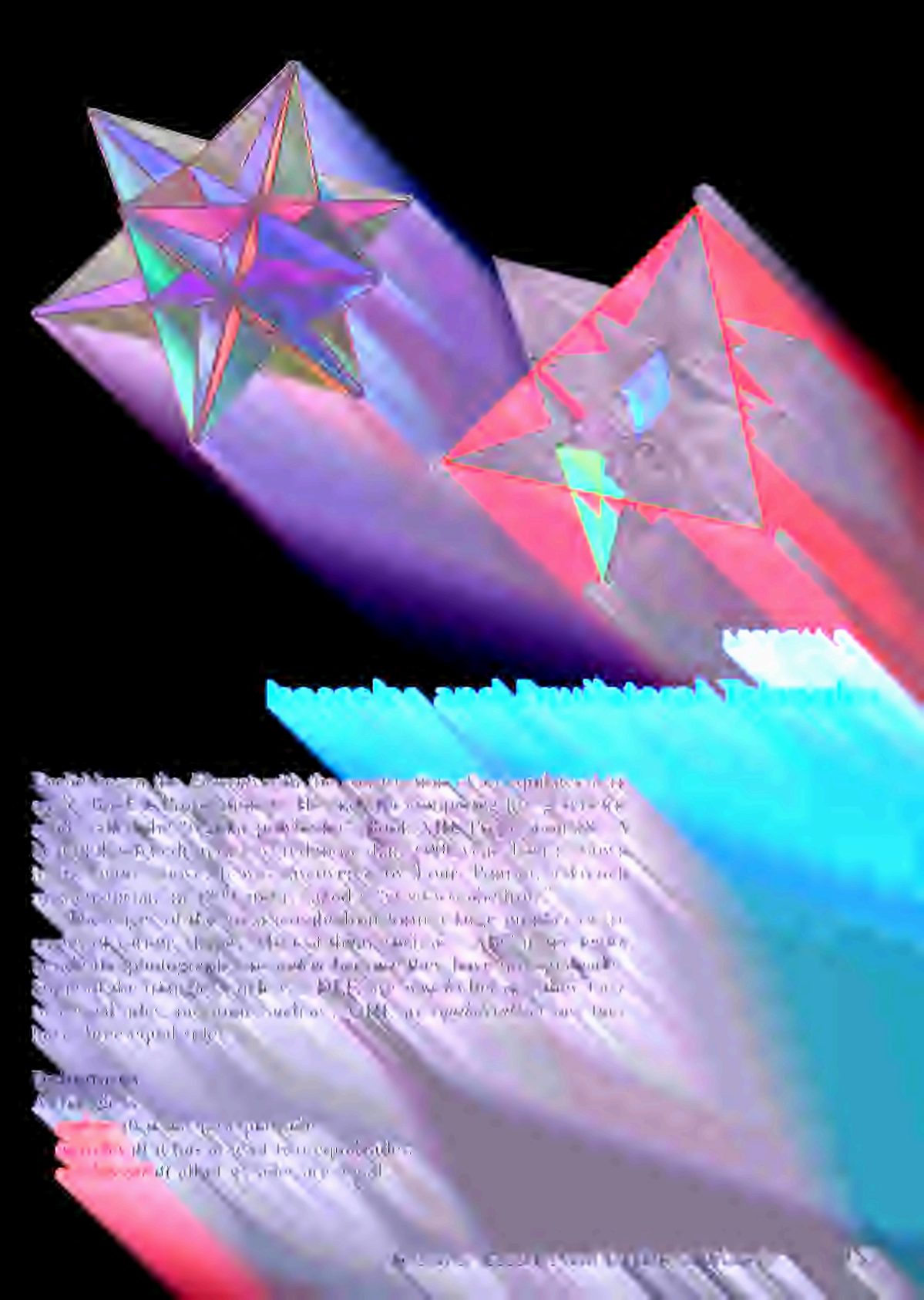
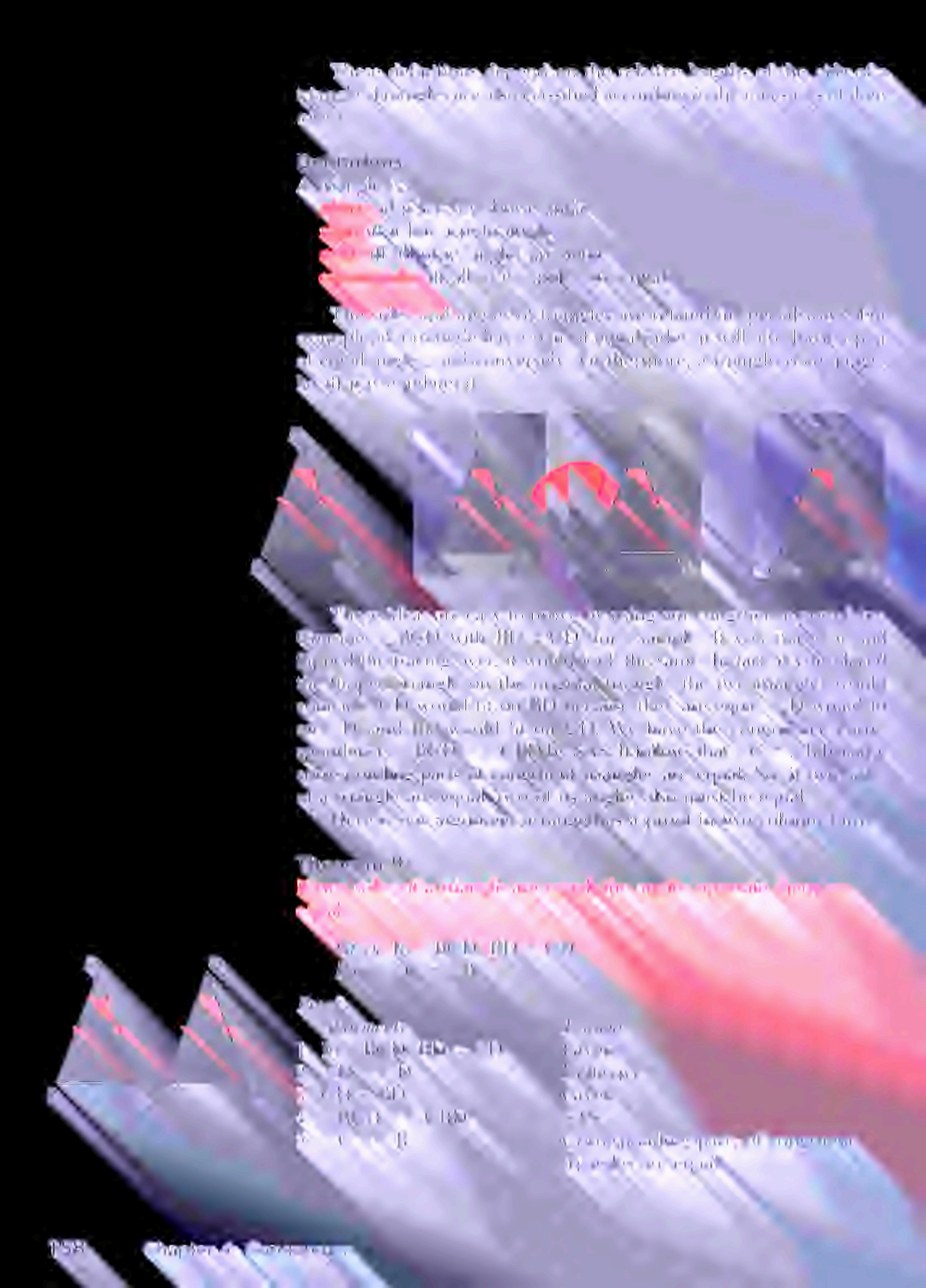


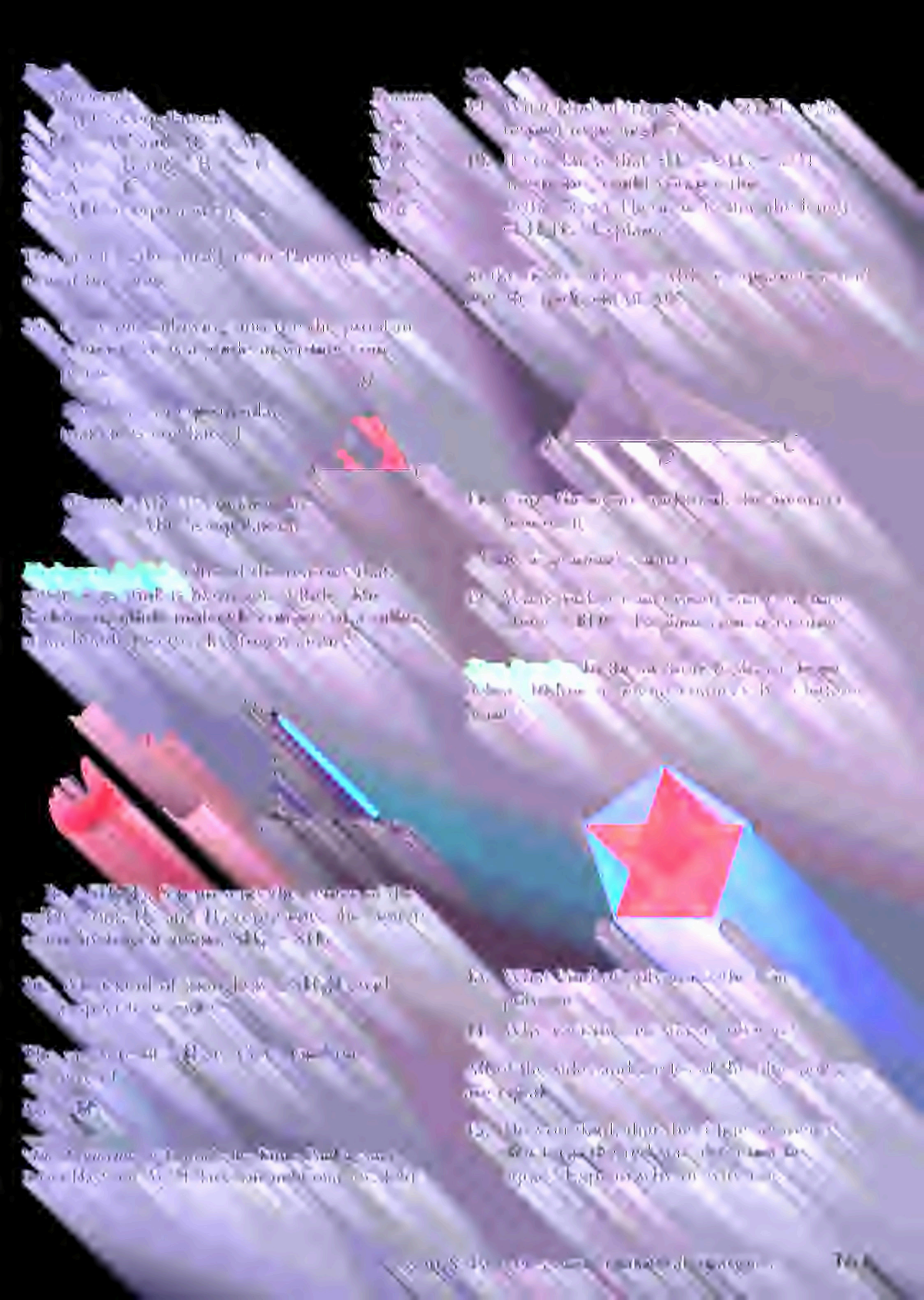


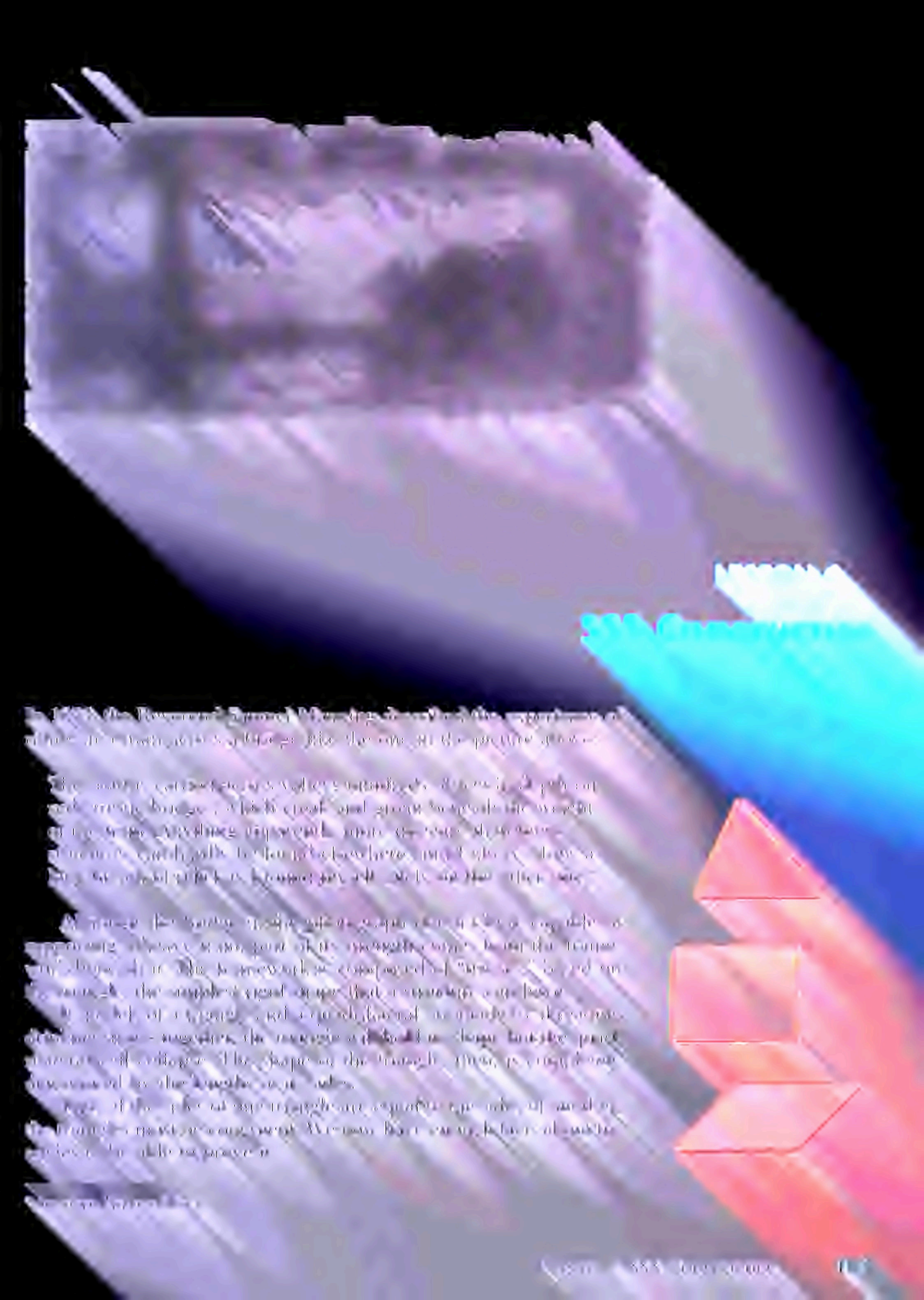








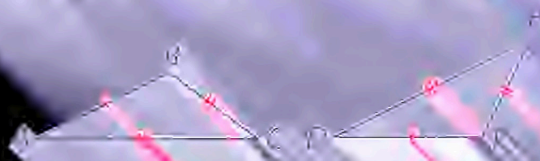




Theorem 7.5: The SAS Theorem

If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent.

We will prove that if two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the other angles of the triangles are also congruent. Then the other corresponding parts will be congruent by CPCTC. Here is the proof.



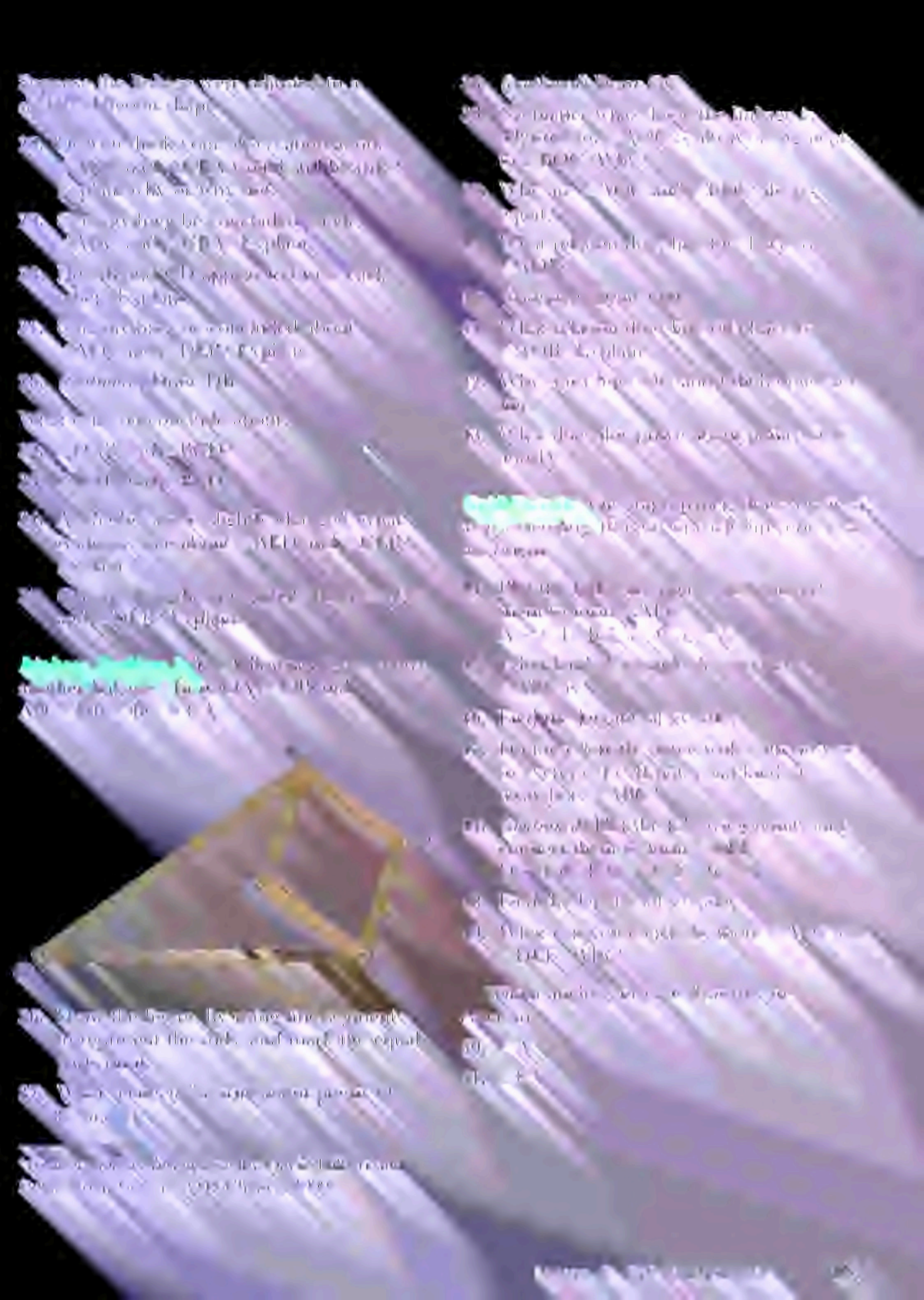
Given: $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $\angle B \cong \angle E$

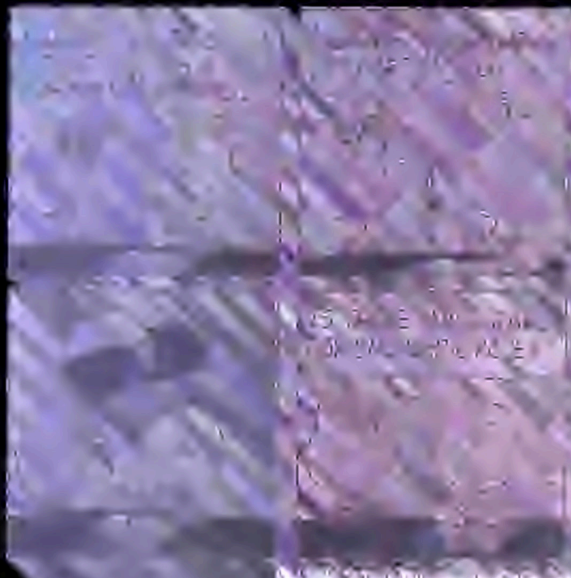
Prove: $\triangle ABC \cong \triangle DEF$

Proof





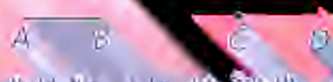




Connections

Question Prop. 2

Problem in the original



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Construction 3

Problem in the original



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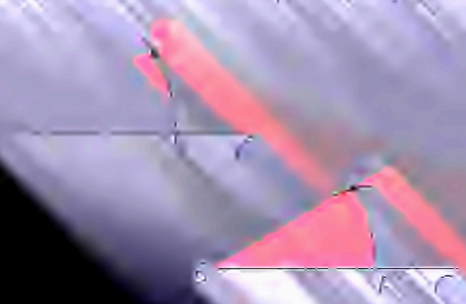

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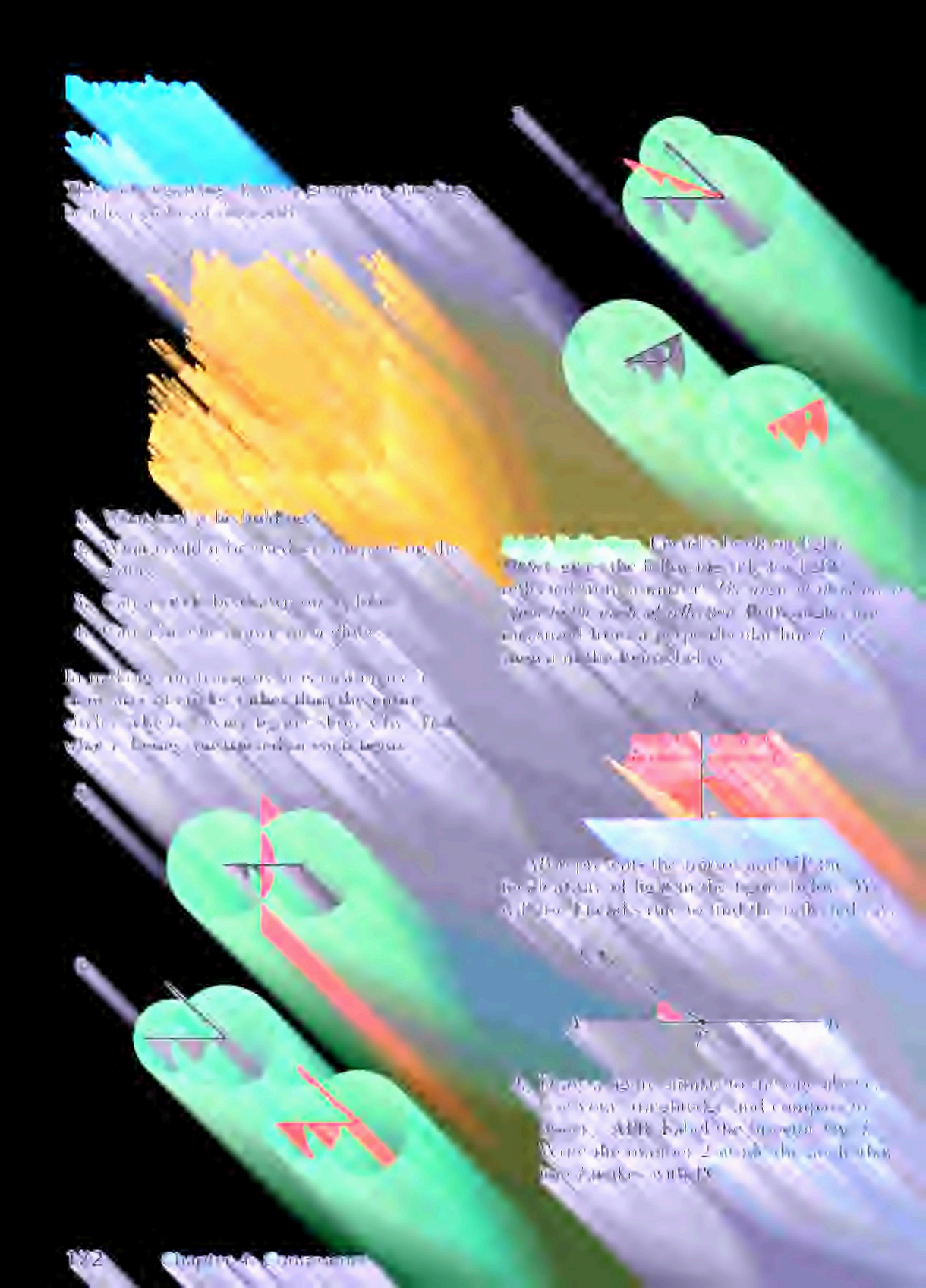

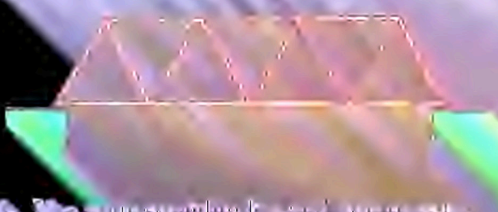





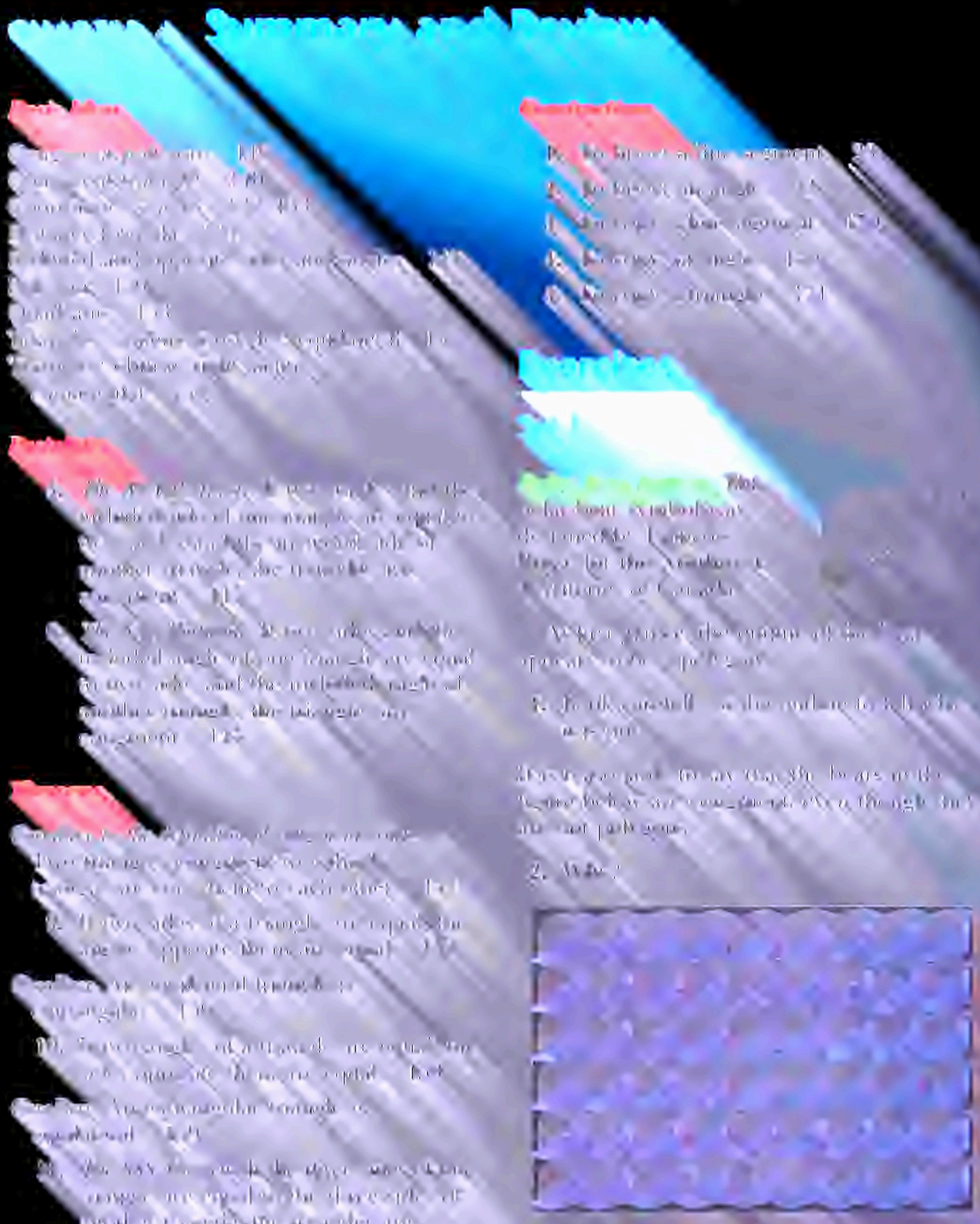



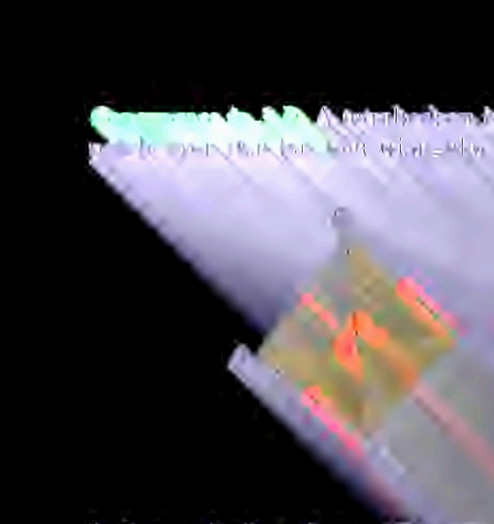

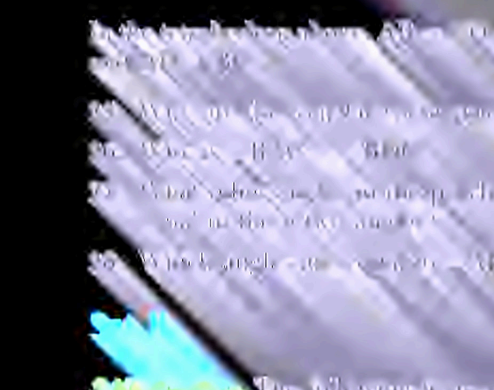

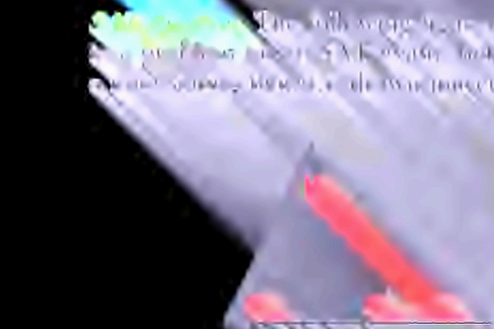

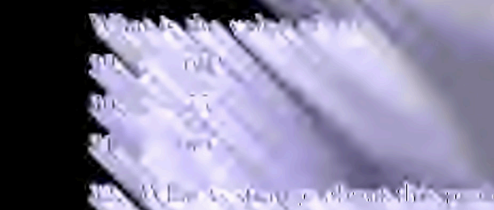

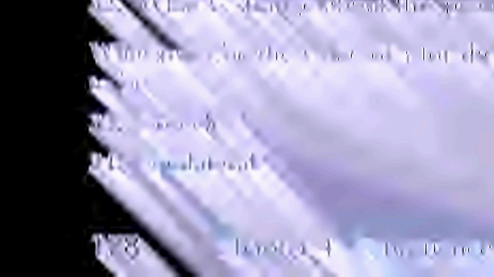

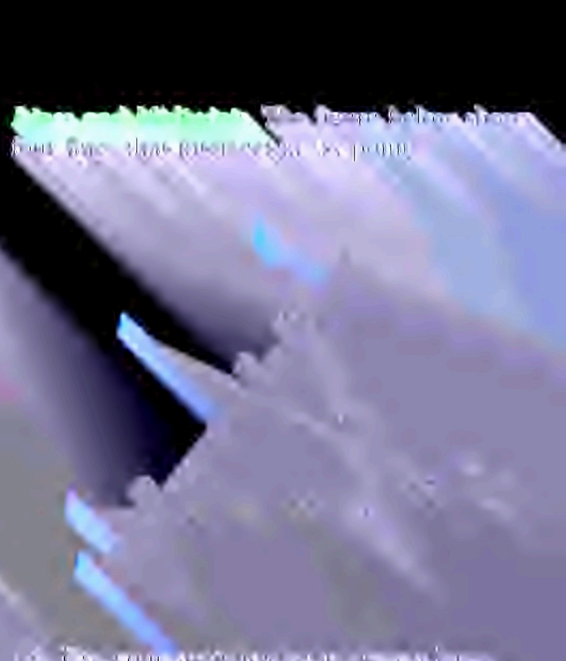

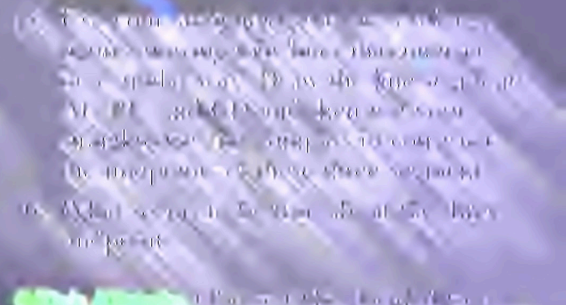

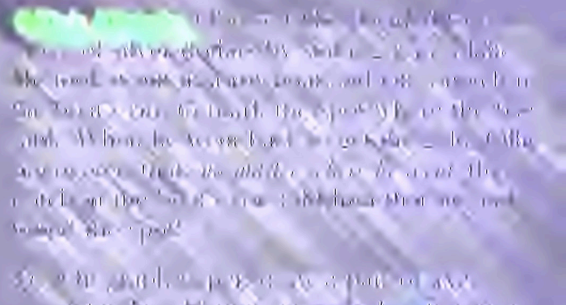

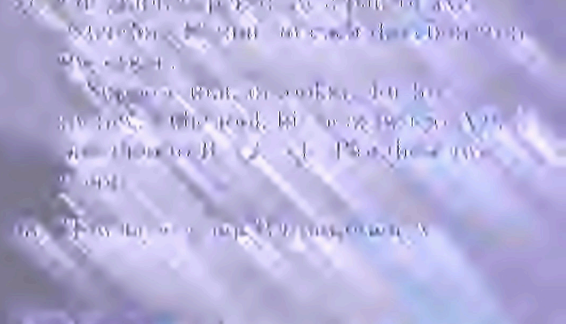



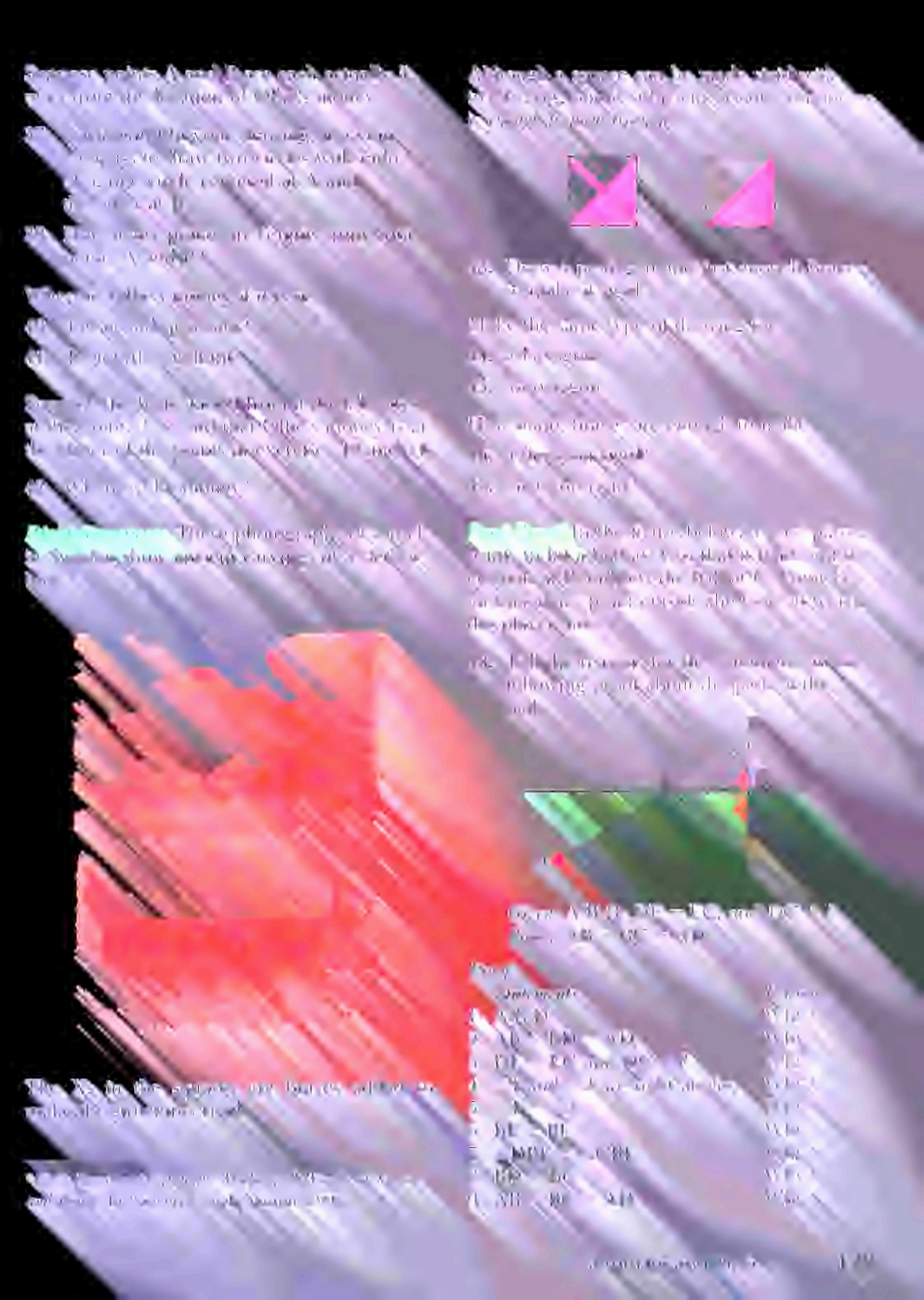



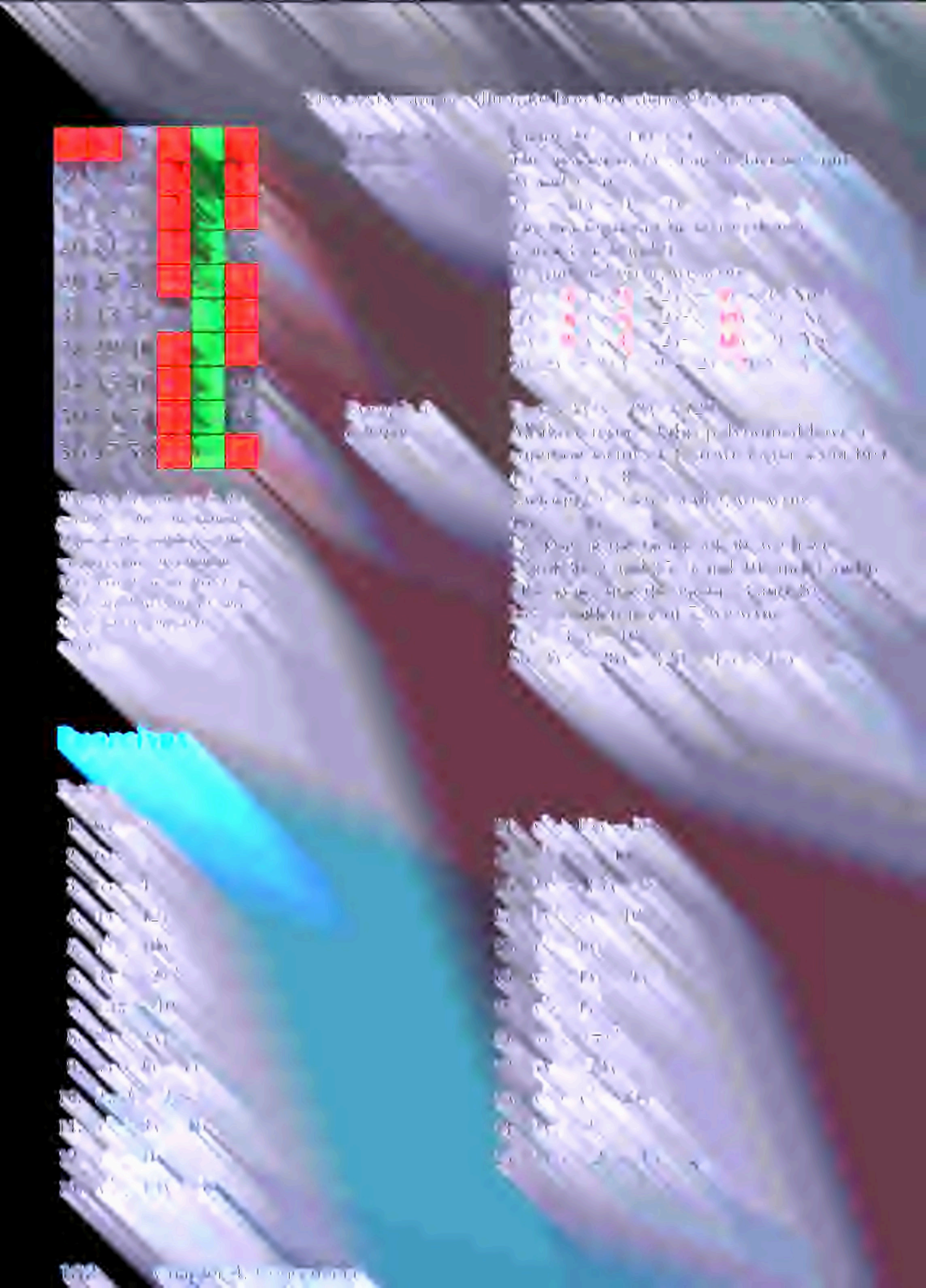

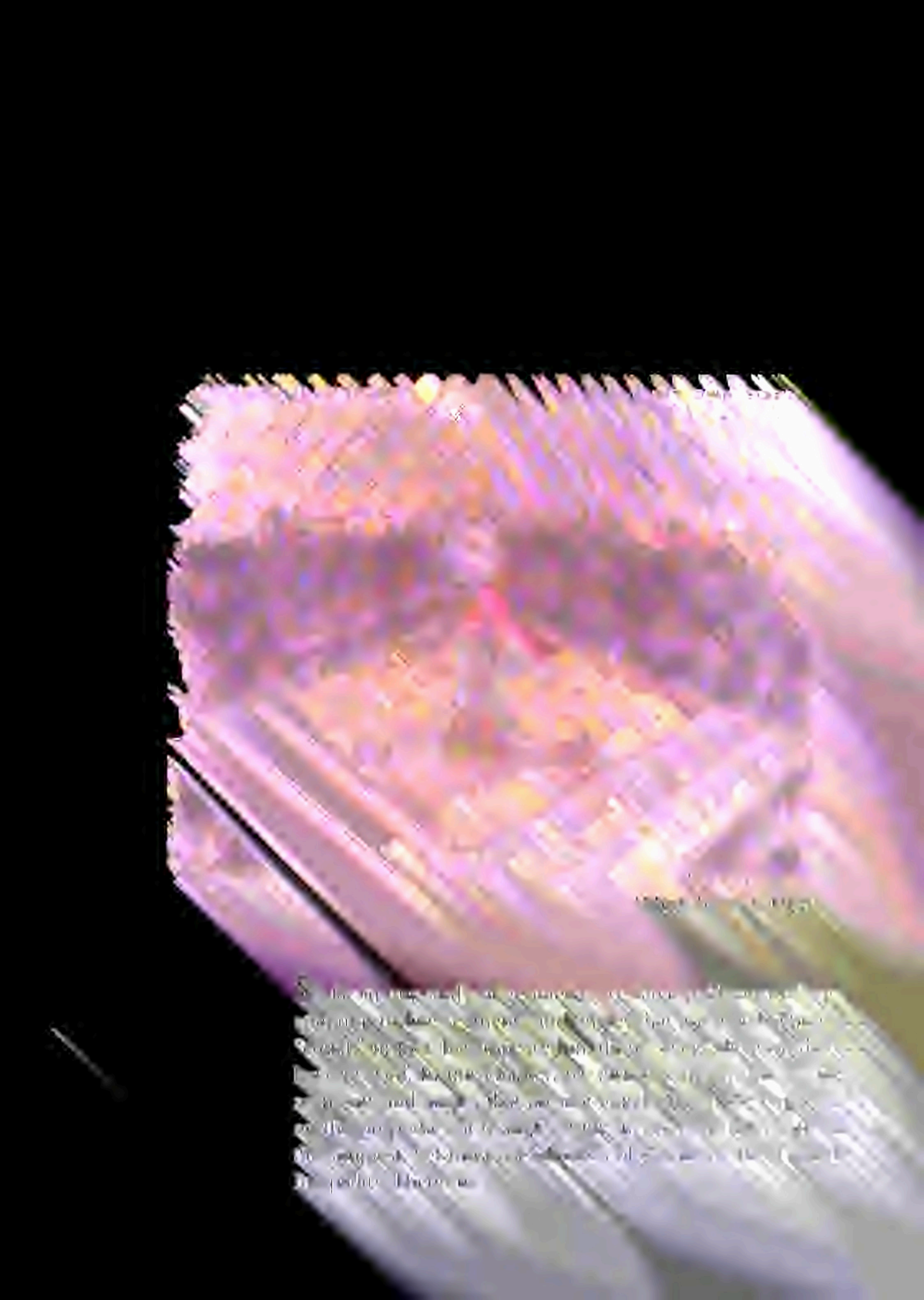

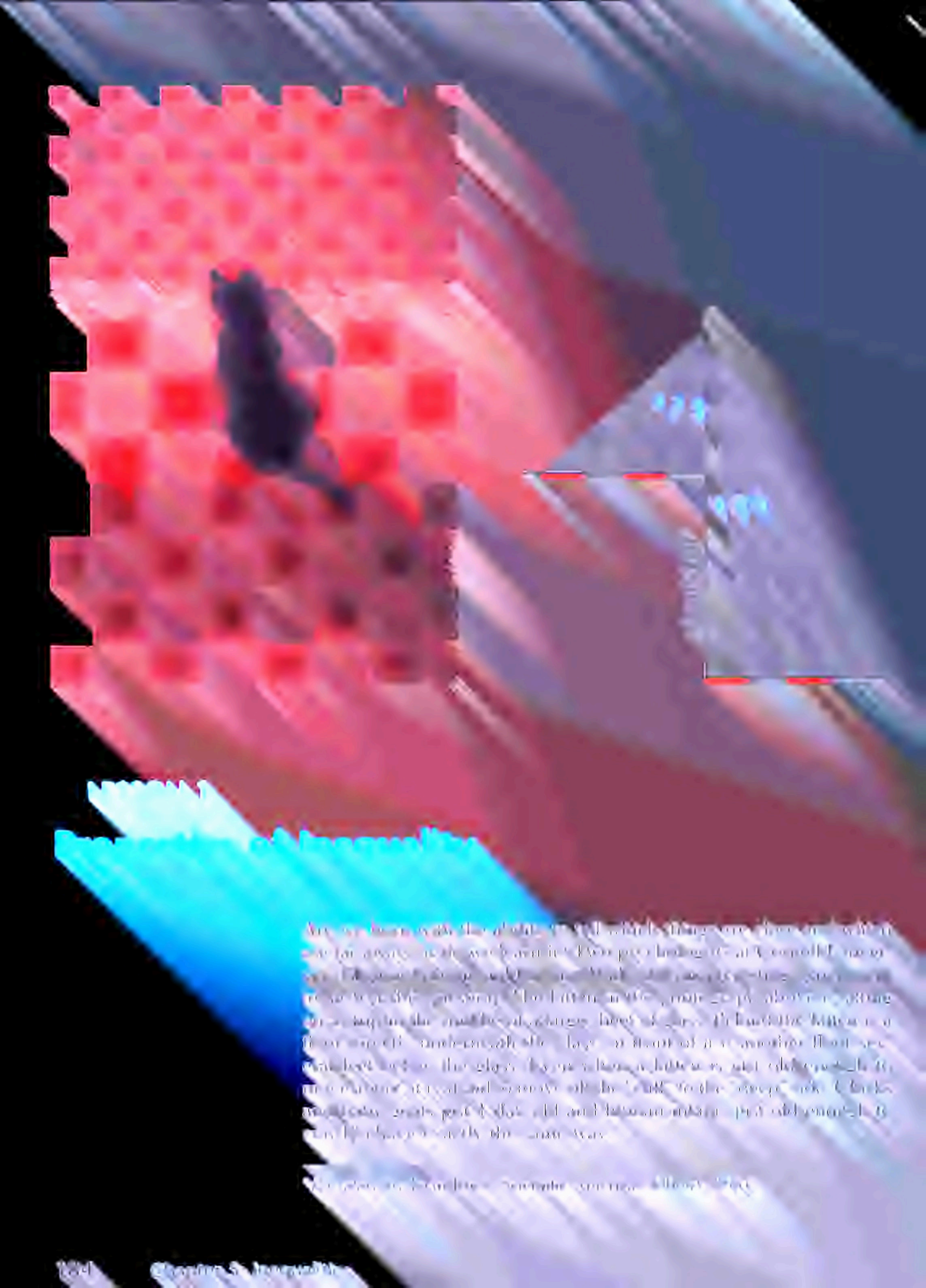

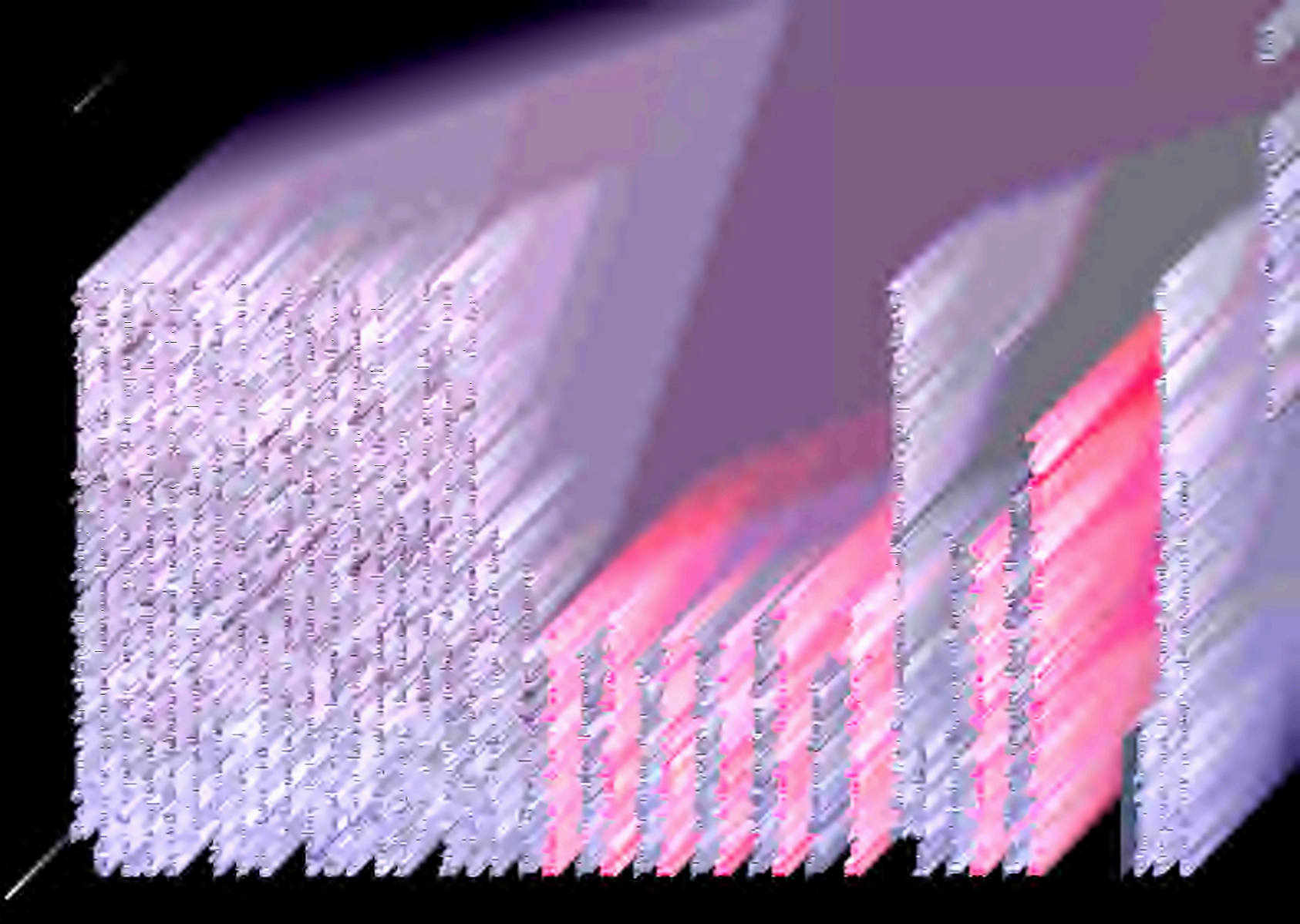

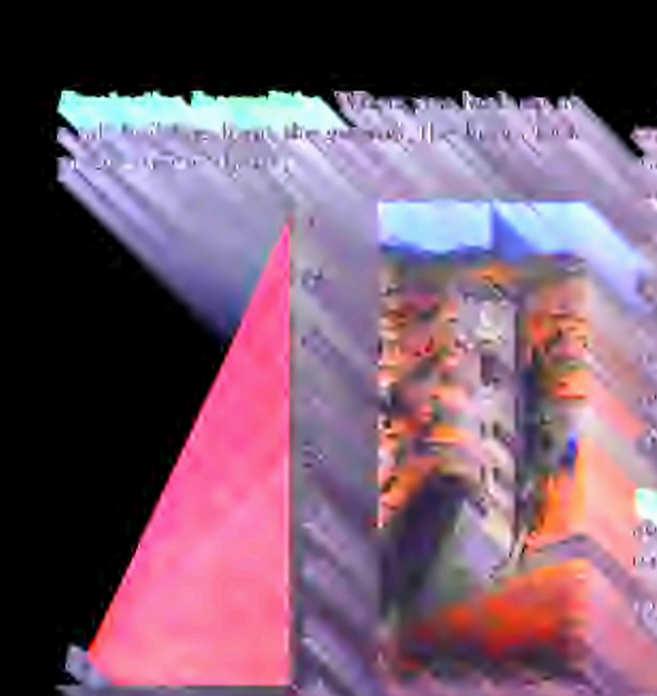



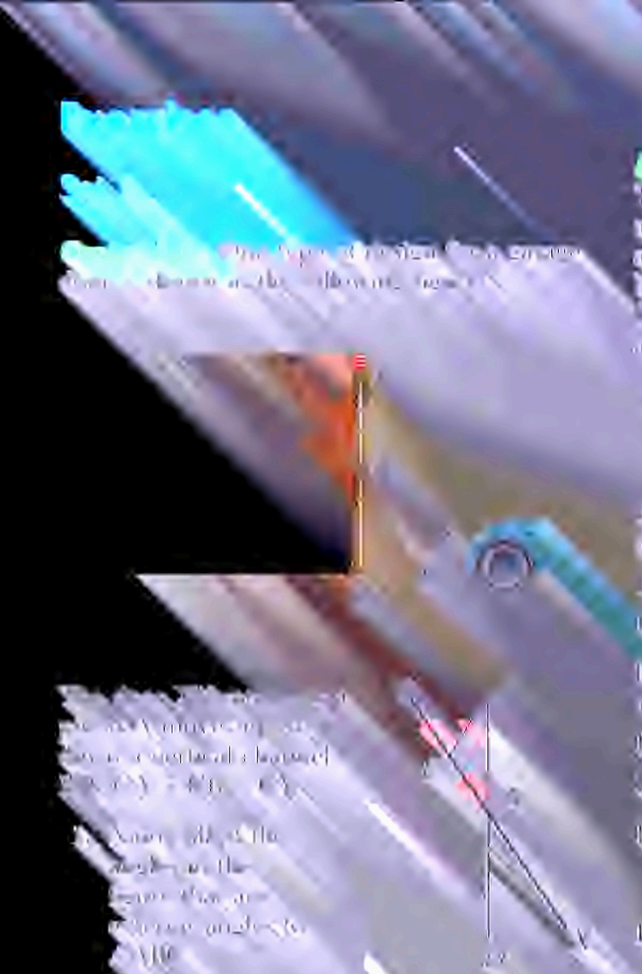





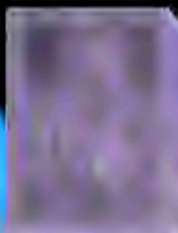



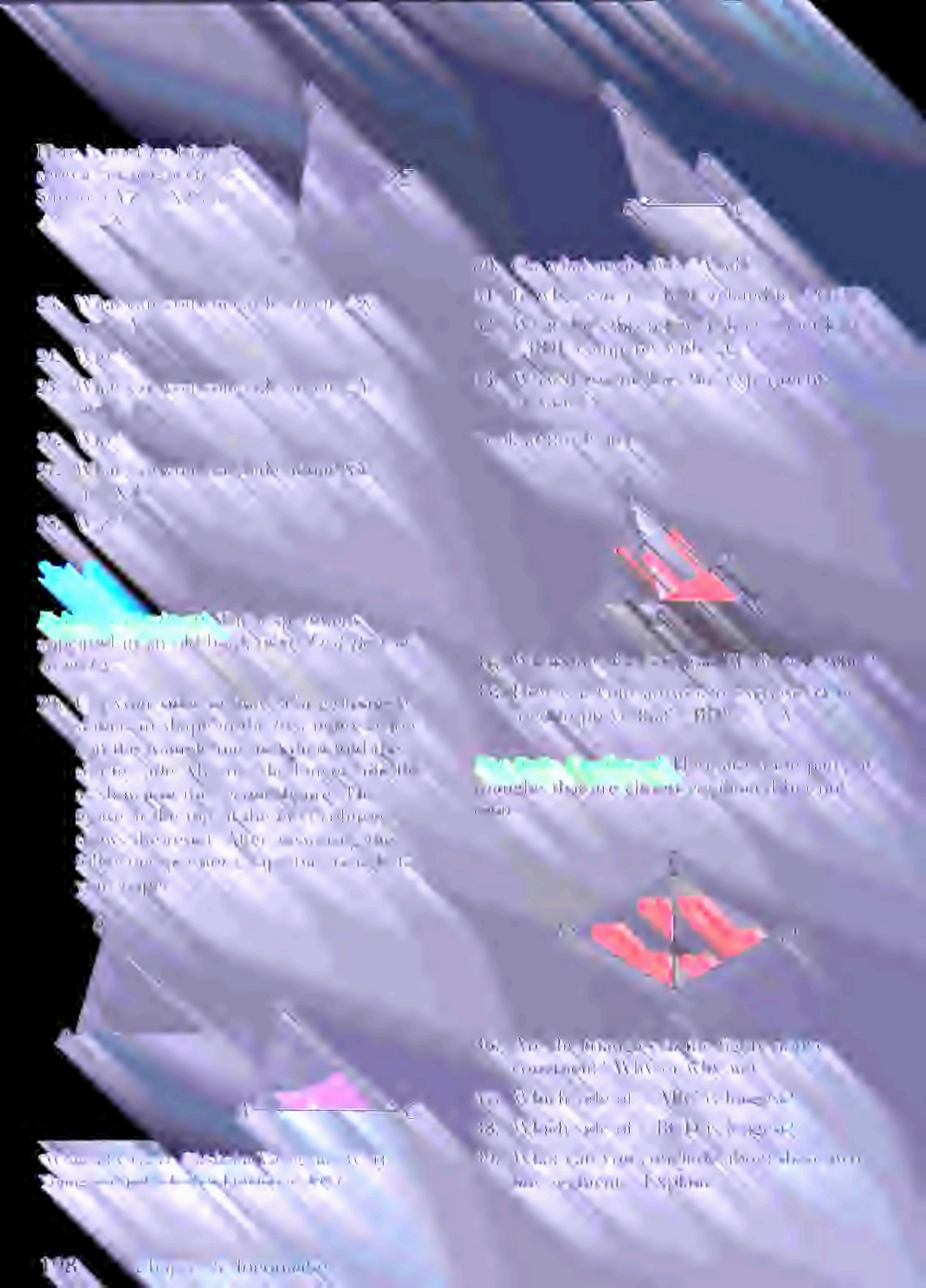







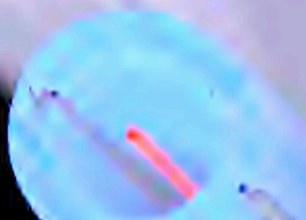

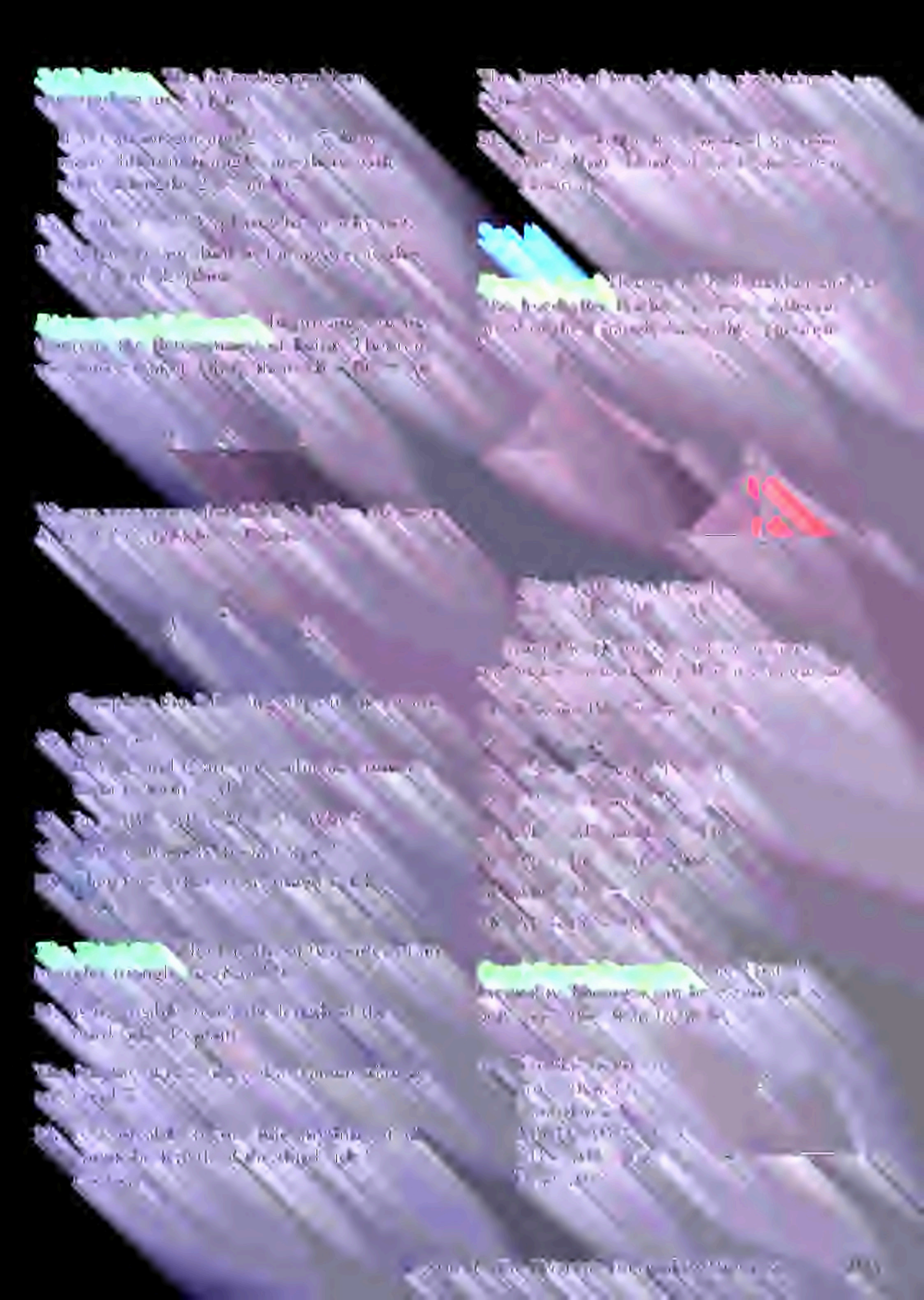

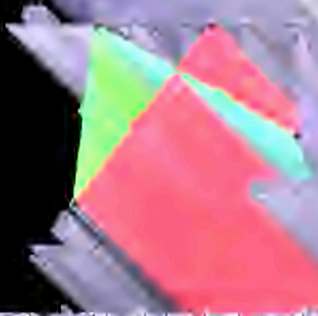

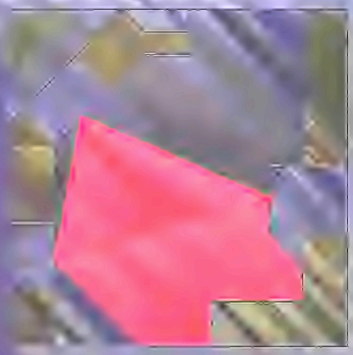



Summary and Review

Key Ideas

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Exercises

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Reasoning

Sets



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Where Is It?

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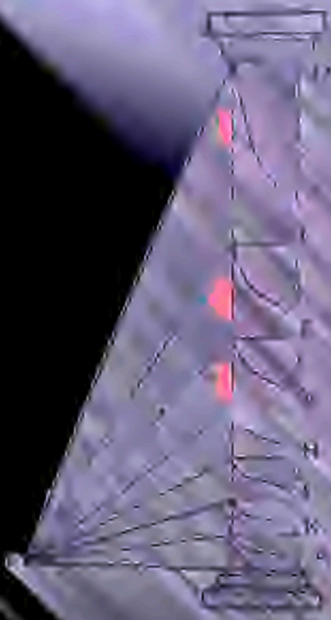
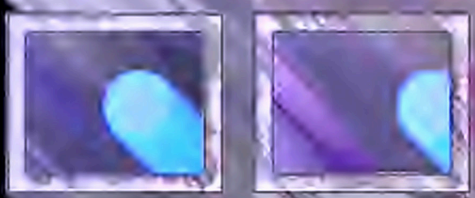
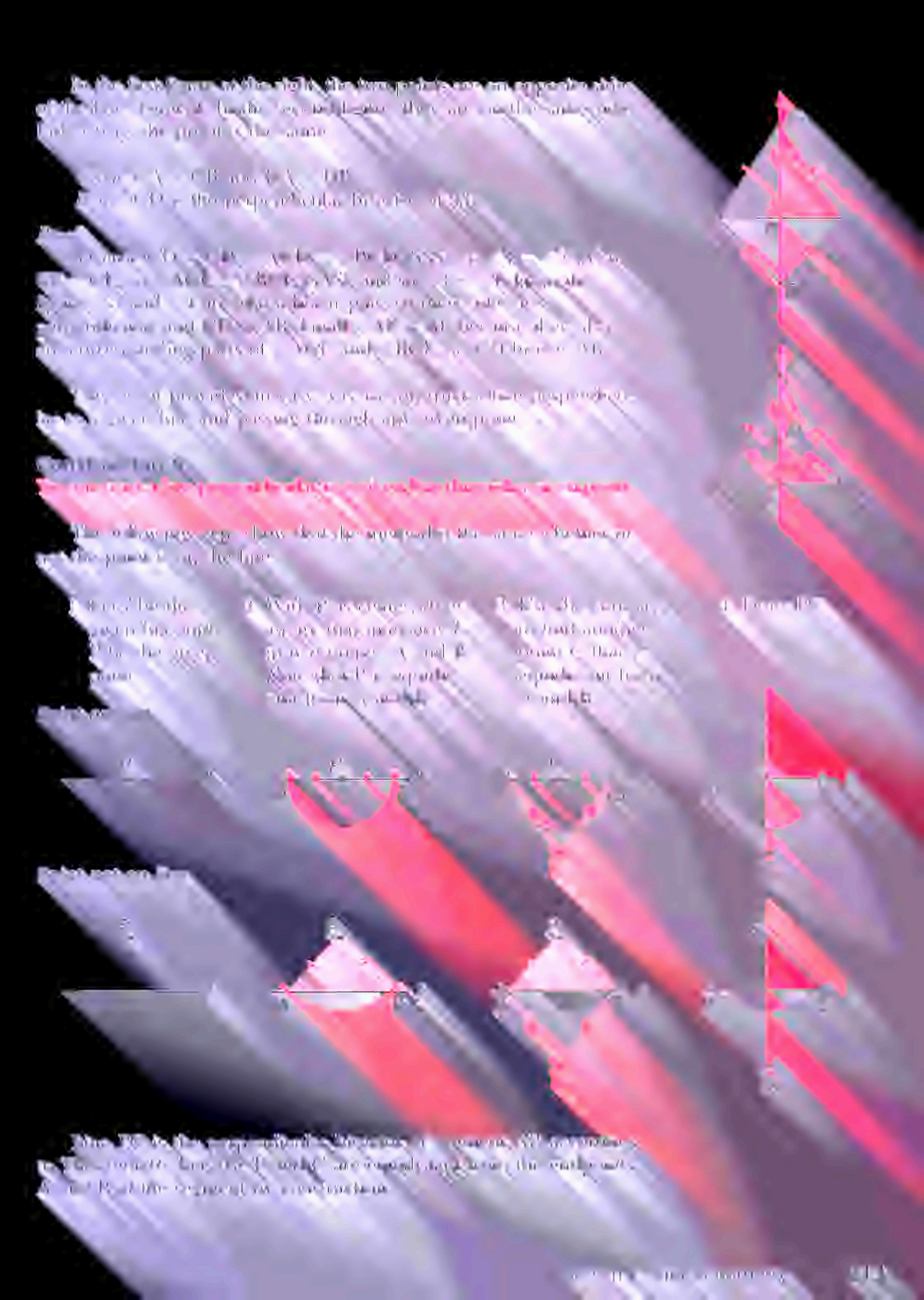


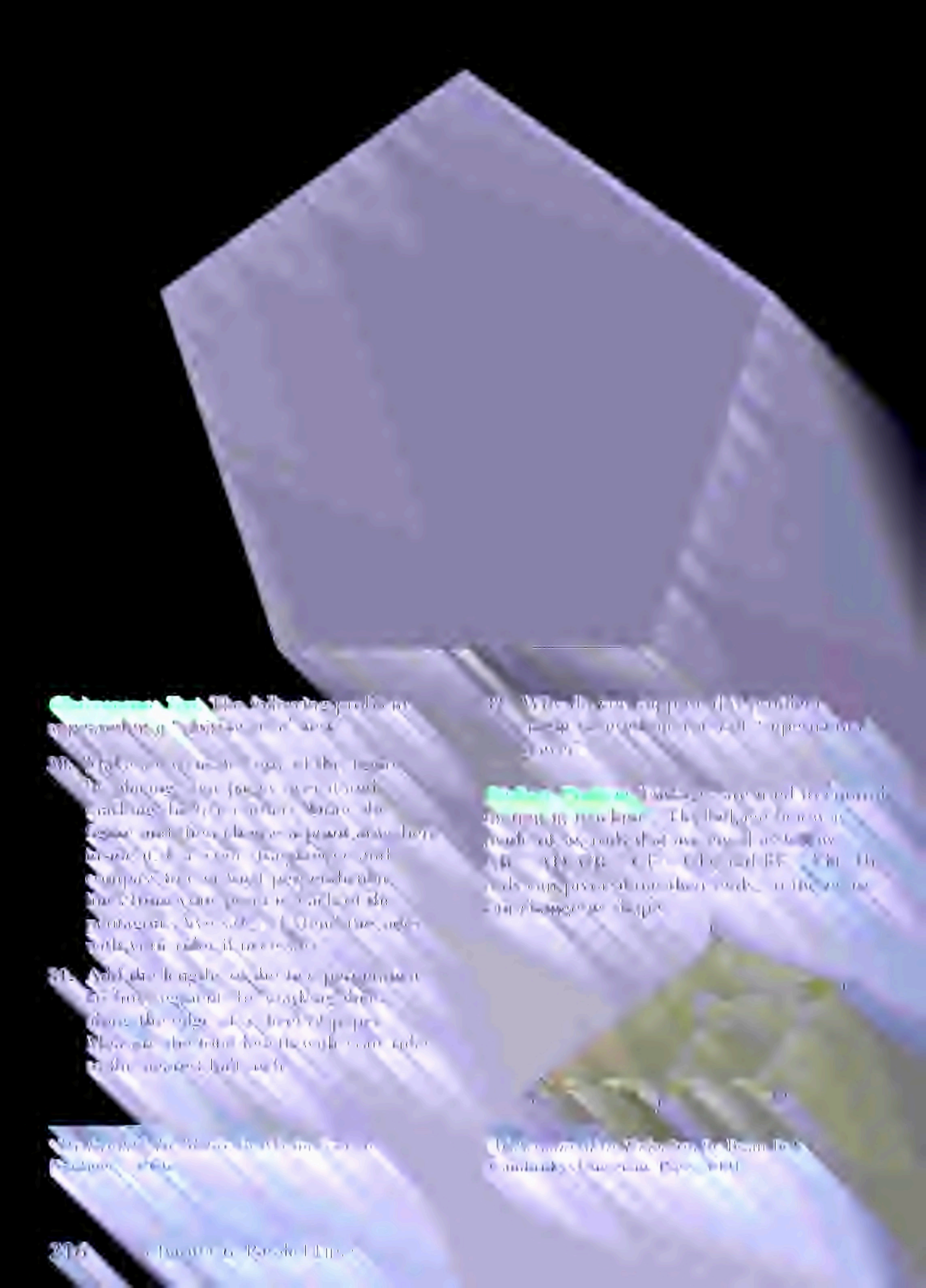




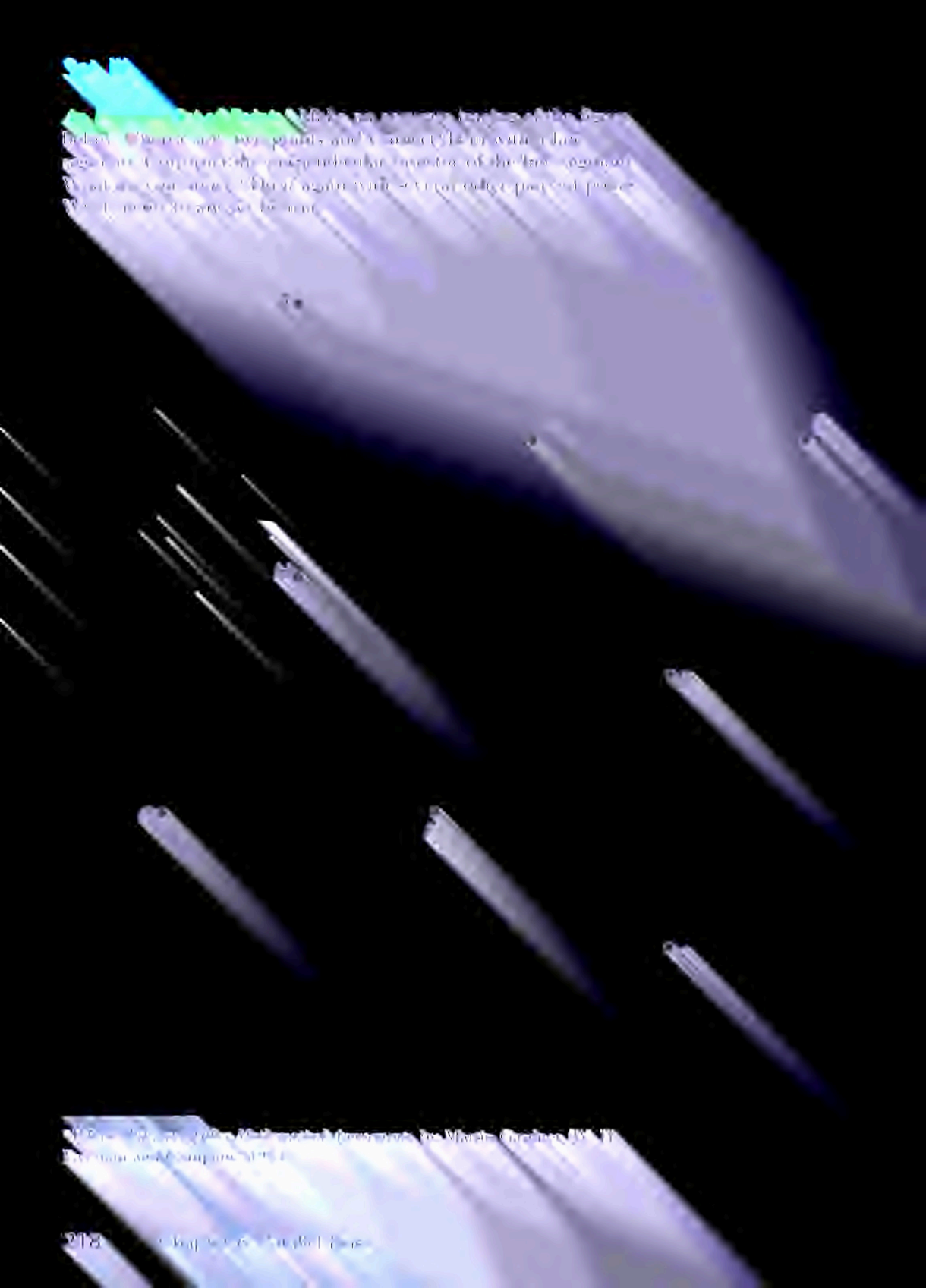









EXERCISES
PROBLEM SOLVING
A
B



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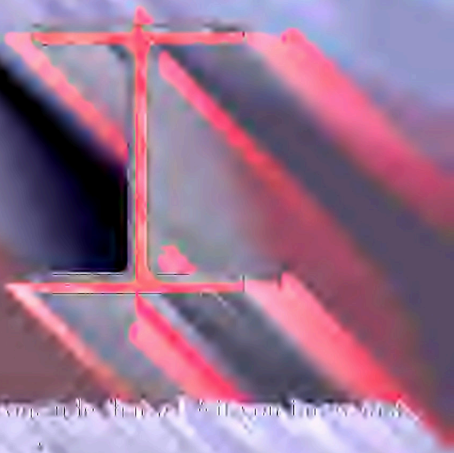
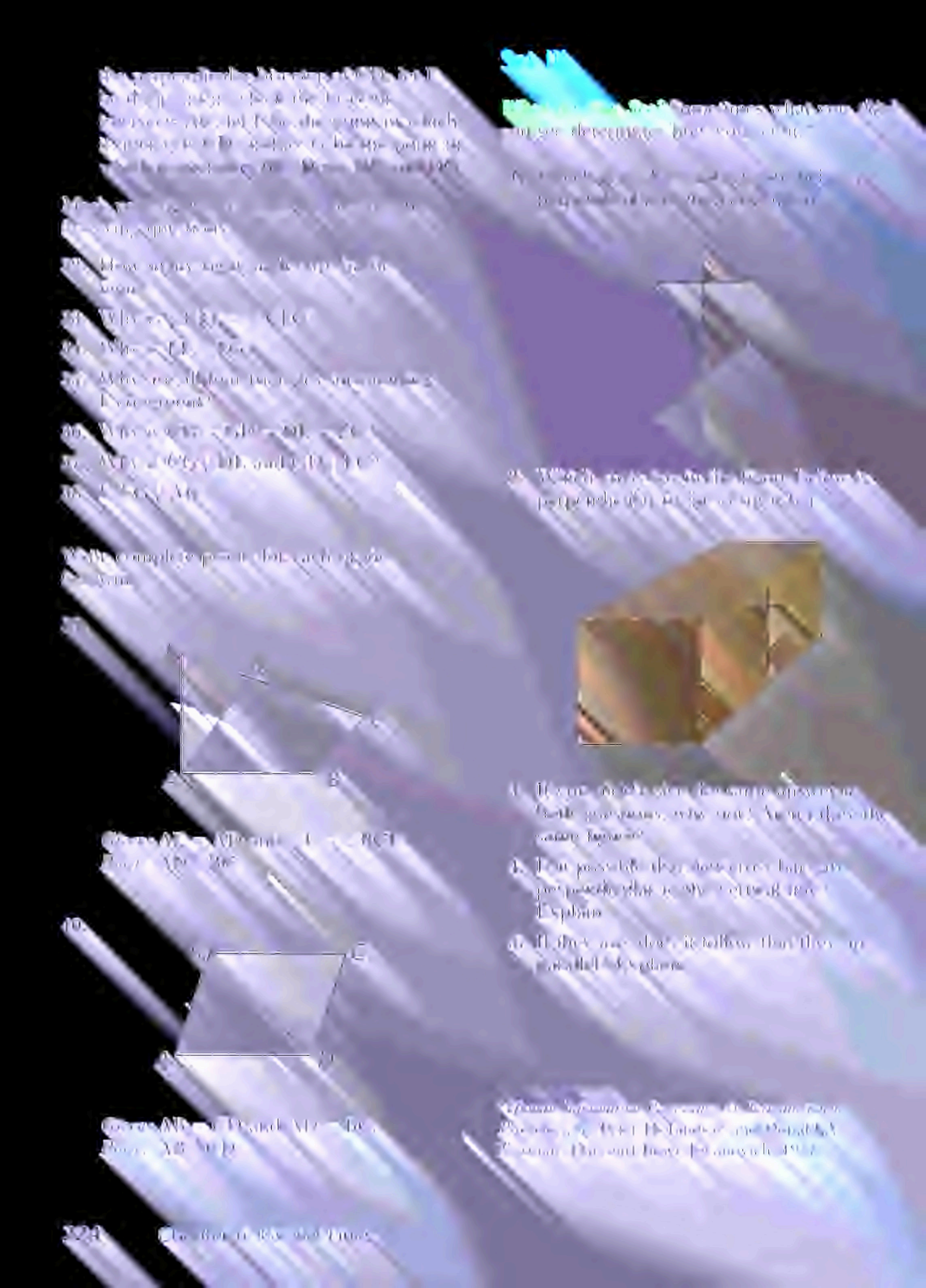






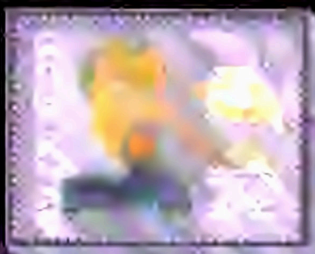


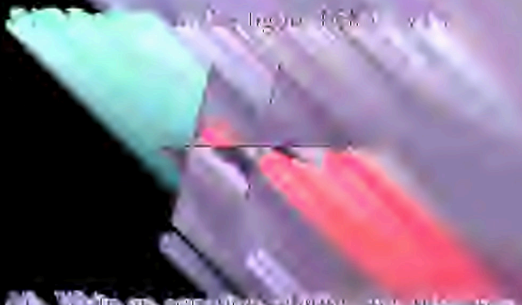


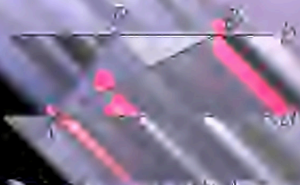


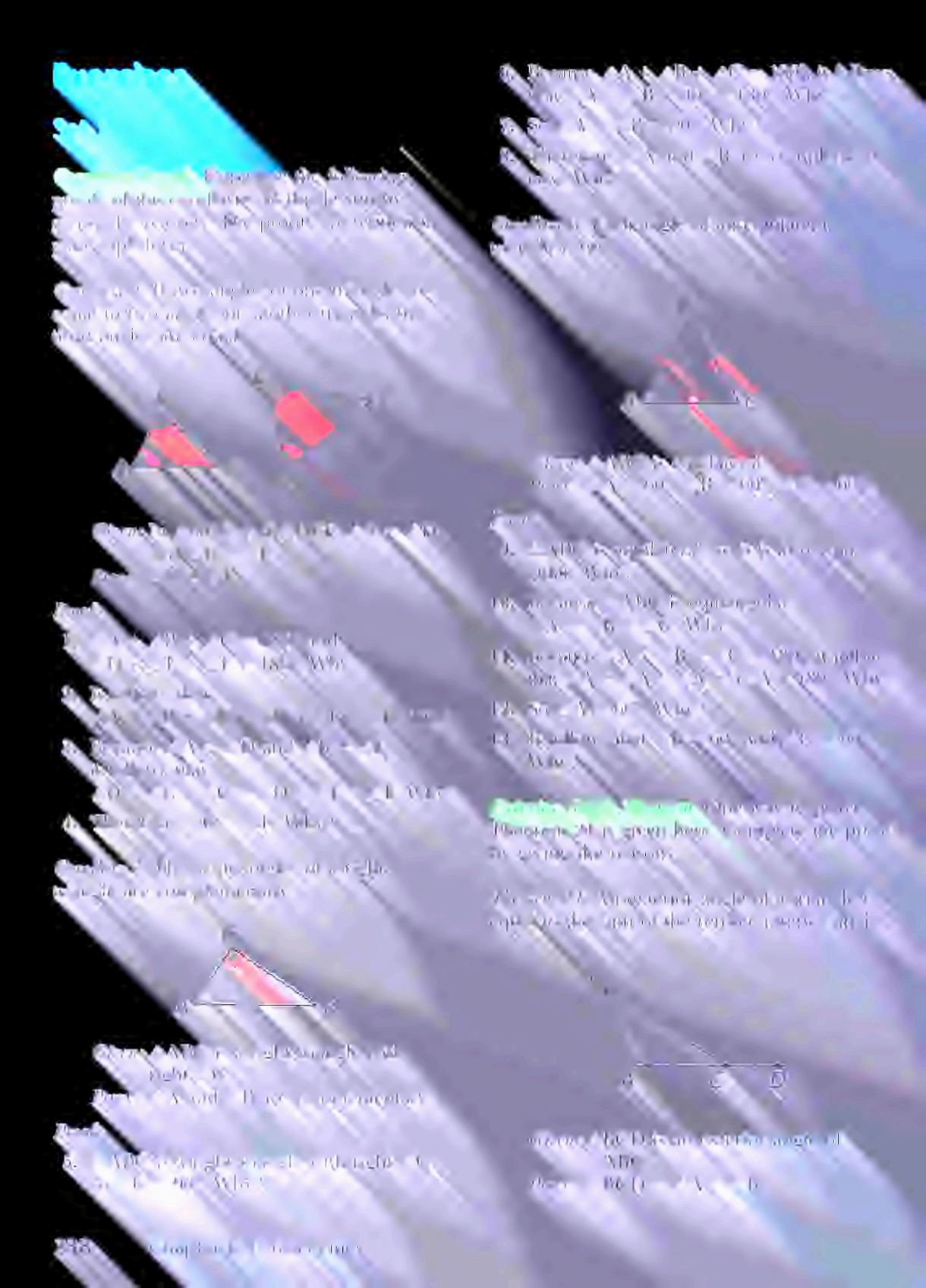


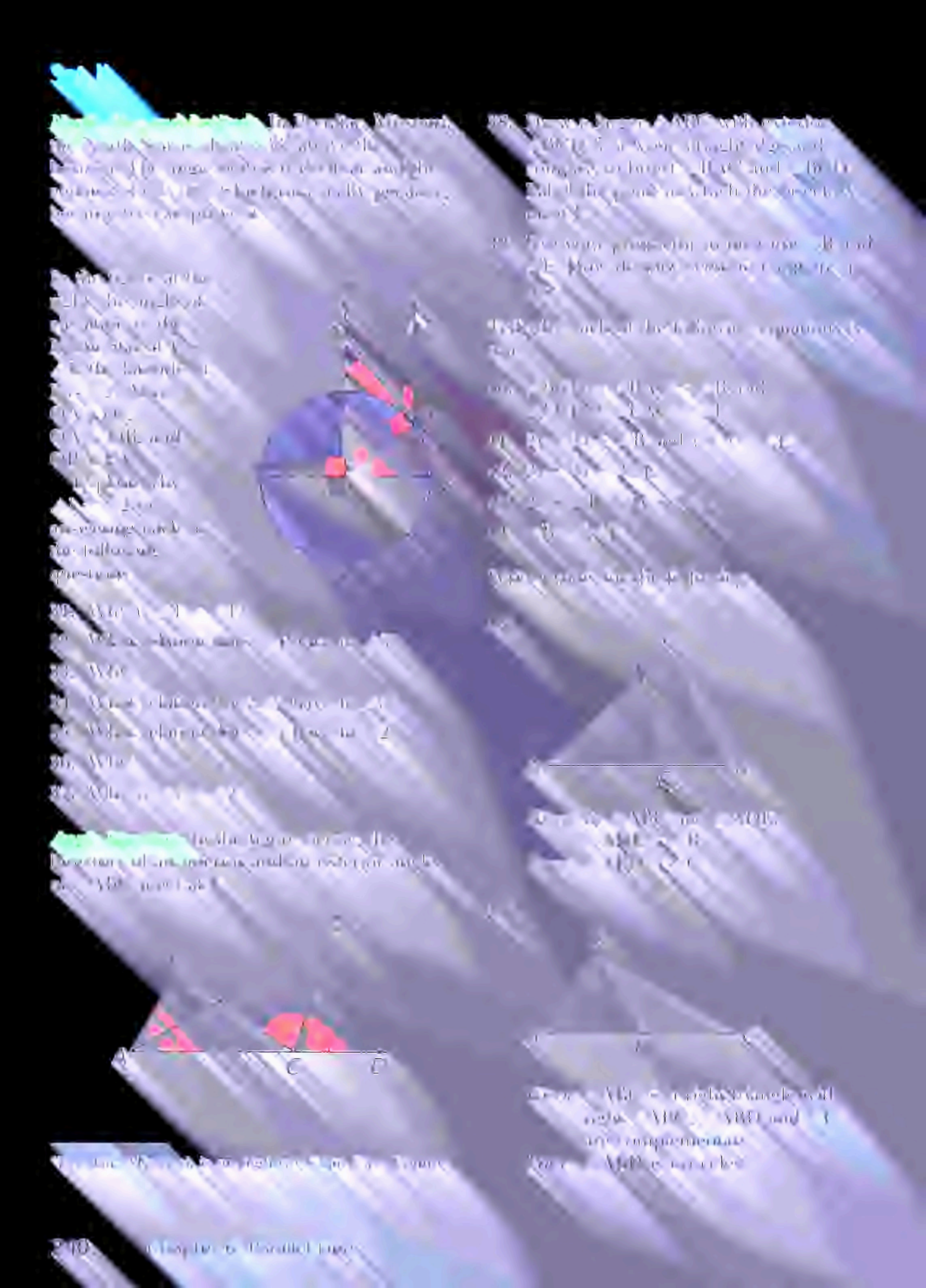


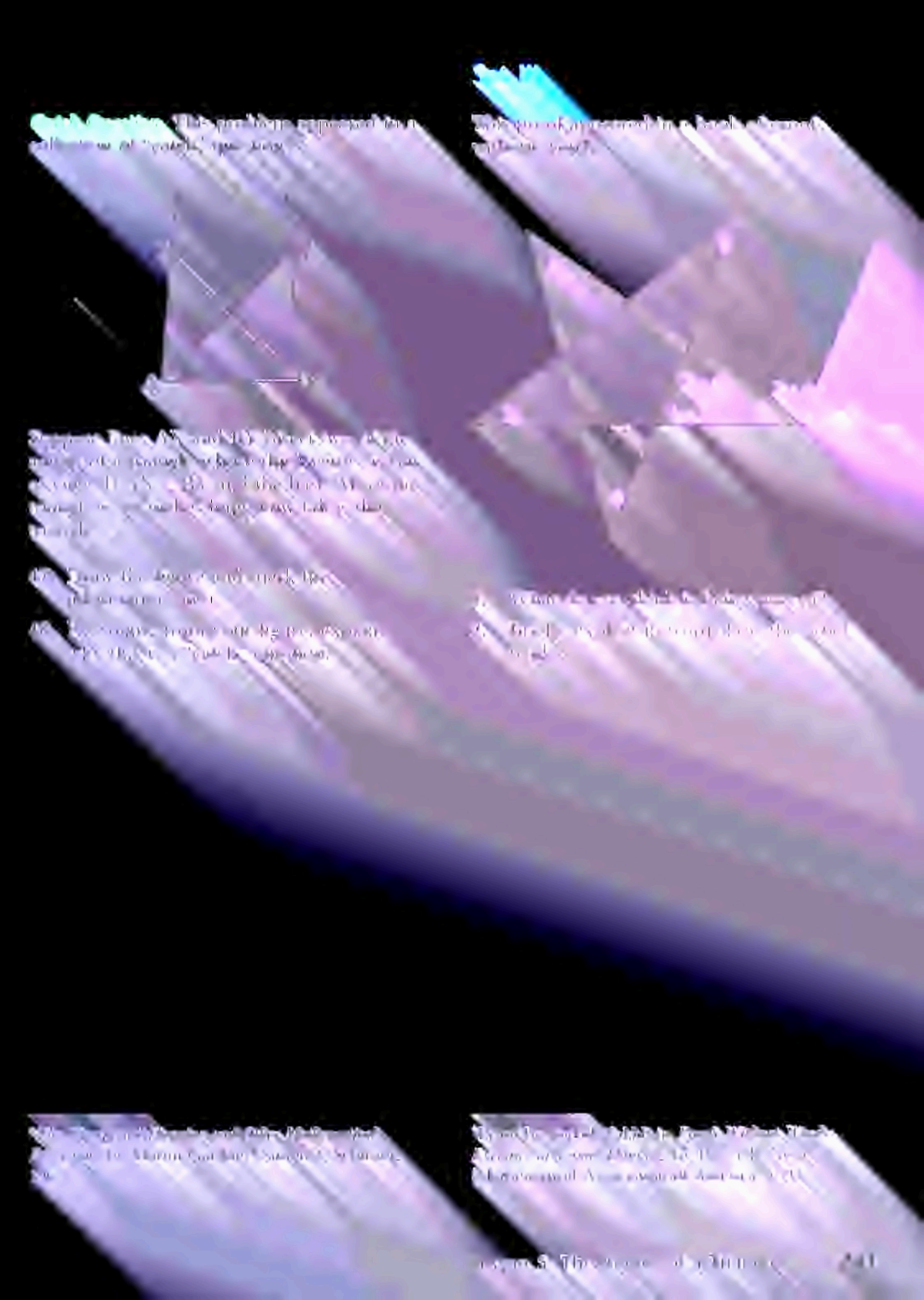


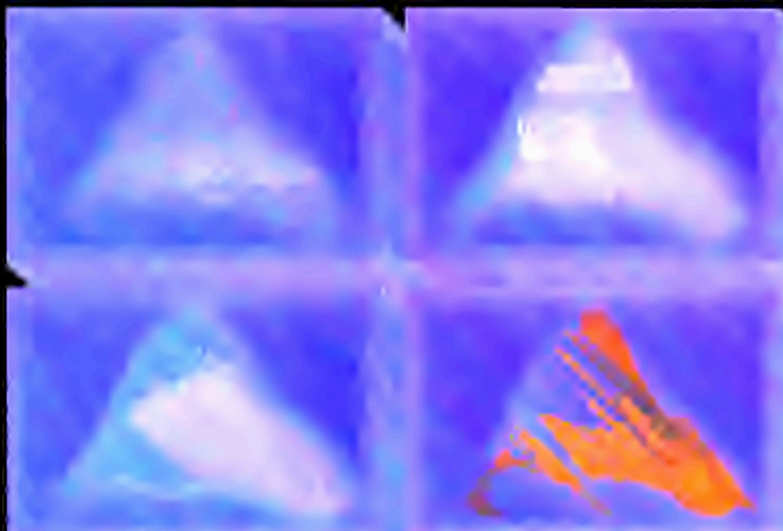













B
C

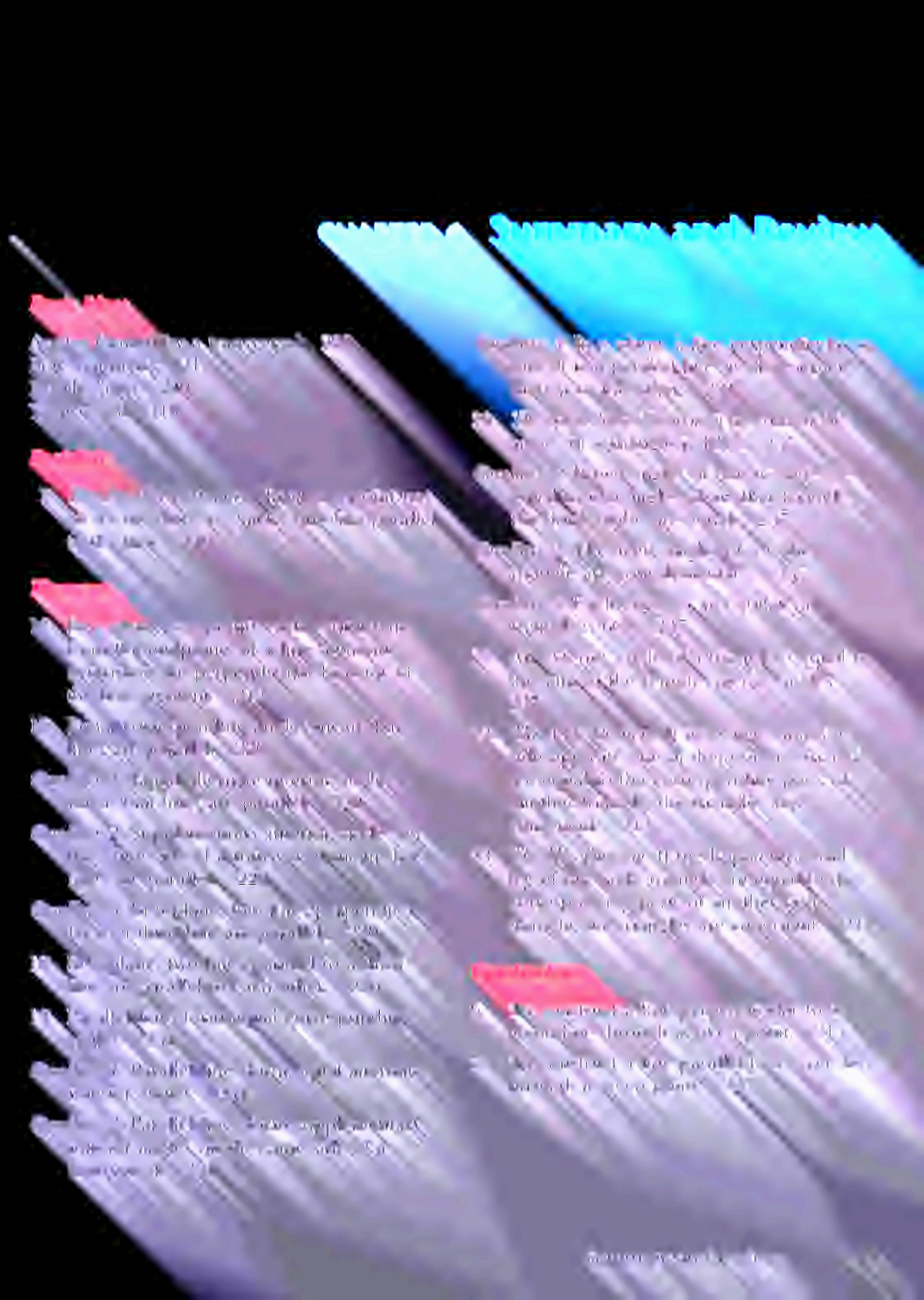

B
C

As the sequel of the next theorem, [illegible] is the converse of the [illegible] theorem applies only to right triangles.

Theorem 22. [illegible]



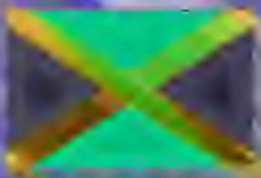


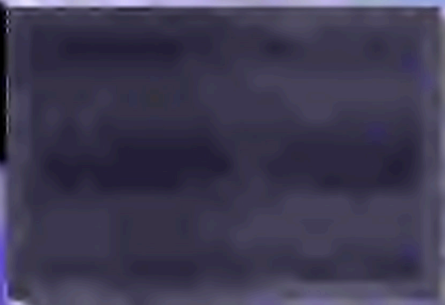


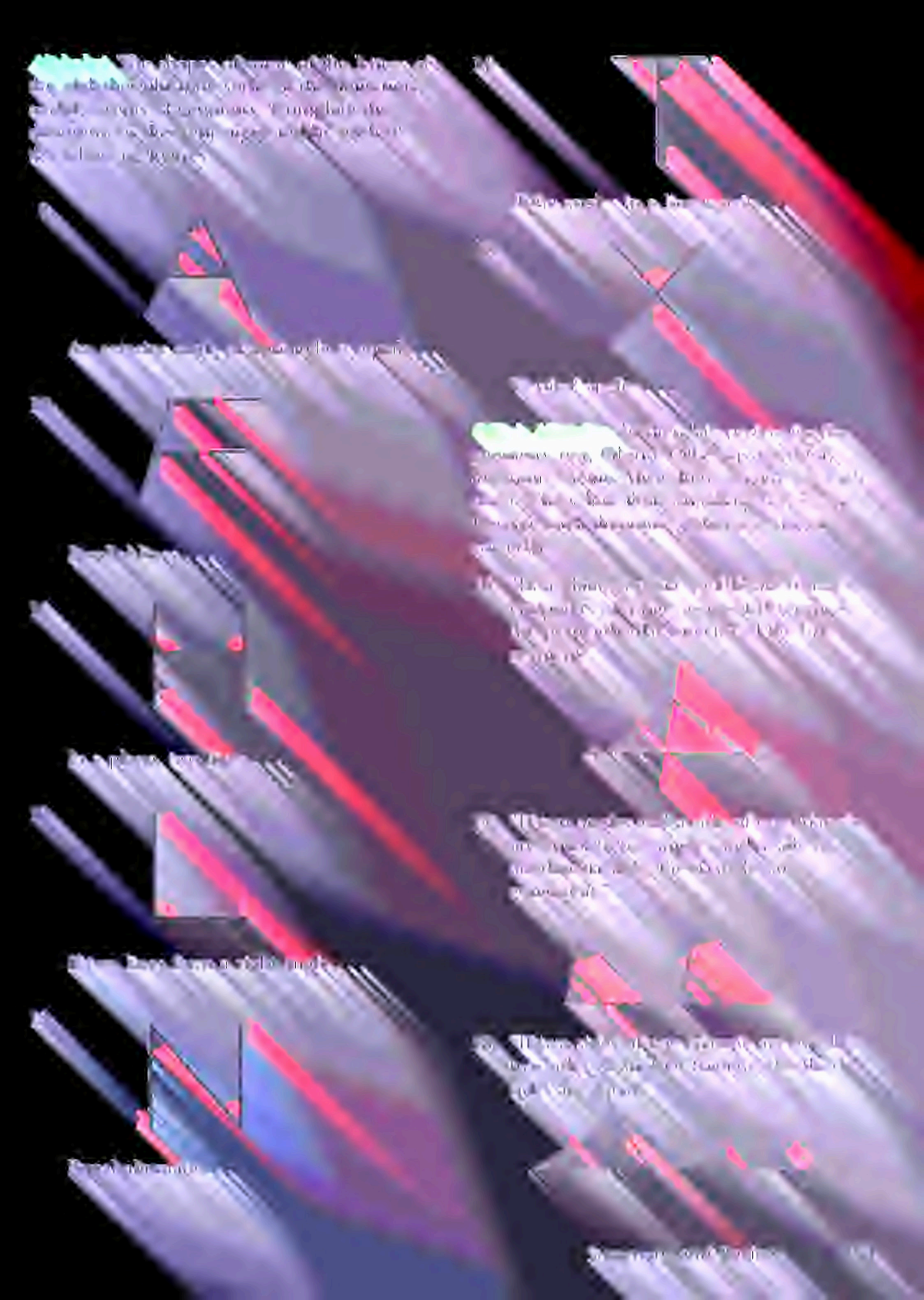




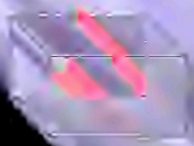








19. [illegible]

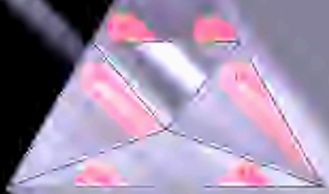


[illegible]

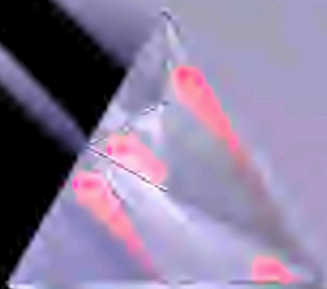
20. [illegible]



21. [illegible]



22. [illegible]



[illegible]

23. What do all of the triangular pieces in all three figures have in common?

24. How many pieces of each figure are [illegible] triangles?

[illegible]

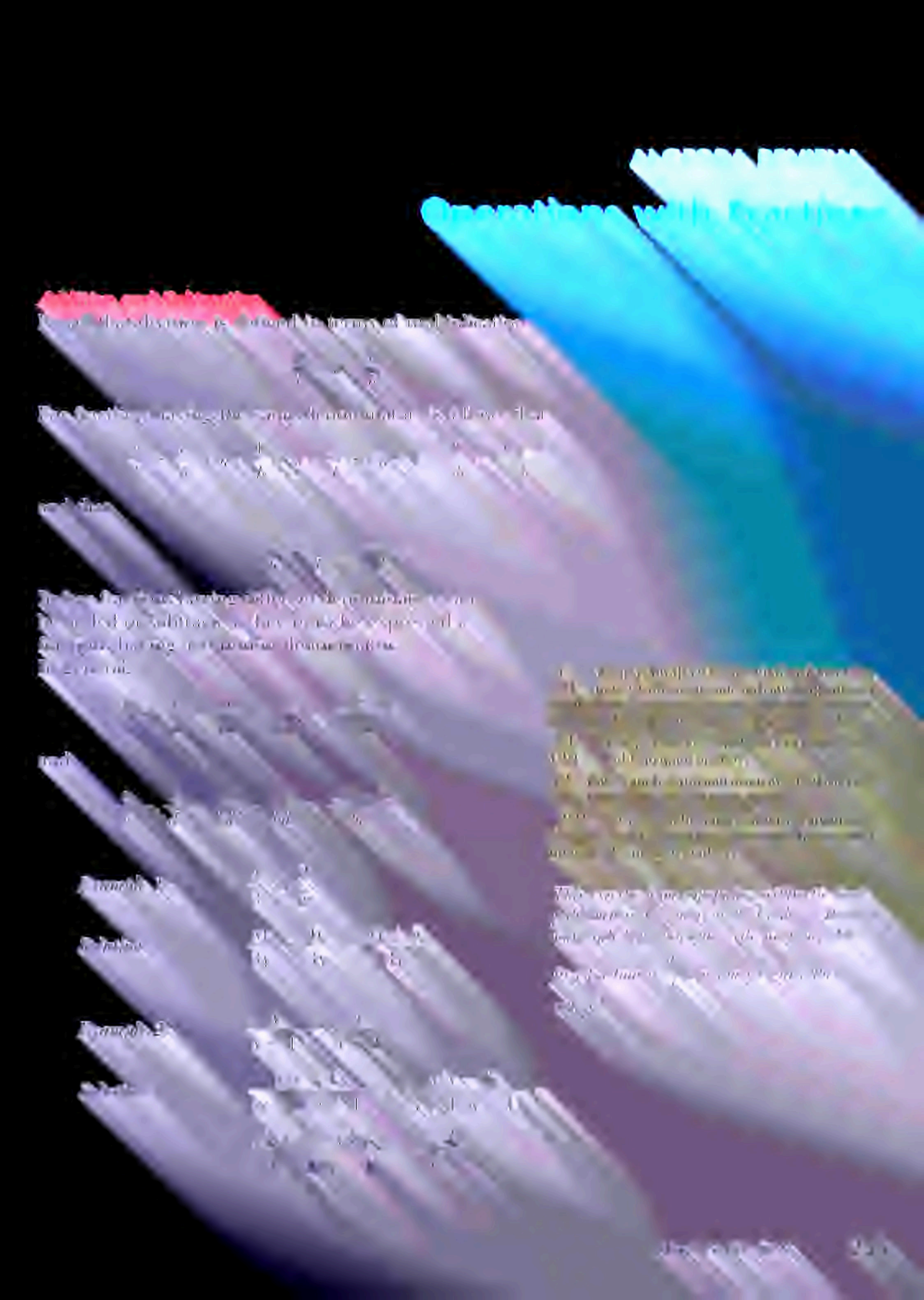


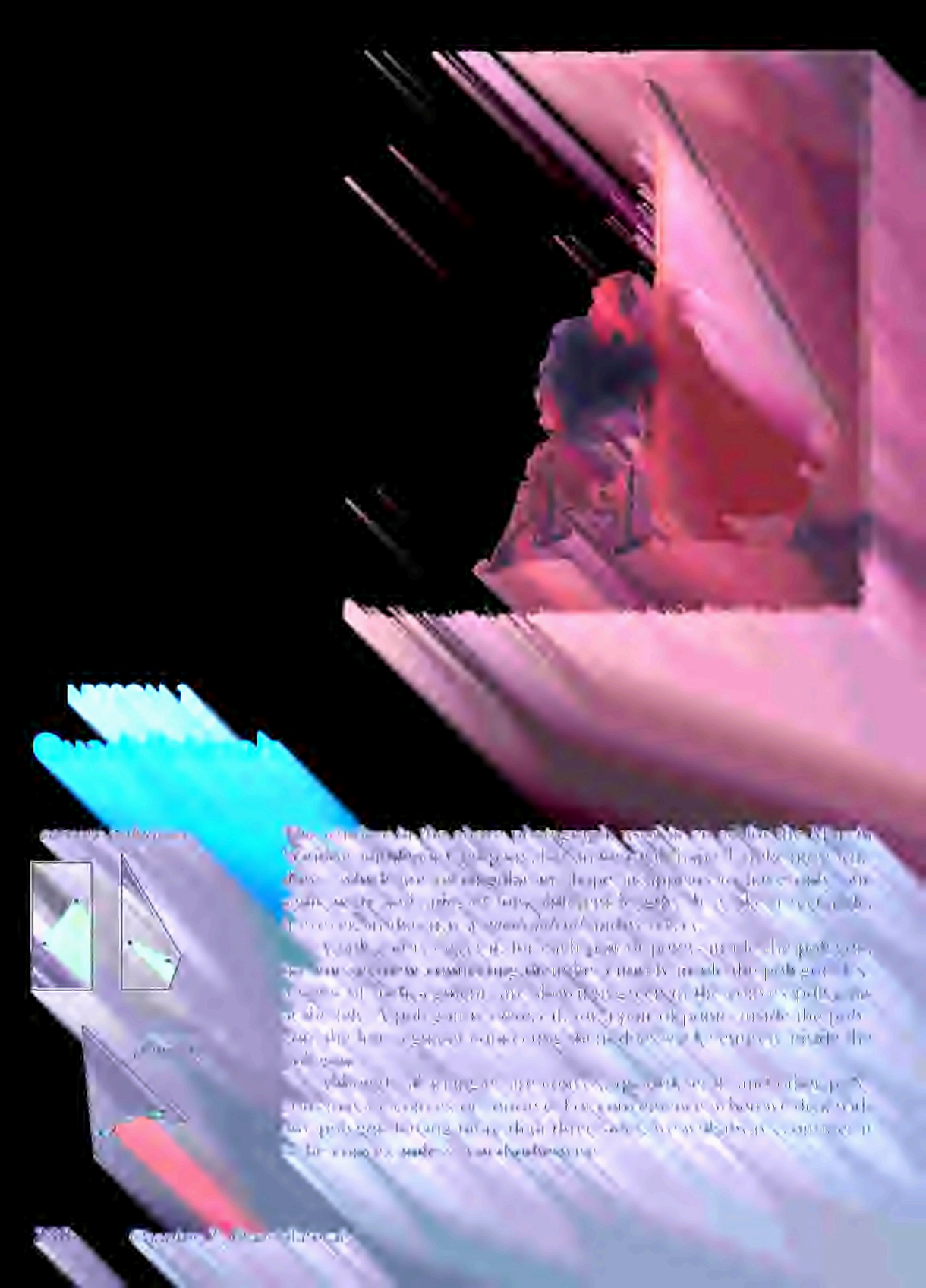
25. [illegible]

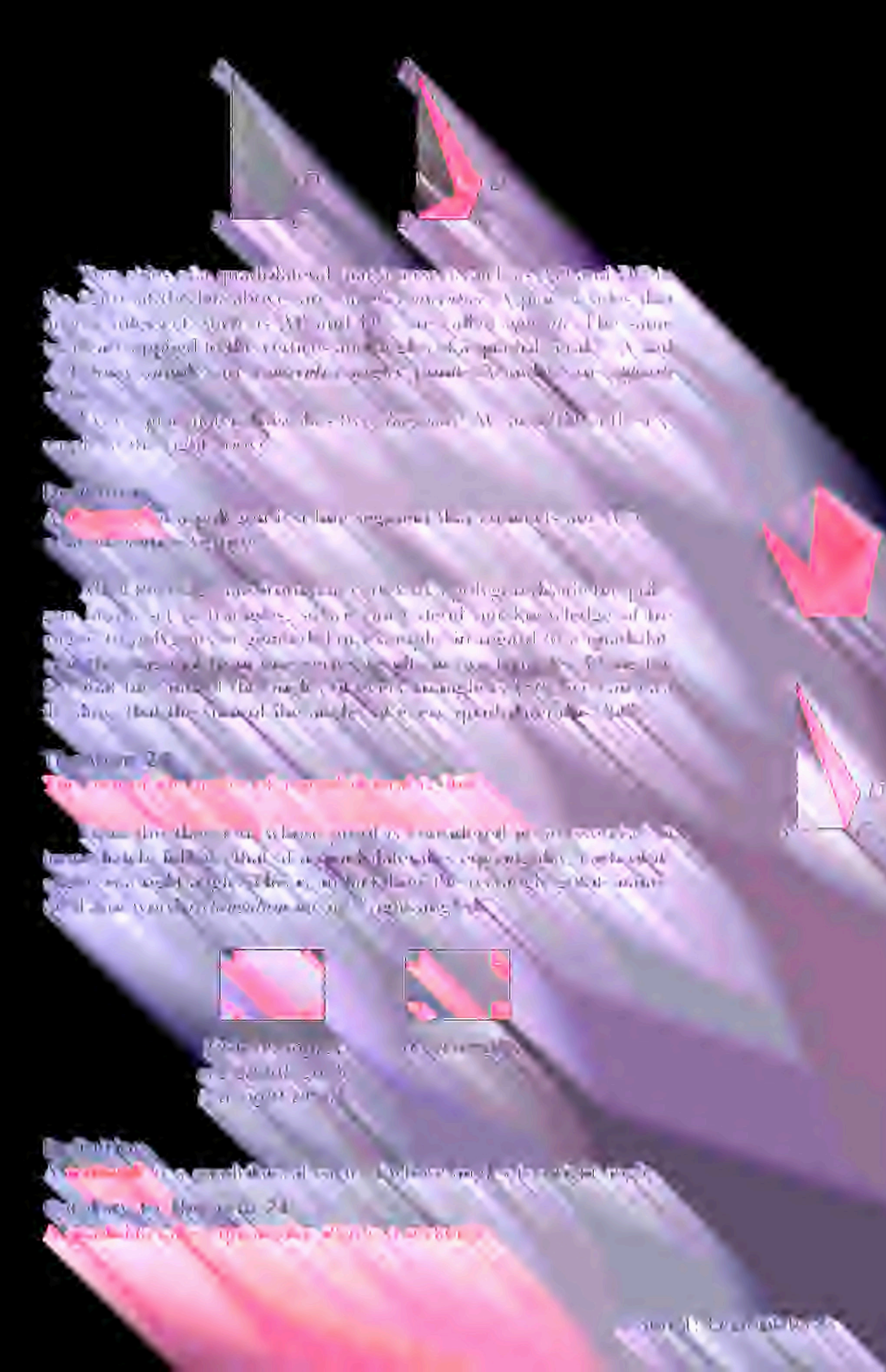
26. [illegible]

27. What can you conclude about [illegible] and [illegible] Why?

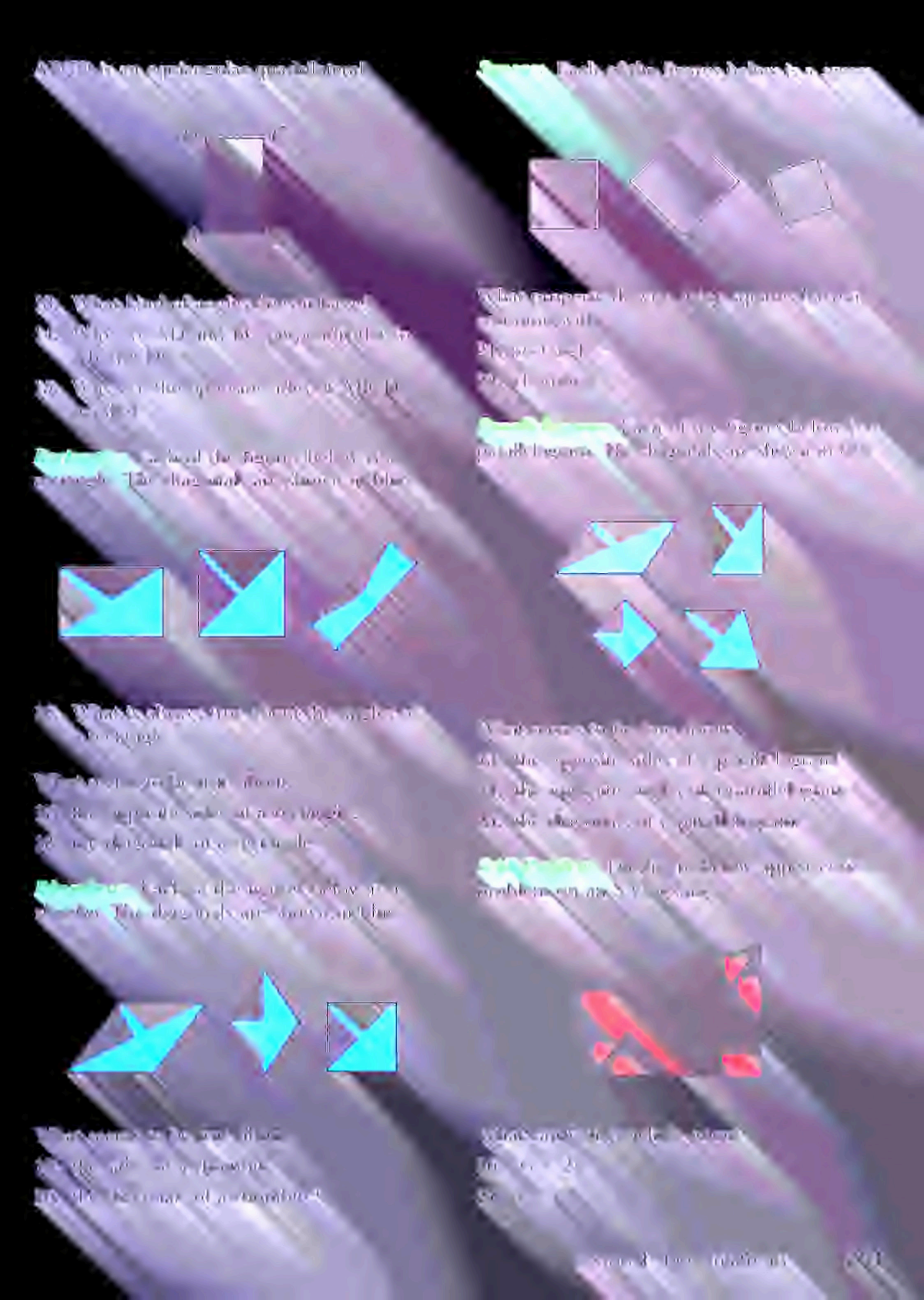
28. What can you conclude about [illegible] Why?

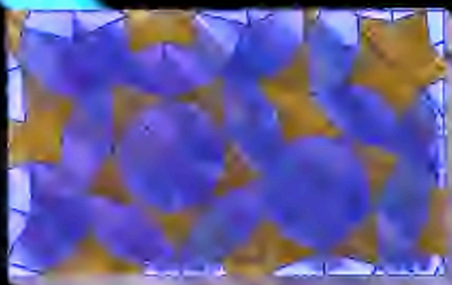








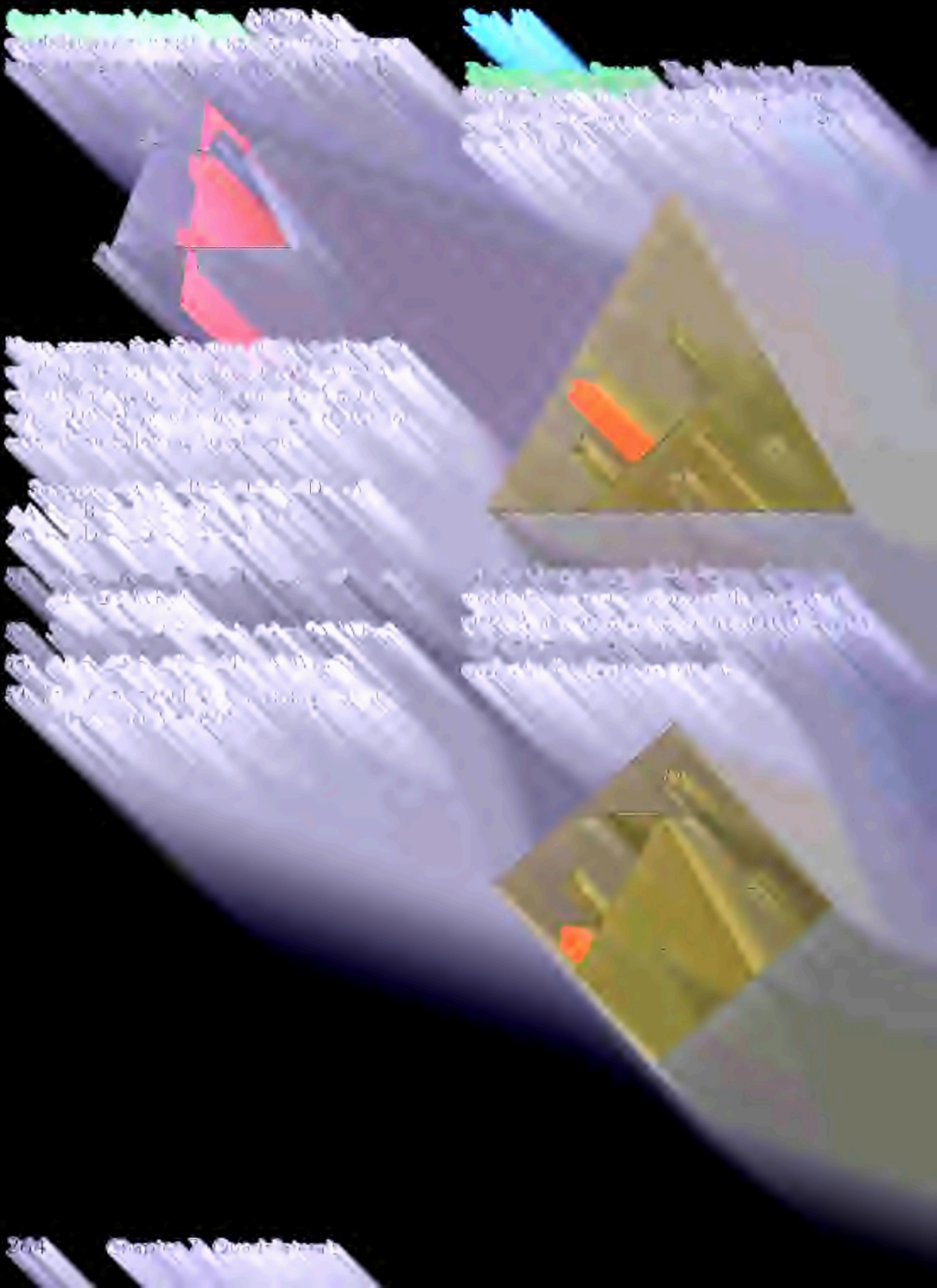


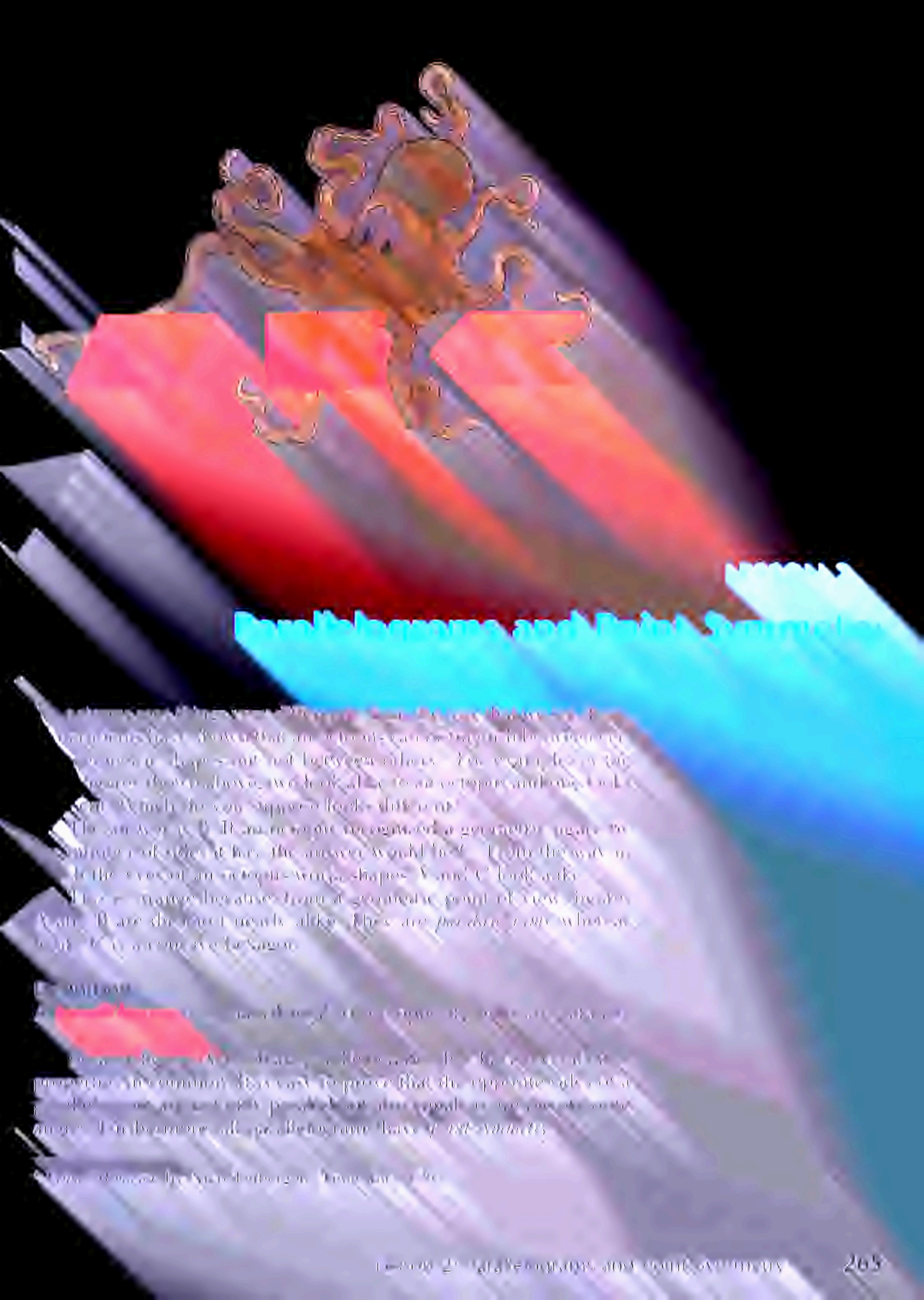


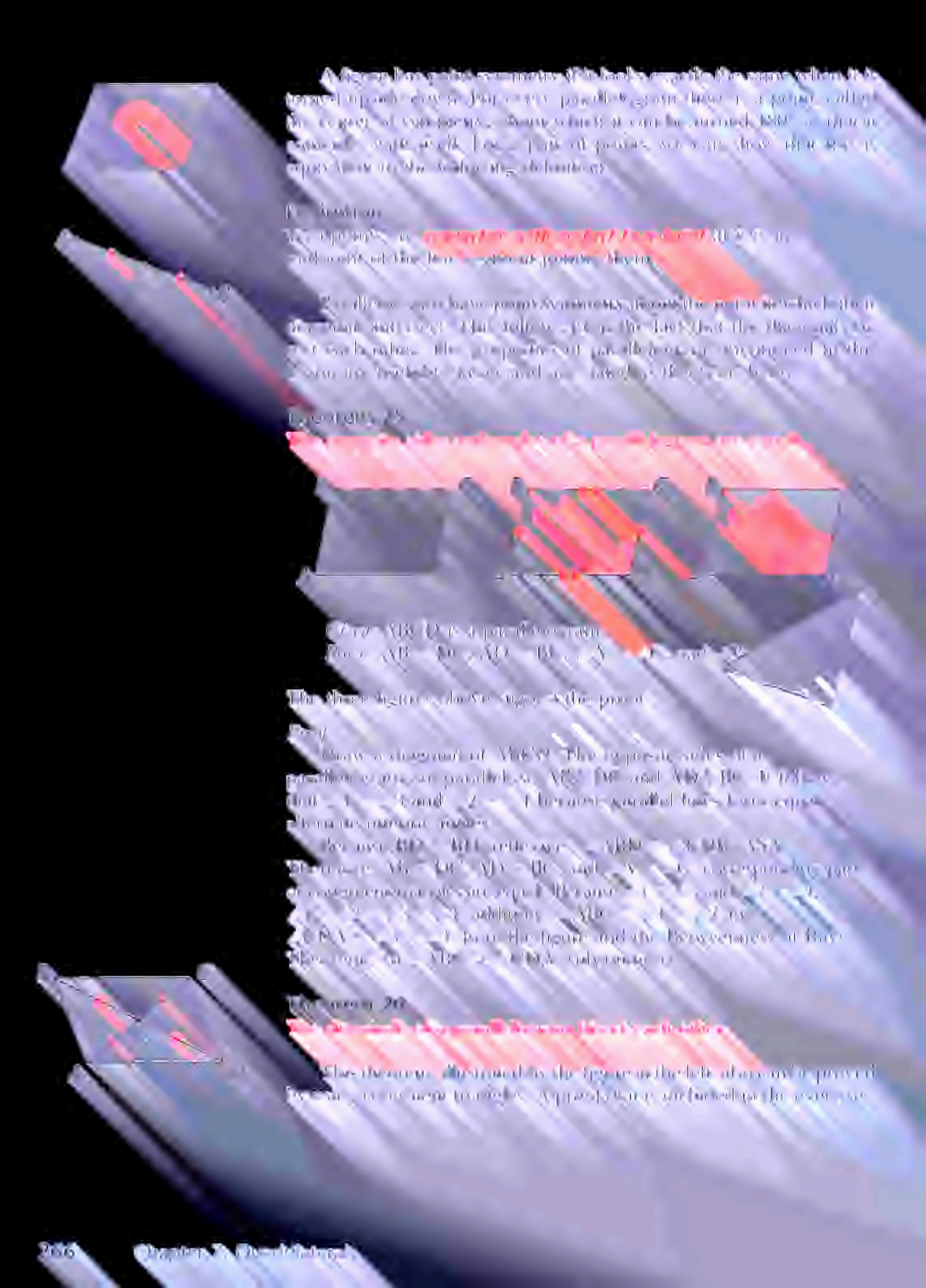






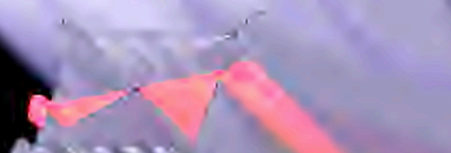












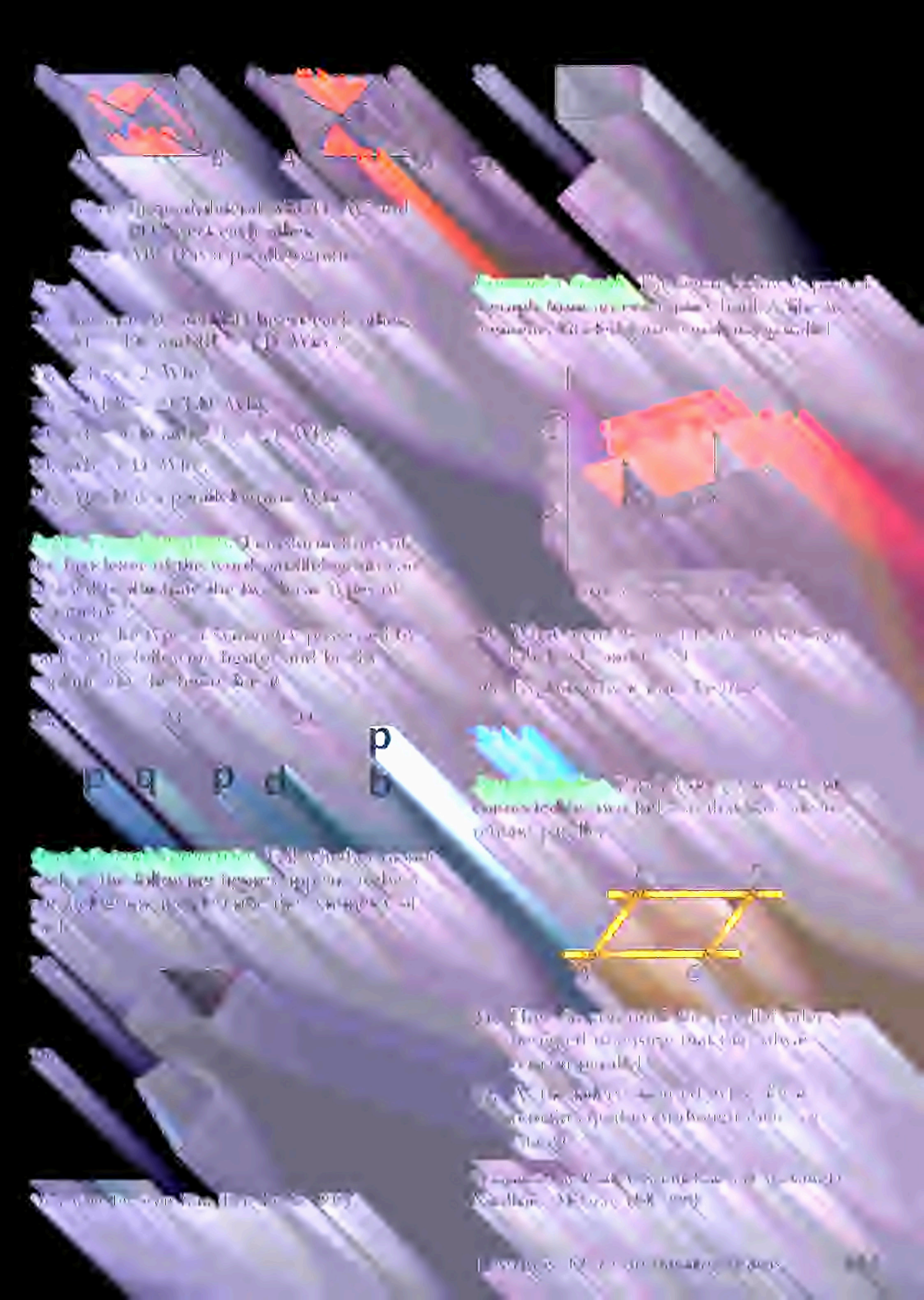


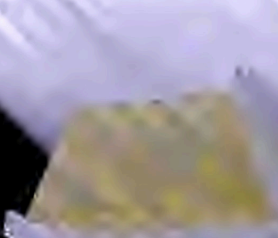










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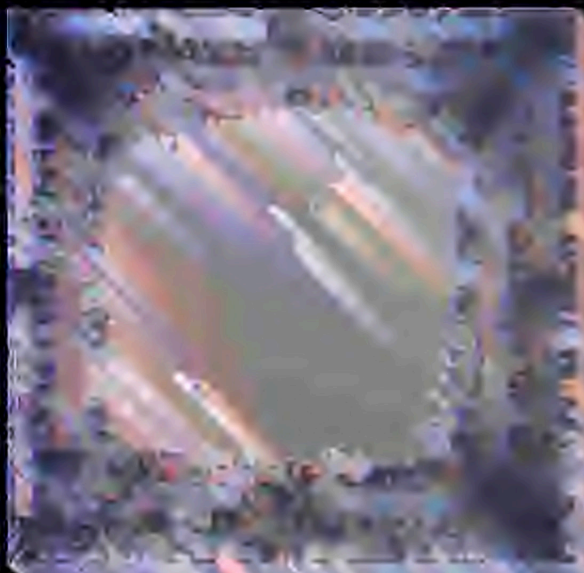


Rectangles, Rhombuses, and Squares













[illegible]

Theorem 15.

[illegible]

[illegible]

Theorem 16.

[illegible]

Exercises

Exercises for Theorem 15.

[illegible]

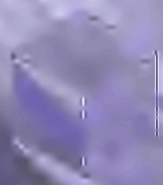


1. [illegible]
2. [illegible]

[illegible]

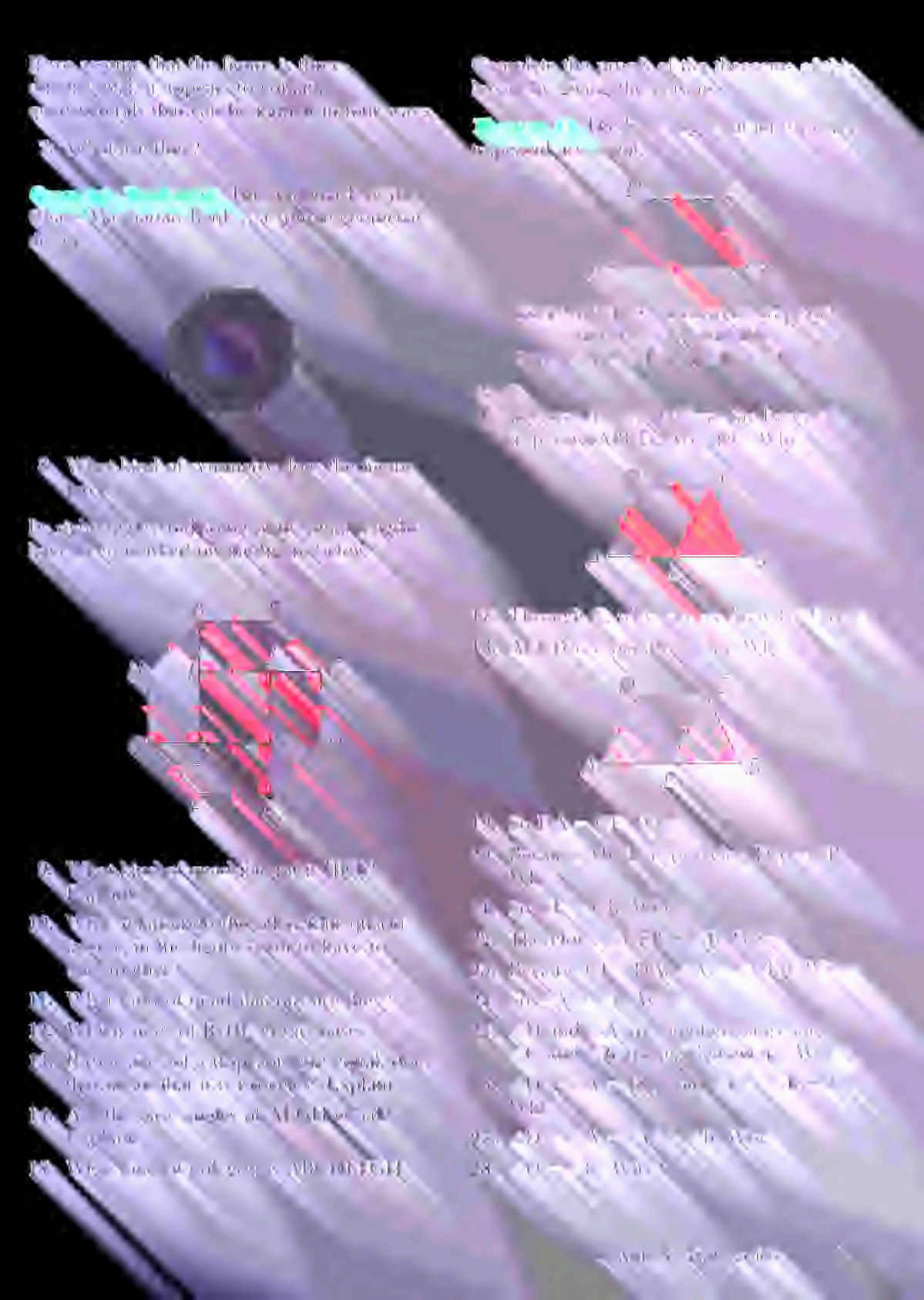
3. [illegible]
4. [illegible]
5. [illegible]

[illegible]



[illegible]

6. [illegible]

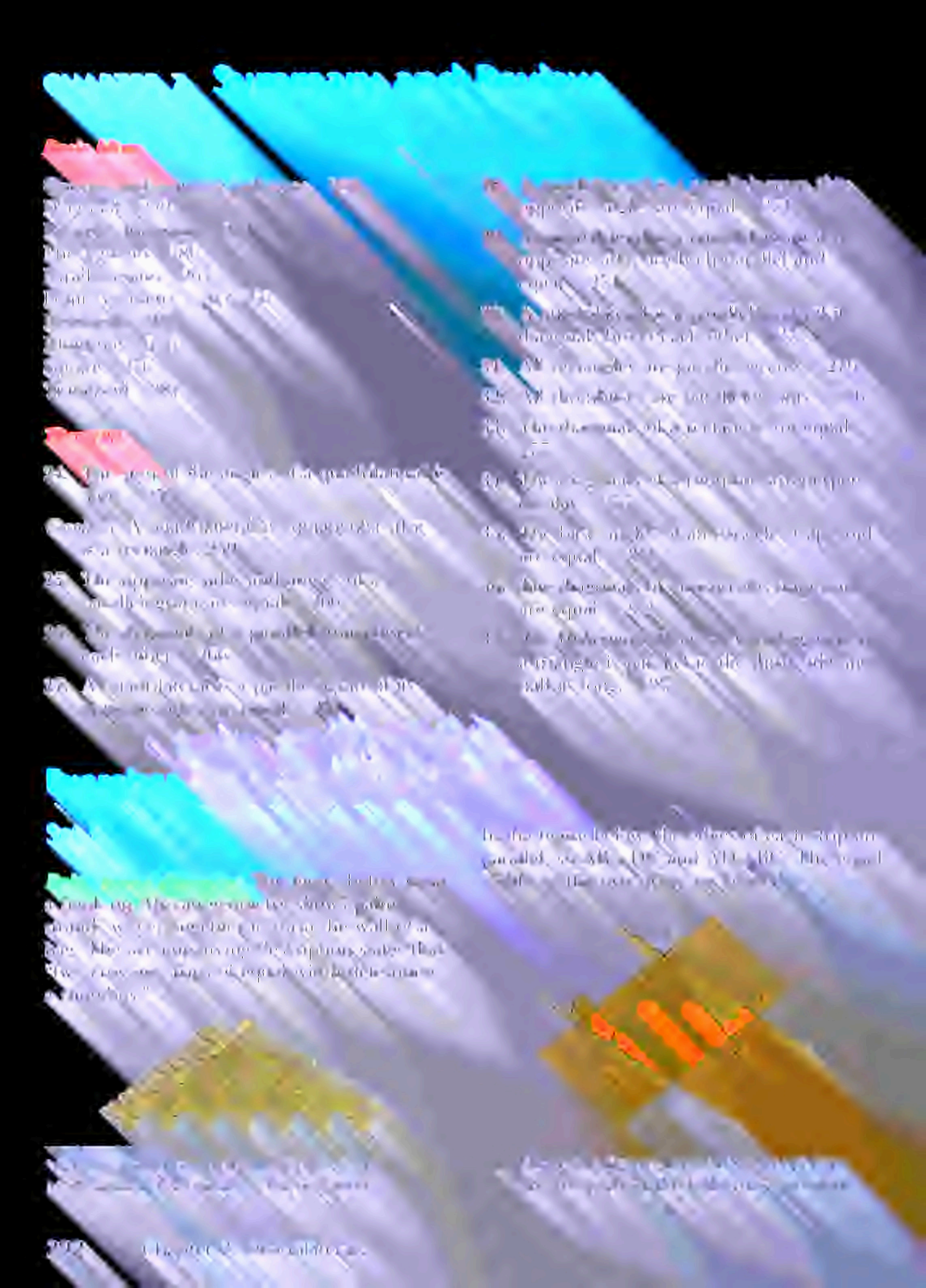


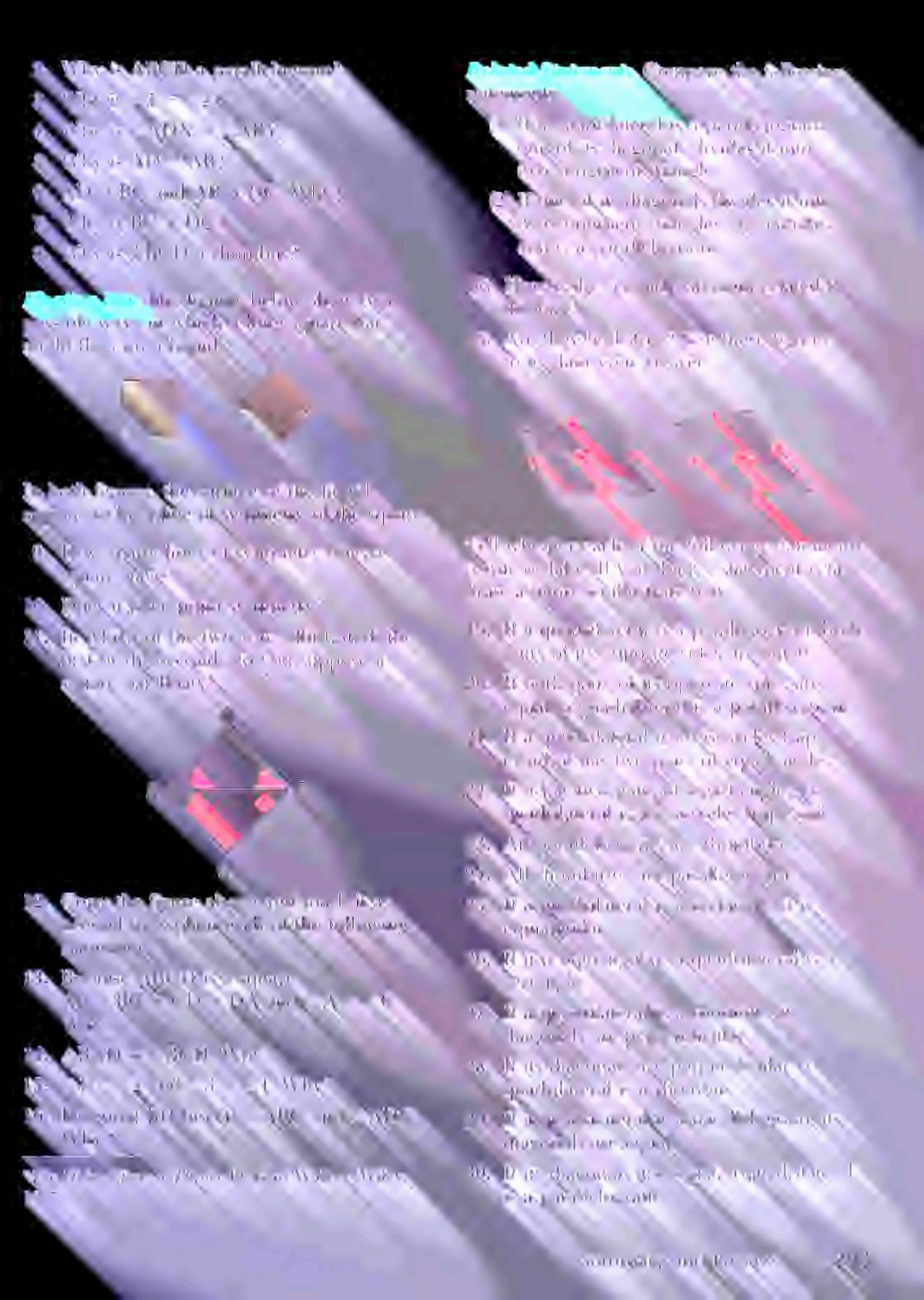


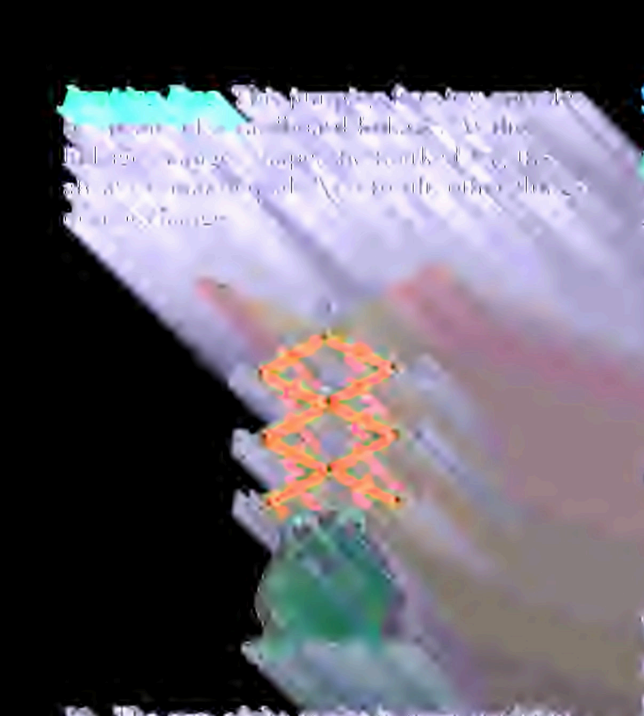





Olympia
Salem
Boise







Square Roots

The Square Root of a Number Squared

If $x \geq 0$, then

$$\sqrt{x^2} = x$$

Square Root Property

If $x^2 = a$, then $x = \sqrt{a}$ or $x = -\sqrt{a}$.

Example: Find the square roots of 81.

$x^2 = 81$

$x = \sqrt{81}$ or $x = -\sqrt{81}$

$x = 9$ or $x = -9$

Example: Solve $(x-3)^2 = 16$.

$x - 3 = \sqrt{16}$ or $x - 3 = -\sqrt{16}$

$x - 3 = 4$ or $x - 3 = -4$

$x = 7$ or $x = -1$

Product Property of Square Roots

If $a \geq 0$ and $b \geq 0$, then $\sqrt{ab} = \sqrt{a}\sqrt{b}$.

Example: $\sqrt{50} = \sqrt{25 \cdot 2} = \sqrt{25}\sqrt{2} = 5\sqrt{2}$

Quotient Property of Square Roots

If $a \geq 0$ and $b > 0$, then $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$.

Example: $\frac{\sqrt{72}}{\sqrt{2}} = \sqrt{\frac{72}{2}} = \sqrt{36} = 6$

Simplified Radical Form

A radical is in simplified form when:

1. No perfect square factors other than 1 remain under the radical.
2. No fractions remain under the radical.
3. No radicals appear in the denominator of a fraction.

Example: $\sqrt{72} = \sqrt{36 \cdot 2} = 6\sqrt{2}$

Example: $\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2}$

Example: $\frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{5\sqrt{2}}{2}$

Adding and Subtracting Radicals

Only like radicals can be added or subtracted. Like radicals have the same radicand and the same index.

Example: $3\sqrt{5} + 2\sqrt{5} = 5\sqrt{5}$

Example: $7\sqrt{3} - 4\sqrt{3} = 3\sqrt{3}$

Example: $\sqrt{8} + \sqrt{18} = 2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2}$

Multiplying Radicals

$\sqrt{a}\sqrt{b} = \sqrt{ab}$

Example: $\sqrt{3}(\sqrt{6} + \sqrt{12}) = \sqrt{18} + \sqrt{36} = 3\sqrt{2} + 6$

Example: $(\sqrt{x} + 2)(\sqrt{x} - 5) = x - 5\sqrt{x} + 2\sqrt{x} - 10 = x - 3\sqrt{x} - 10$

Rationalizing the Denominator

To rationalize a denominator of the form $\sqrt{a}$, multiply numerator and denominator by $\sqrt{a}$.

To rationalize a denominator of the form $\sqrt{a} + b$ or $\sqrt{a} - b$, multiply numerator and denominator by the conjugate.

Example: $\frac{1}{\sqrt{3}+1} = \frac{1(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{\sqrt{3}-1}{3-1} = \frac{\sqrt{3}-1}{2}$

Higher Index Radicals

$\sqrt[3]{8} = 2$ because $2^3 = 8$

$\sqrt[4]{16} = 2$ because $2^4 = 16$

$\sqrt[3]{-27} = -3$ because $(-3)^3 = -27$

$\sqrt[n]{a^n} = a$ when n is odd

$\sqrt[n]{a^n} = |a|$ when n is even

$\sqrt[3]{x^3} = x$

$\sqrt[4]{x^4} = |x|$

$\sqrt[5]{x^5} = x$

$\sqrt[3]{ab} = \sqrt[3]{a}\sqrt[3]{b}$

$\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$

$\sqrt[3]{a^3 b} = a\sqrt[3]{b}$

Fractional Exponents and Radicals

$x^{\frac{1}{2}} = \sqrt{x}$

$x^{\frac{1}{3}} = \sqrt[3]{x}$

$x^{\frac{2}{3}} = (\sqrt[3]{x})^2$

$x^{\frac{3}{2}} = (\sqrt{x})^3$

$x^{\frac{m}{n}} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$

$x^{-\frac{1}{2}} = \frac{1}{\sqrt{x}}$

$x^{-\frac{m}{n}} = \frac{1}{x^{\frac{m}{n}}}$

$x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} = x$

$x^{\frac{1}{3}} \cdot x^{\frac{1}{3}} \cdot x^{\frac{1}{3}} = x$

$x^{\frac{1}{2}} \cdot x^{\frac{1}{3}} = x^{\frac{5}{6}}$

$\frac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}} = x^{\frac{1}{6}}$

$(x^{\frac{1}{2}})^{\frac{1}{3}} = x^{\frac{1}{6}}$

$(x^{\frac{2}{3}})^{\frac{3}{2}} = x$

$x^{\frac{1}{2}} + x^{\frac{1}{2}} = 2x^{\frac{1}{2}}$

$x^{\frac{1}{2}} - x^{\frac{1}{2}} = 0$

$x^{\frac{1}{2}} + x^{\frac{1}{3}}$ cannot be combined

$(x+y)^{\frac{1}{2}} \neq x^{\frac{1}{2}} + y^{\frac{1}{2}}$

$(xy)^{\frac{1}{2}} = x^{\frac{1}{2}} y^{\frac{1}{2}}$

$\left(\frac{x}{y}\right)^{\frac{1}{2}} = \frac{x^{\frac{1}{2}}}{y^{\frac{1}{2}}}$

$x^{\frac{1}{2}} y^{\frac{1}{3}} = x^{\frac{3}{6}} y^{\frac{2}{6}}$

$x^{\frac{1}{2}} + 2x^{\frac{1}{2}} = 3x^{\frac{1}{2}}$

$x^{\frac{1}{2}} - 2x^{\frac{1}{2}} = -x^{\frac{1}{2}}$

$x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} = x^1$

$x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} = x^{\frac{3}{2}}$

$x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} = x^2$

$x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} = x^{\frac{5}{2}}$

$x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} = x^3$

$x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} = x^{\frac{7}{2}}$

$x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} = x^4$

$x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} = x^{\frac{9}{2}}$

$x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} = x^5$

$x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} = x^{\frac{11}{2}}$

$x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} = x^6$













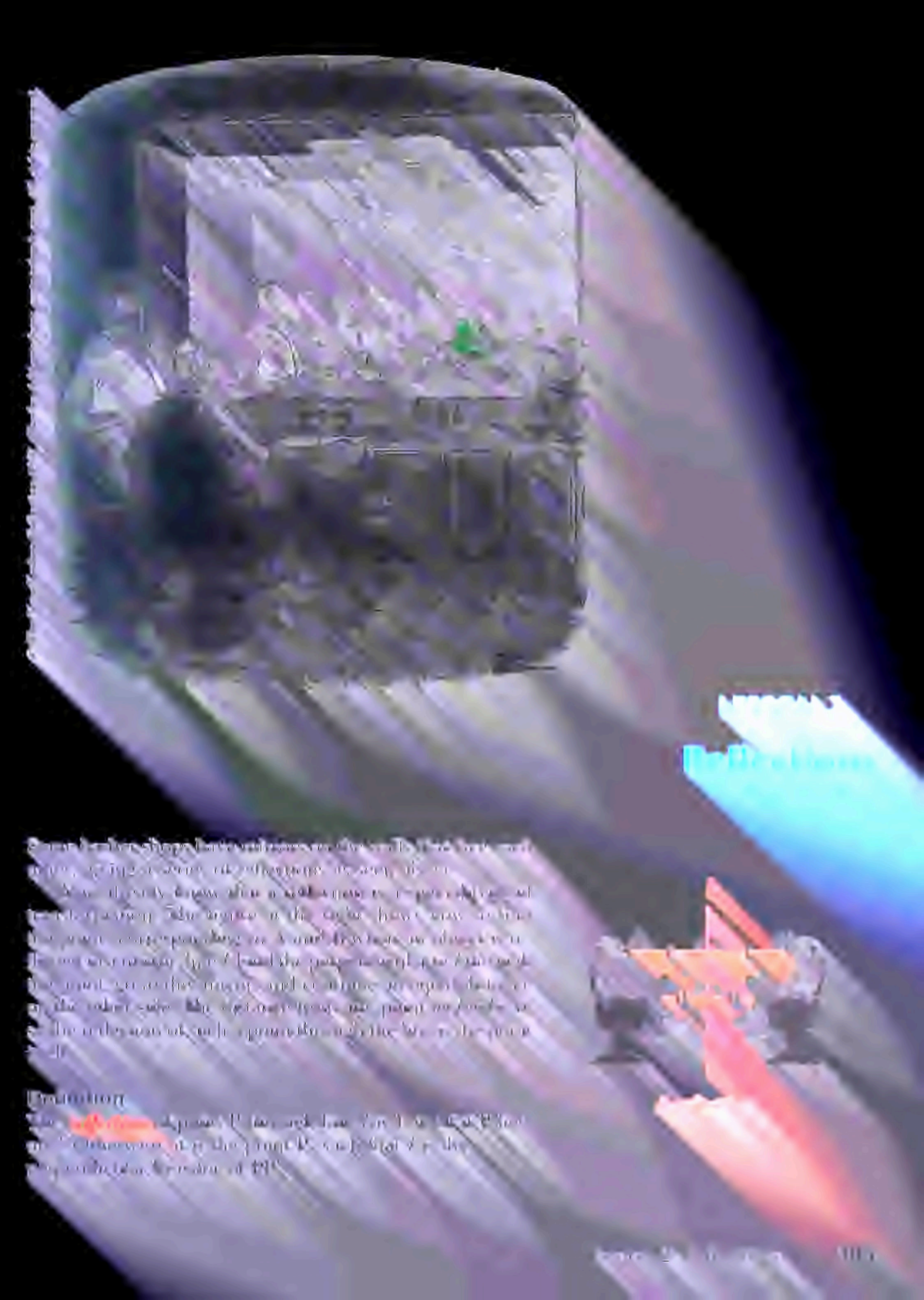








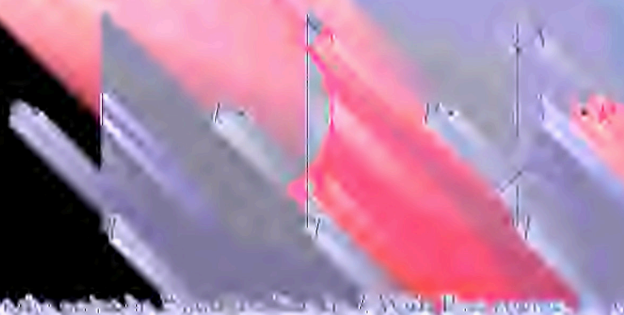




The reflection of a point through a line can be found by using a [illegible]

Construction 8

[illegible]



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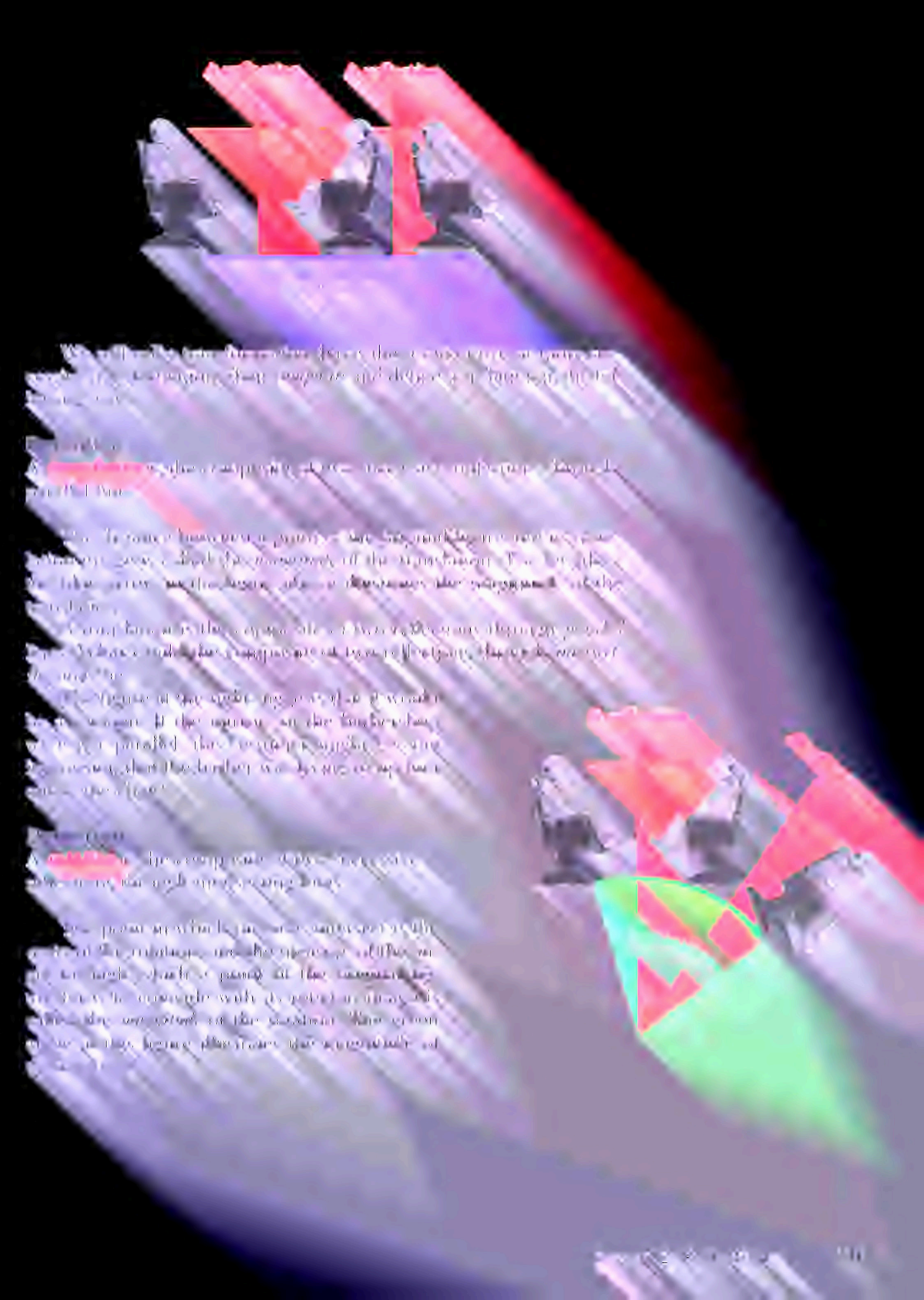

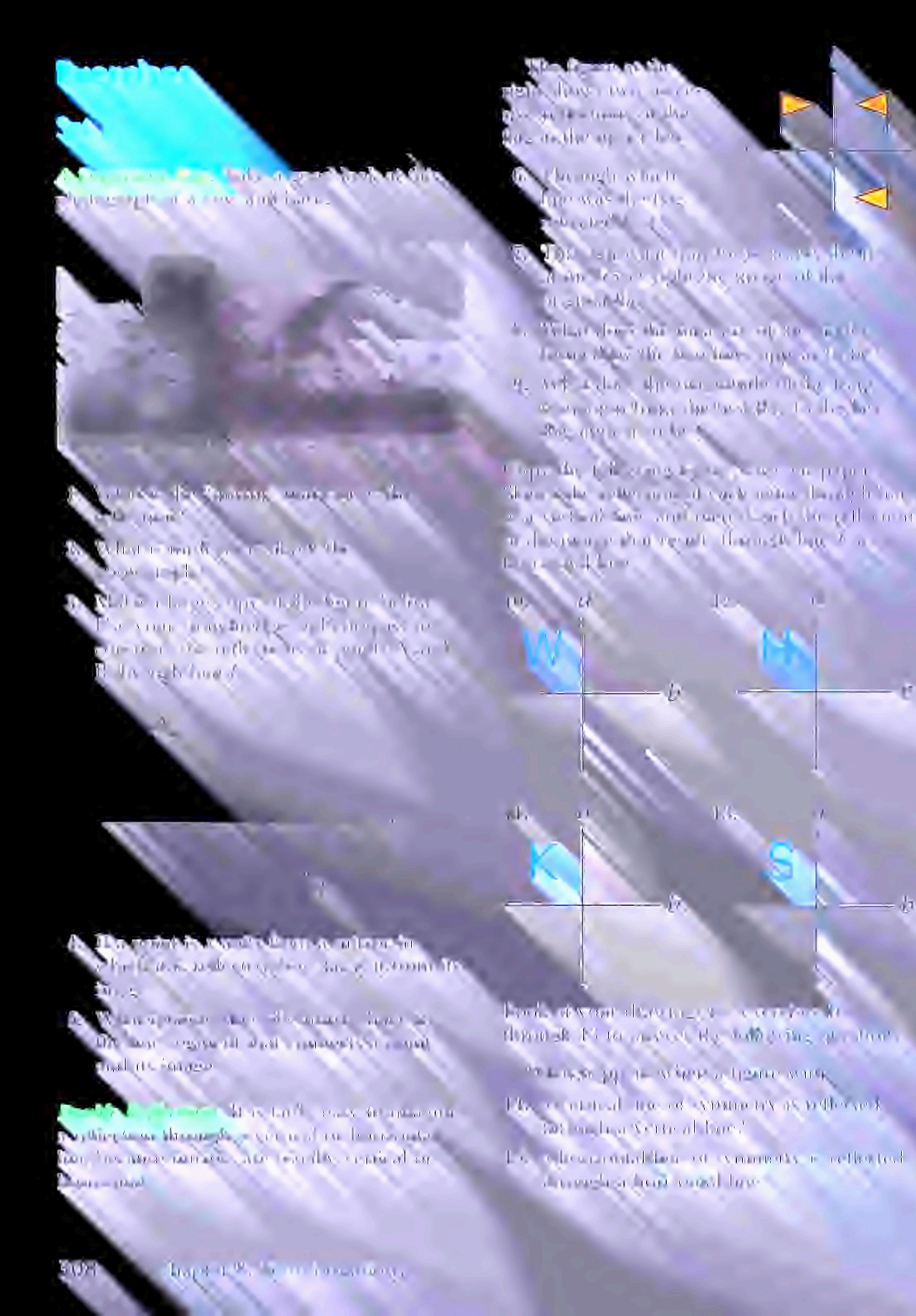



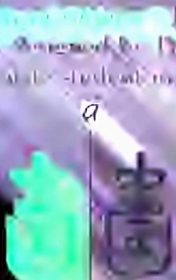
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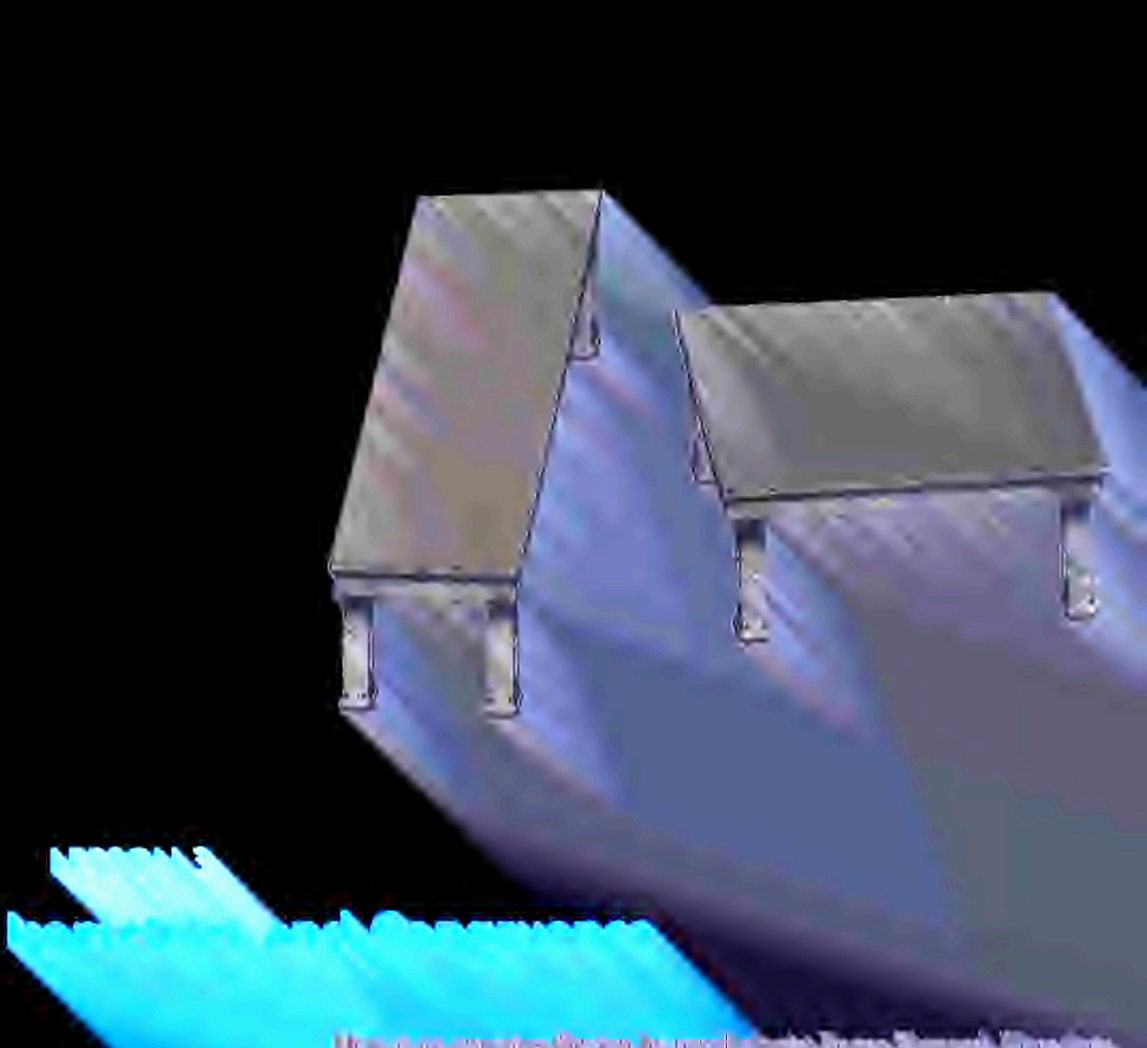

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Definition

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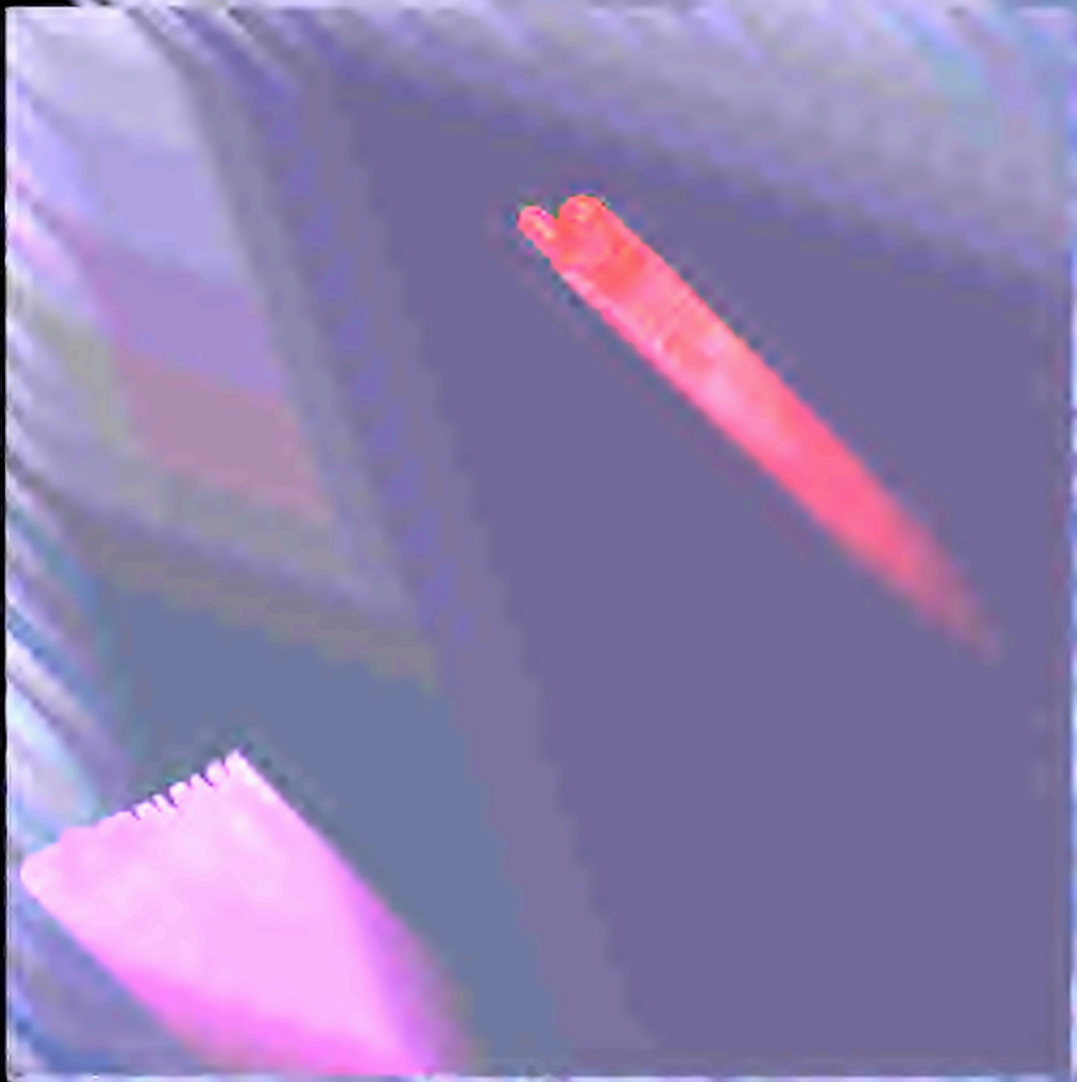
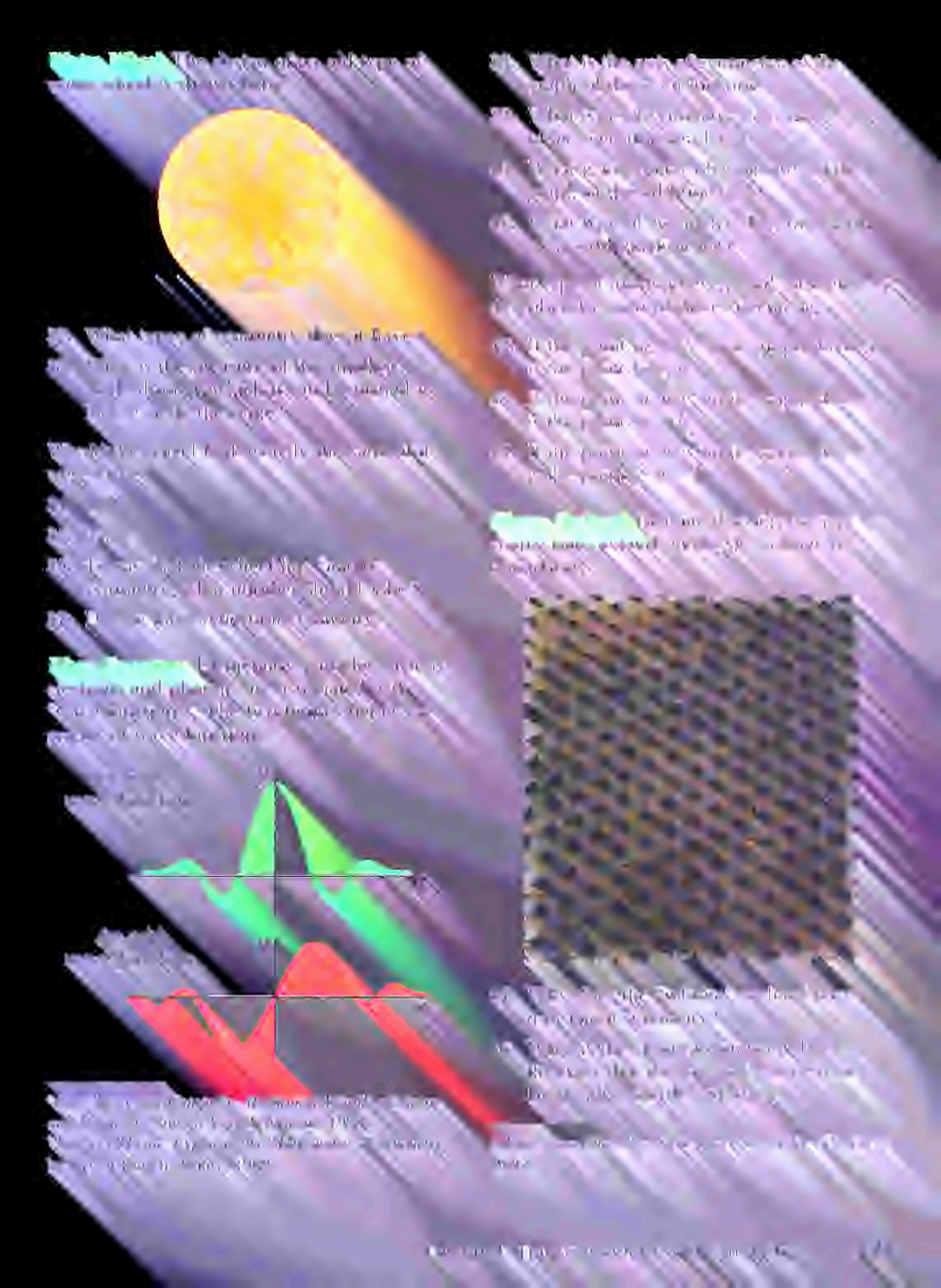
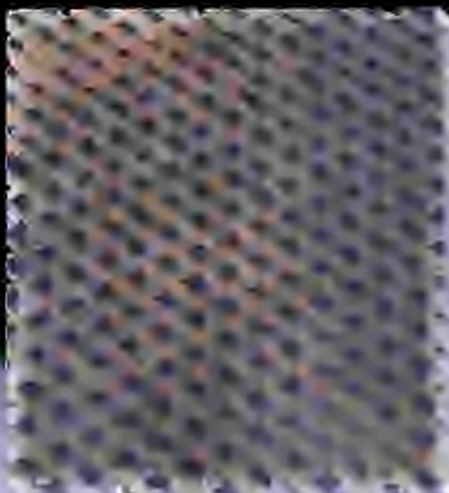








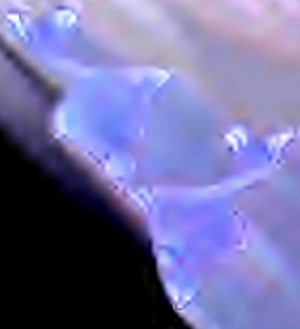




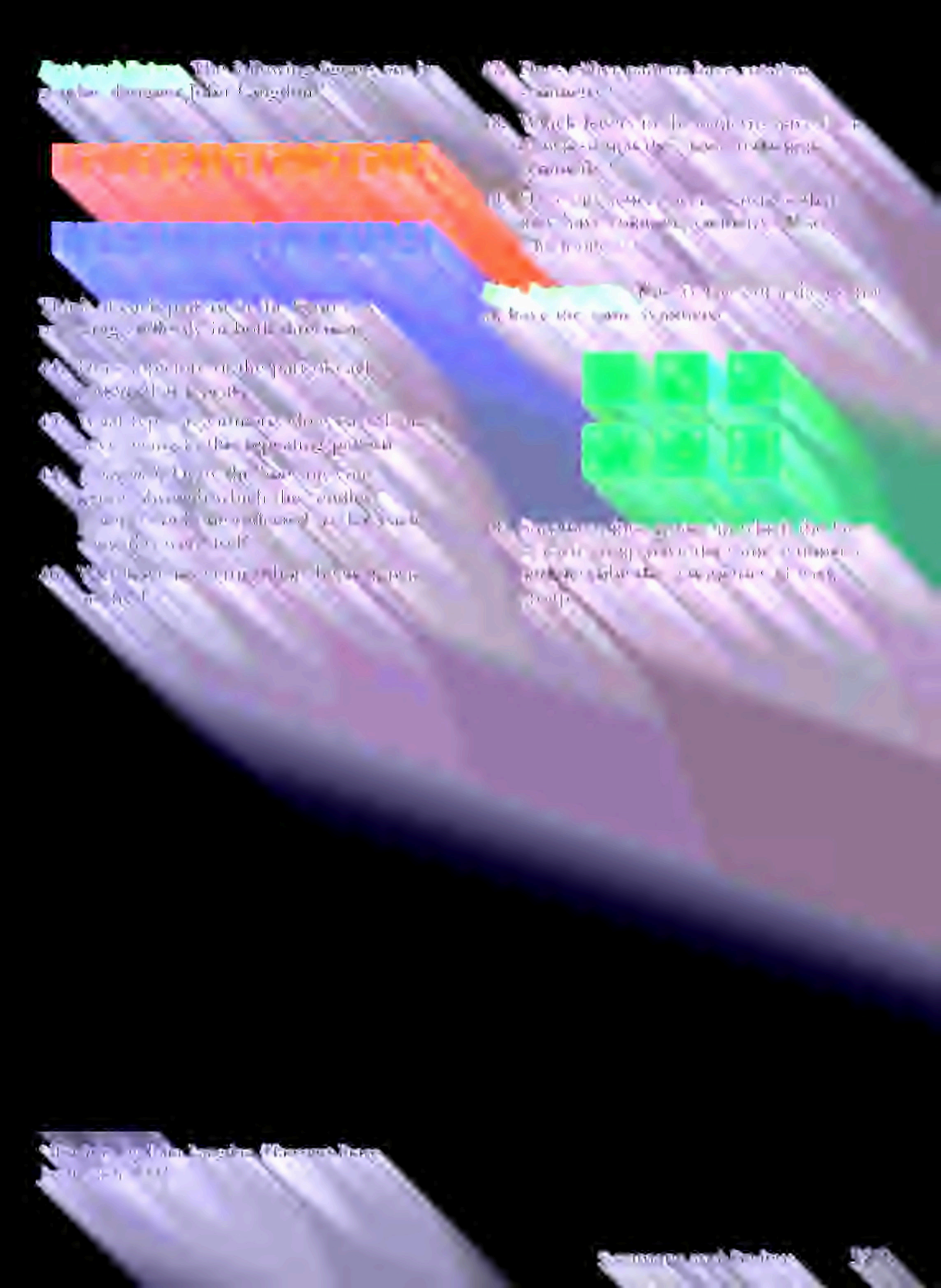


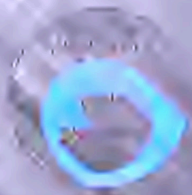


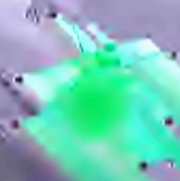


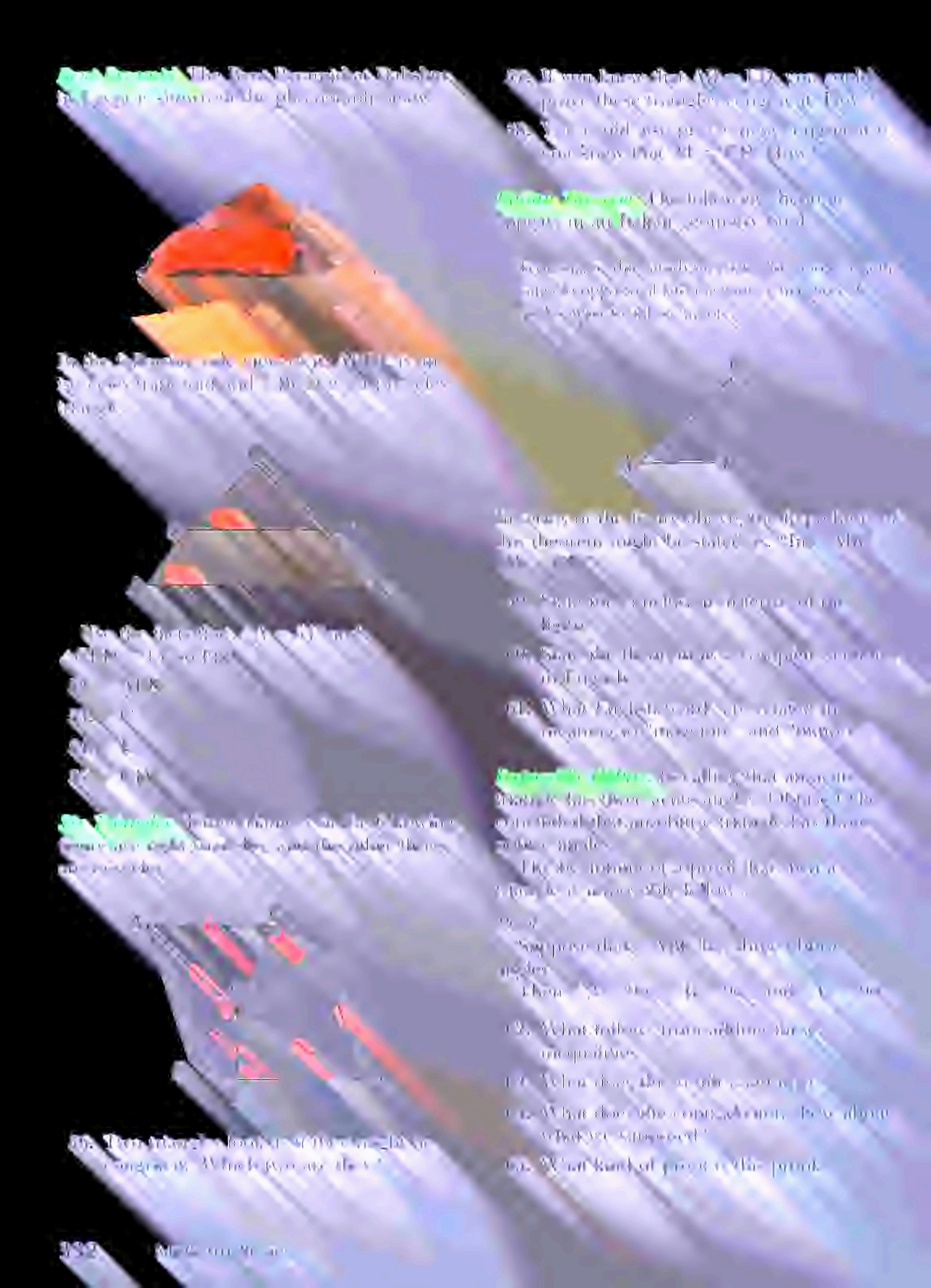


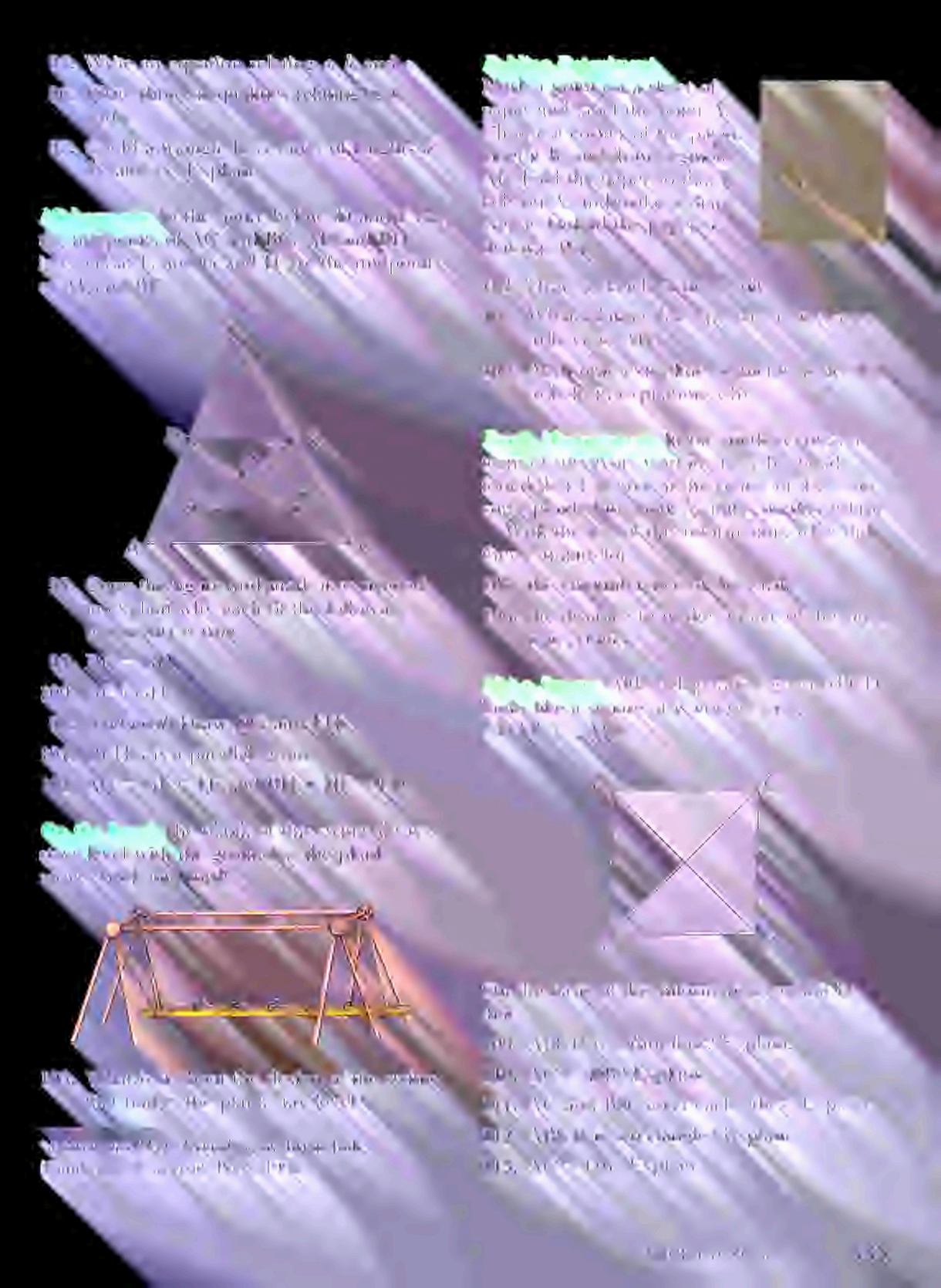


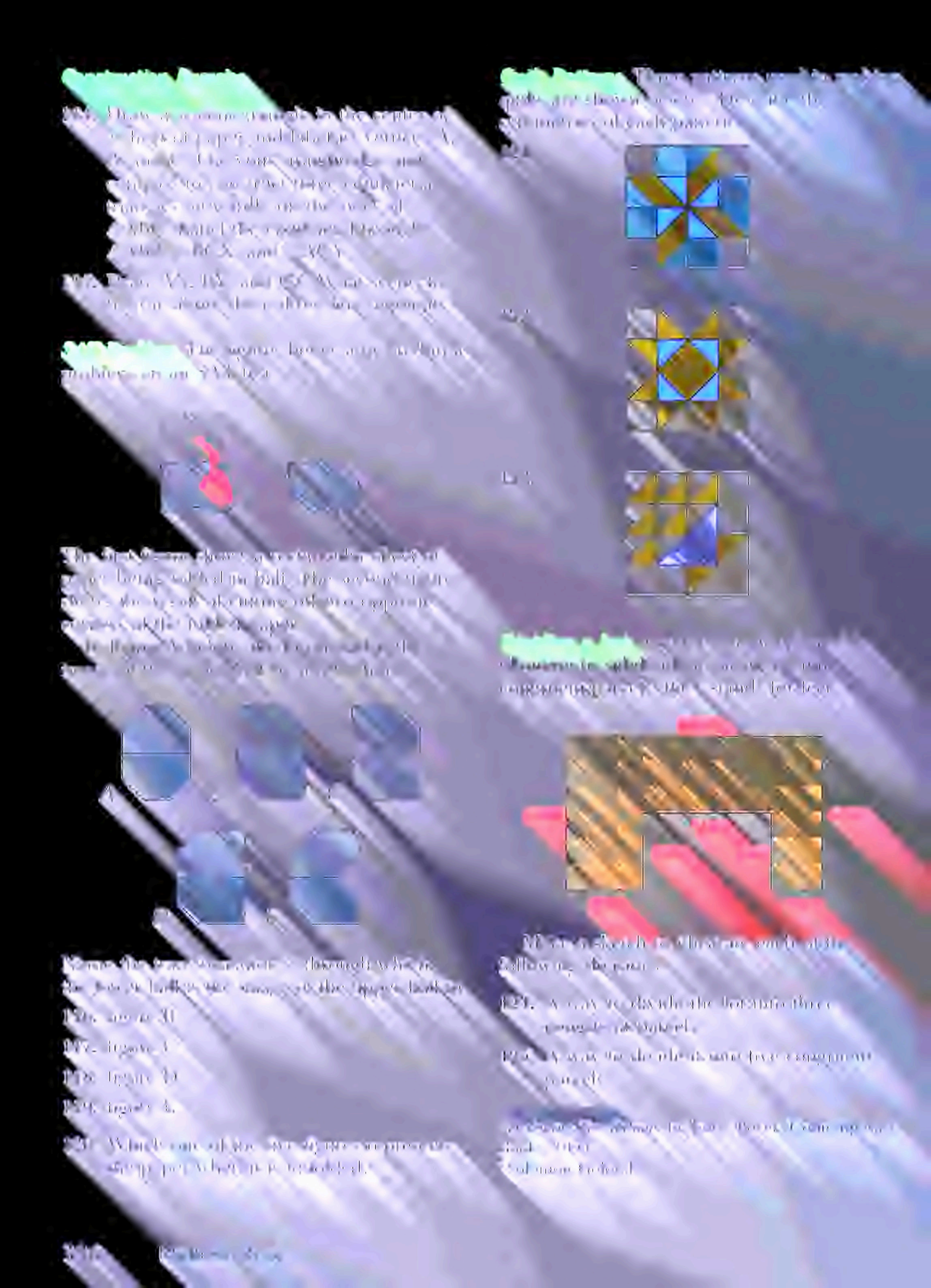


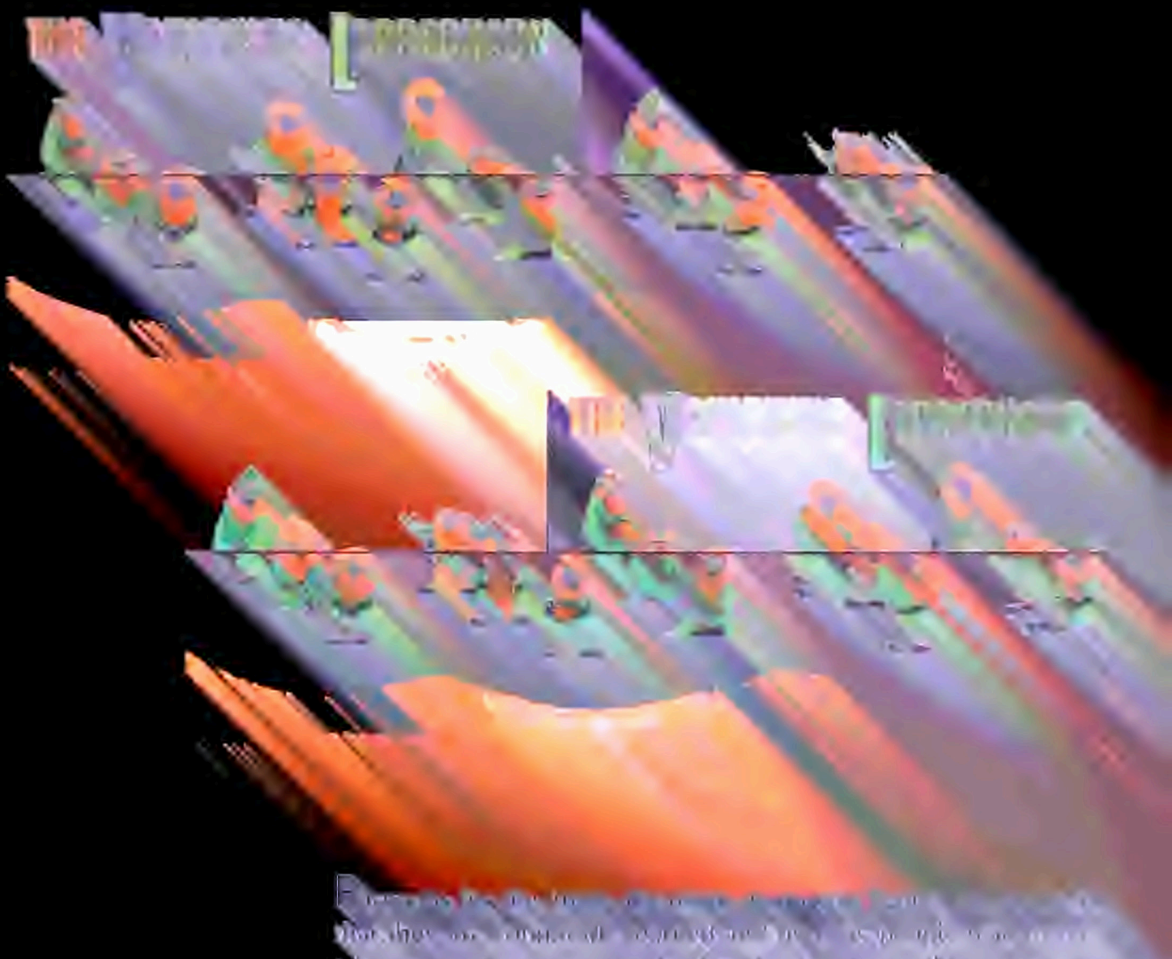












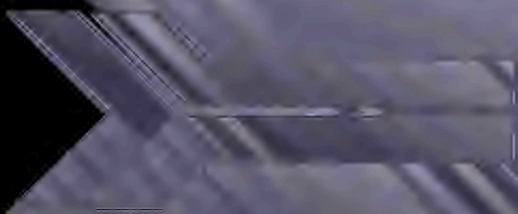
Exercises

Sets

Problem. The Amtrak symbol shown below consists of three [illegible] regions.

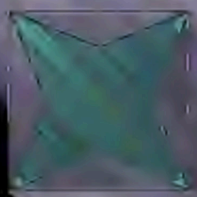


1. Consider a rotation of the figure below [illegible] parallelogram.



2. What type of parallelogram is [illegible]?
3. What [illegible] the Amtrak [illegible]?

Construction. The [illegible] shows a "star" drawn inside a square.



4. Which has the greater perimeter, the "star" or the square "Wiw"?
5. Which has the greater area? Why?
6. [illegible] greater perimeter [illegible] that it has a greater area.

Area Relations. ABCD is a parallelogram with diagonal AC. [illegible] parallelogram [illegible].



[illegible]

[illegible]

Area Relations. [illegible]



[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

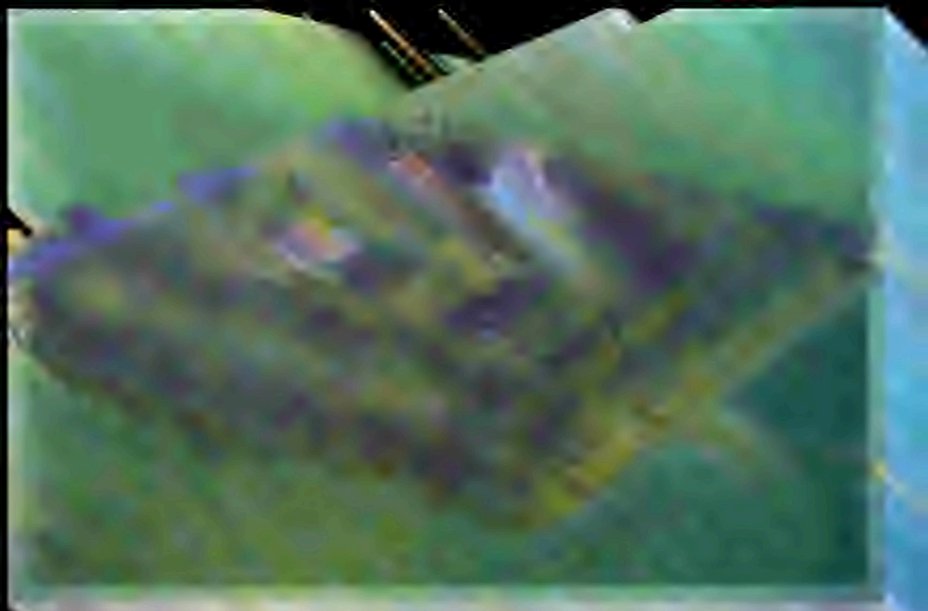
[illegible]

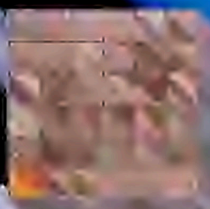
[illegible]

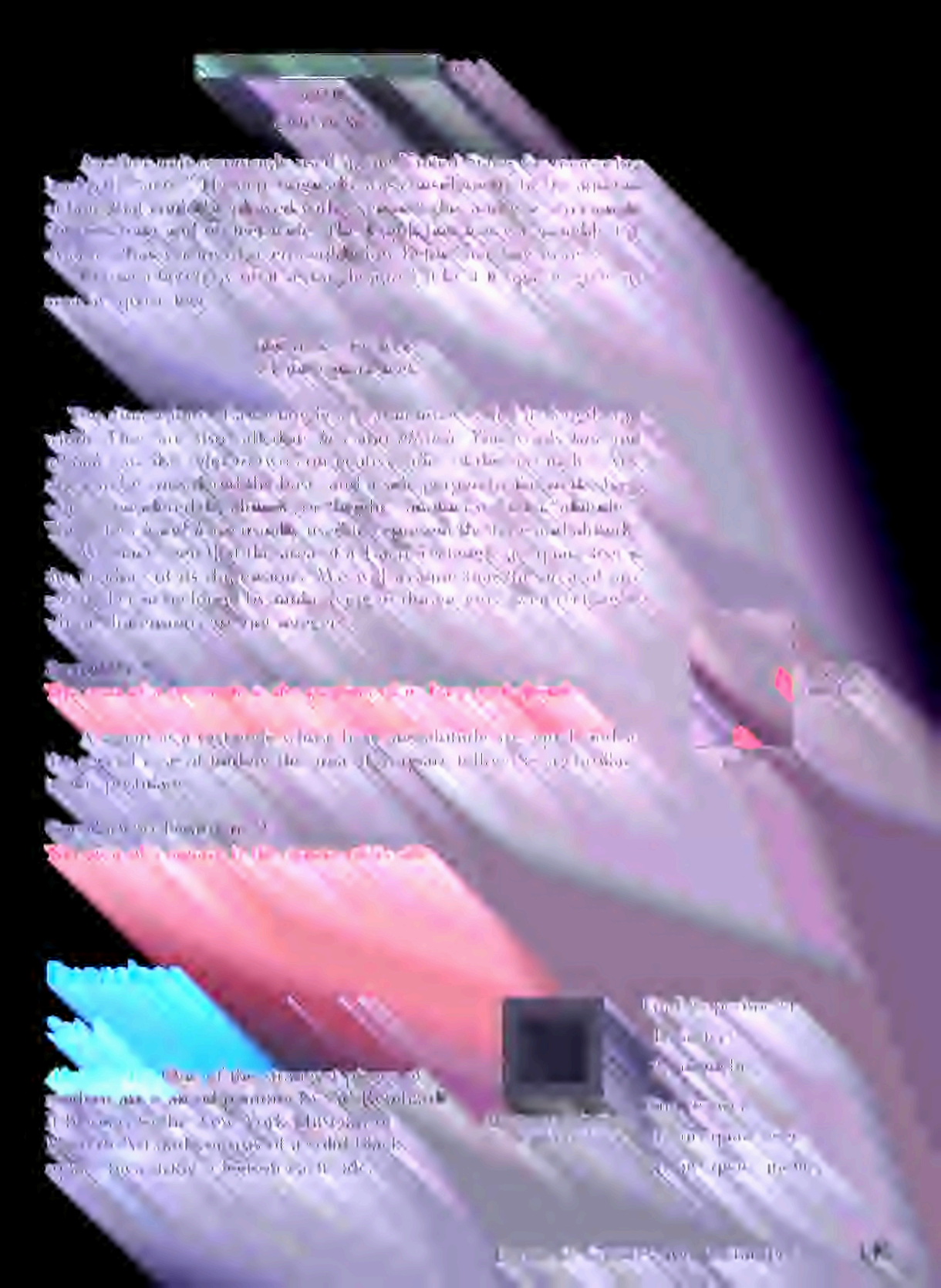




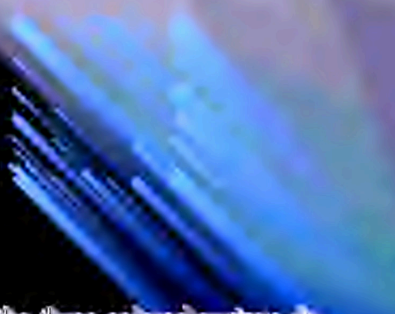








[illegible] [illegible]



5. Which of the three [illegible] do you think has the [illegible]?
6. Find the area of each region.

[illegible] [illegible]

[illegible] [illegible]



17. [illegible]
18. [illegible]
19. [illegible]

[illegible] [illegible]



20. [illegible]
21. [illegible]
22. [illegible]



23. [illegible]

[illegible]

7. [illegible]
8. [illegible]
9. [illegible]
10. [illegible]





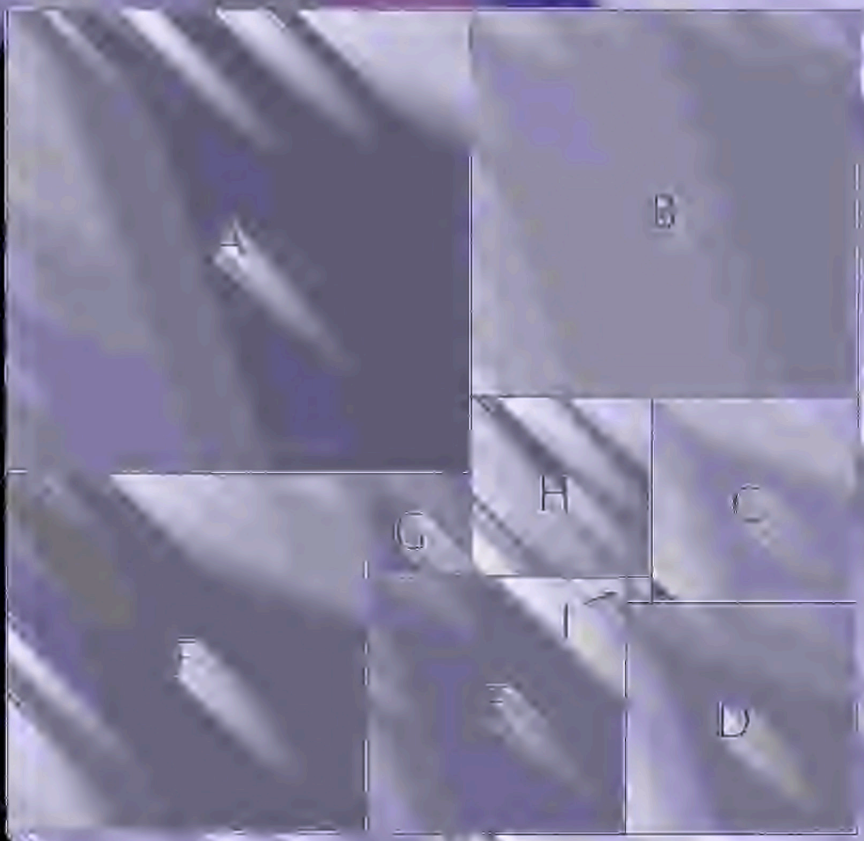



SOS
ANGELES



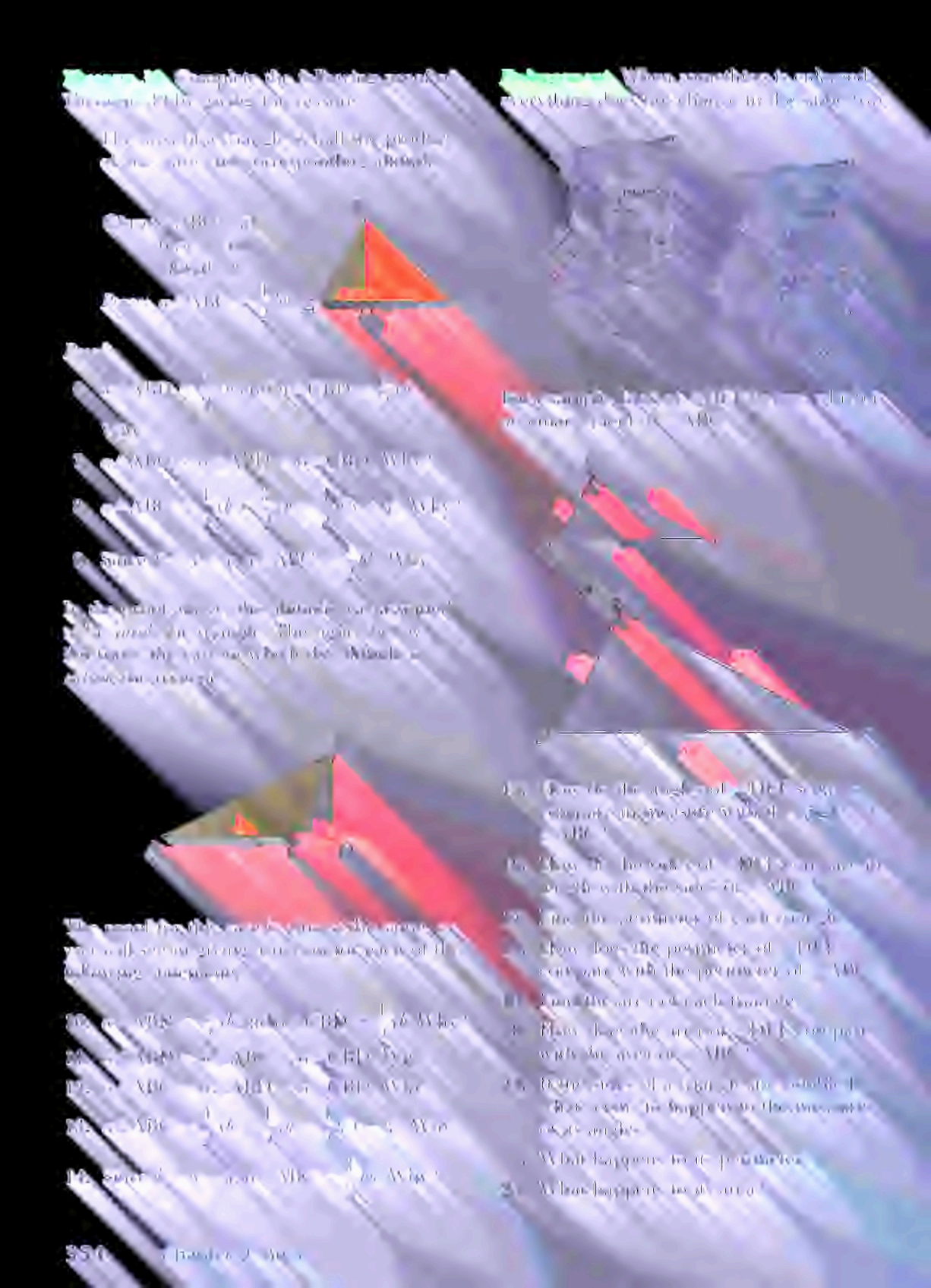





A
B
H
C
G
I
F
E
D

Triangles

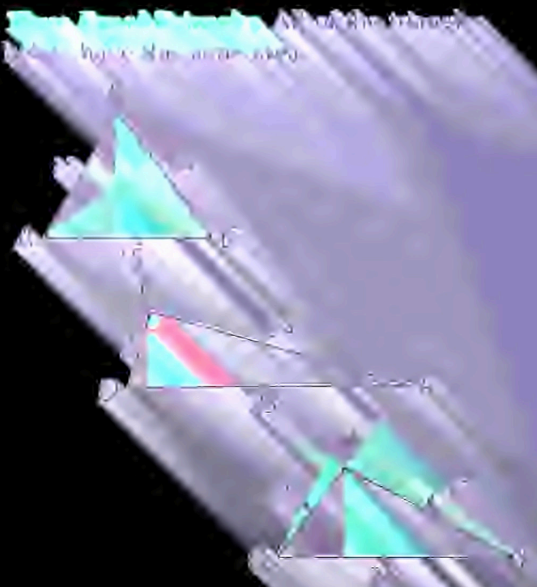




















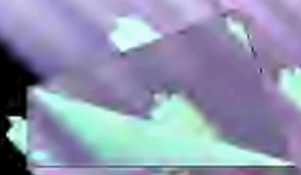






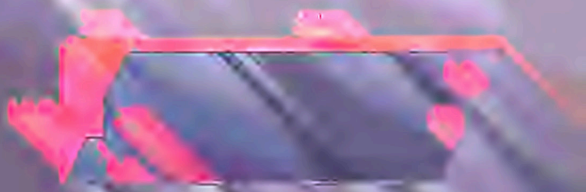






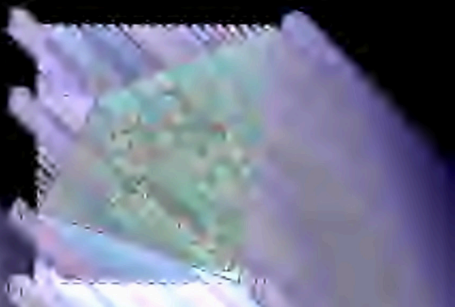










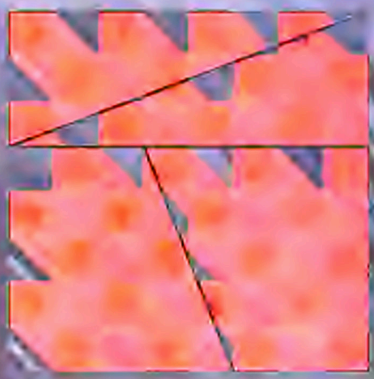



















C
D











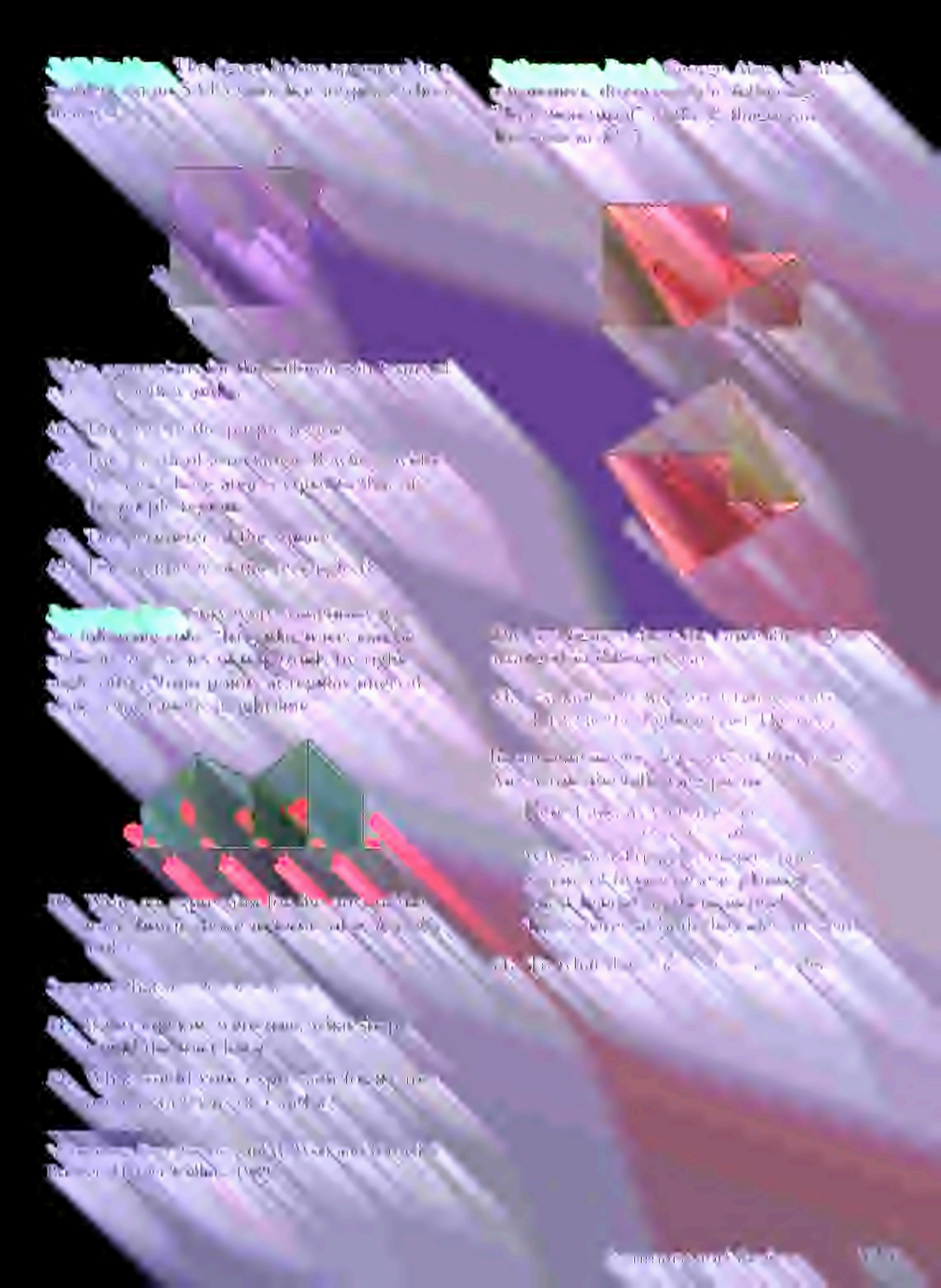


















Ratio and Proportion

[illegible]

[illegible]

[illegible]

[illegible]















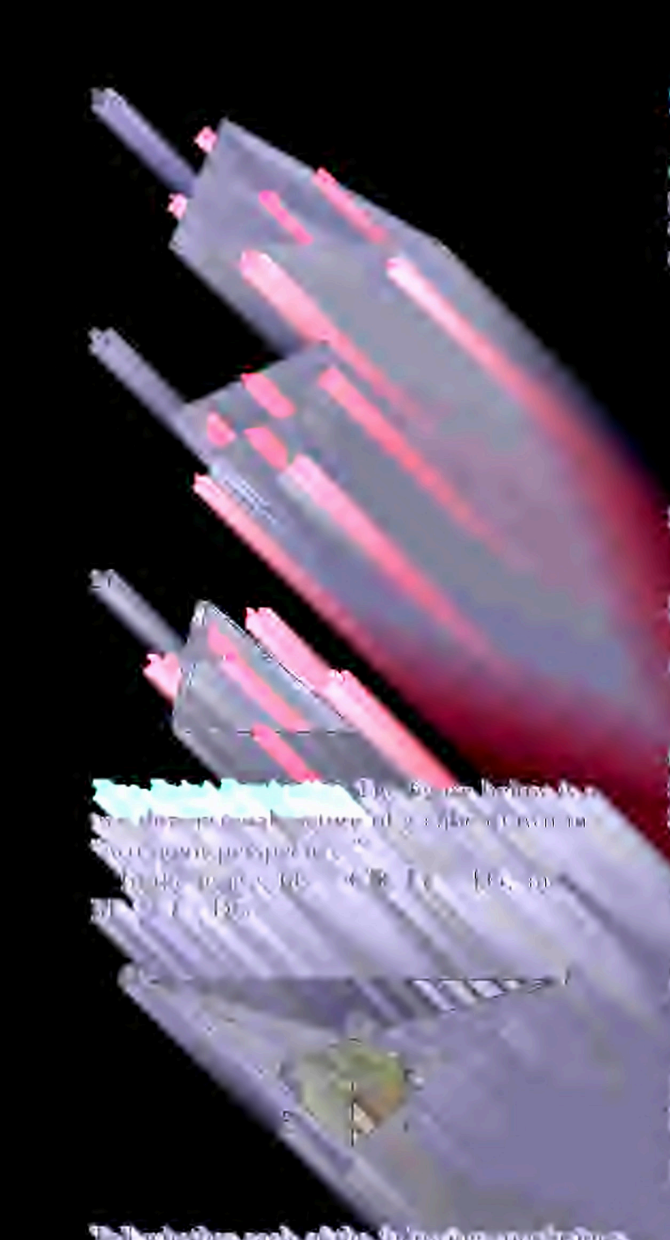


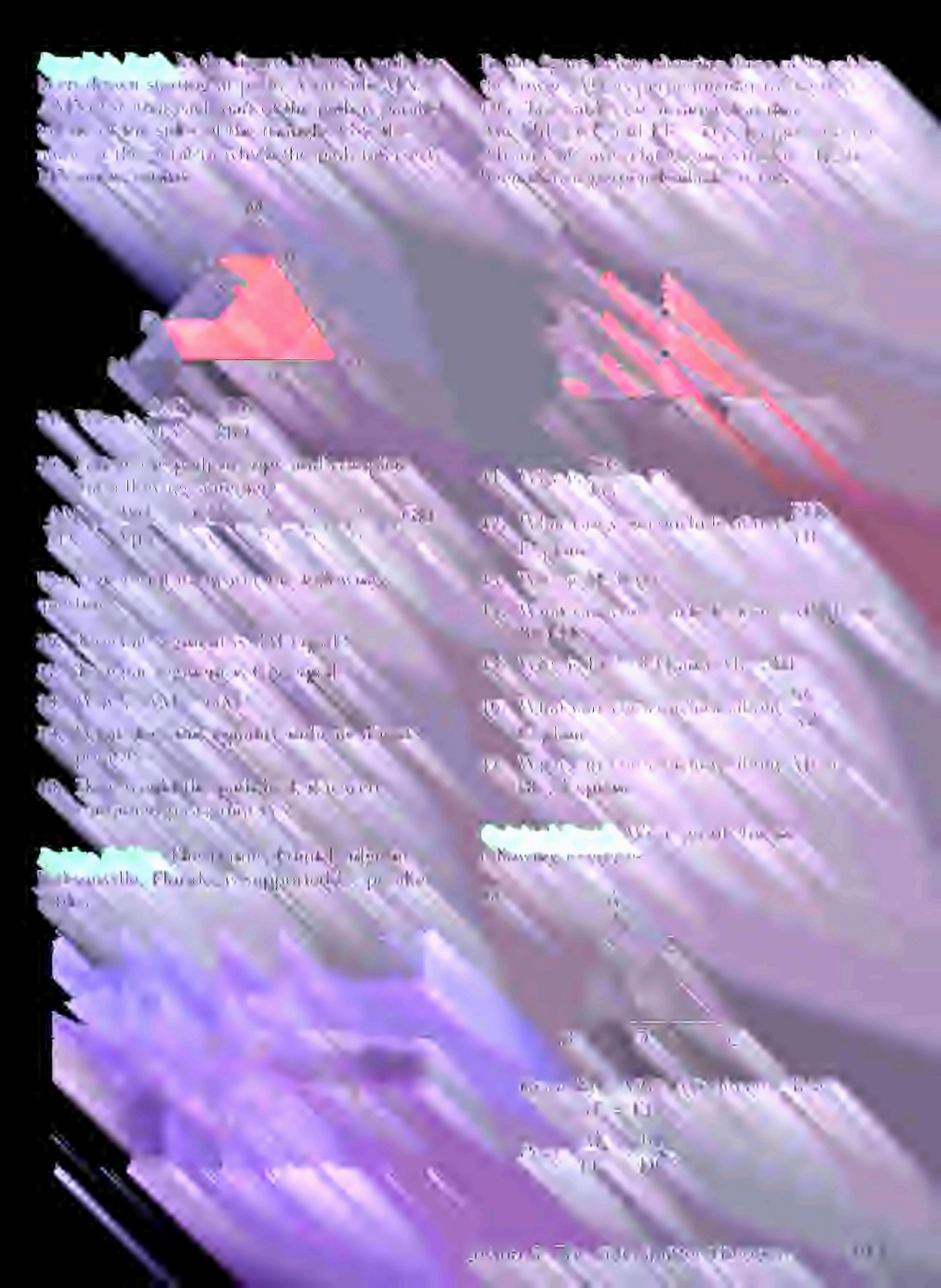




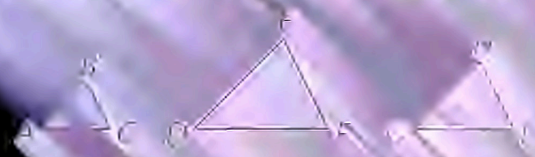


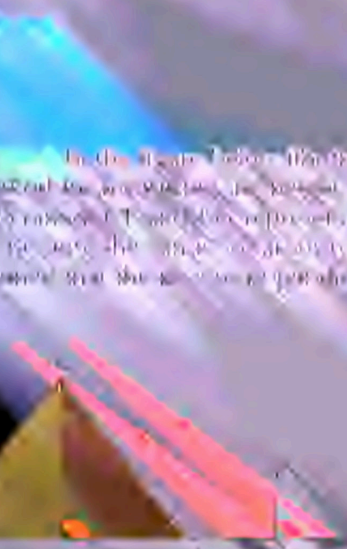












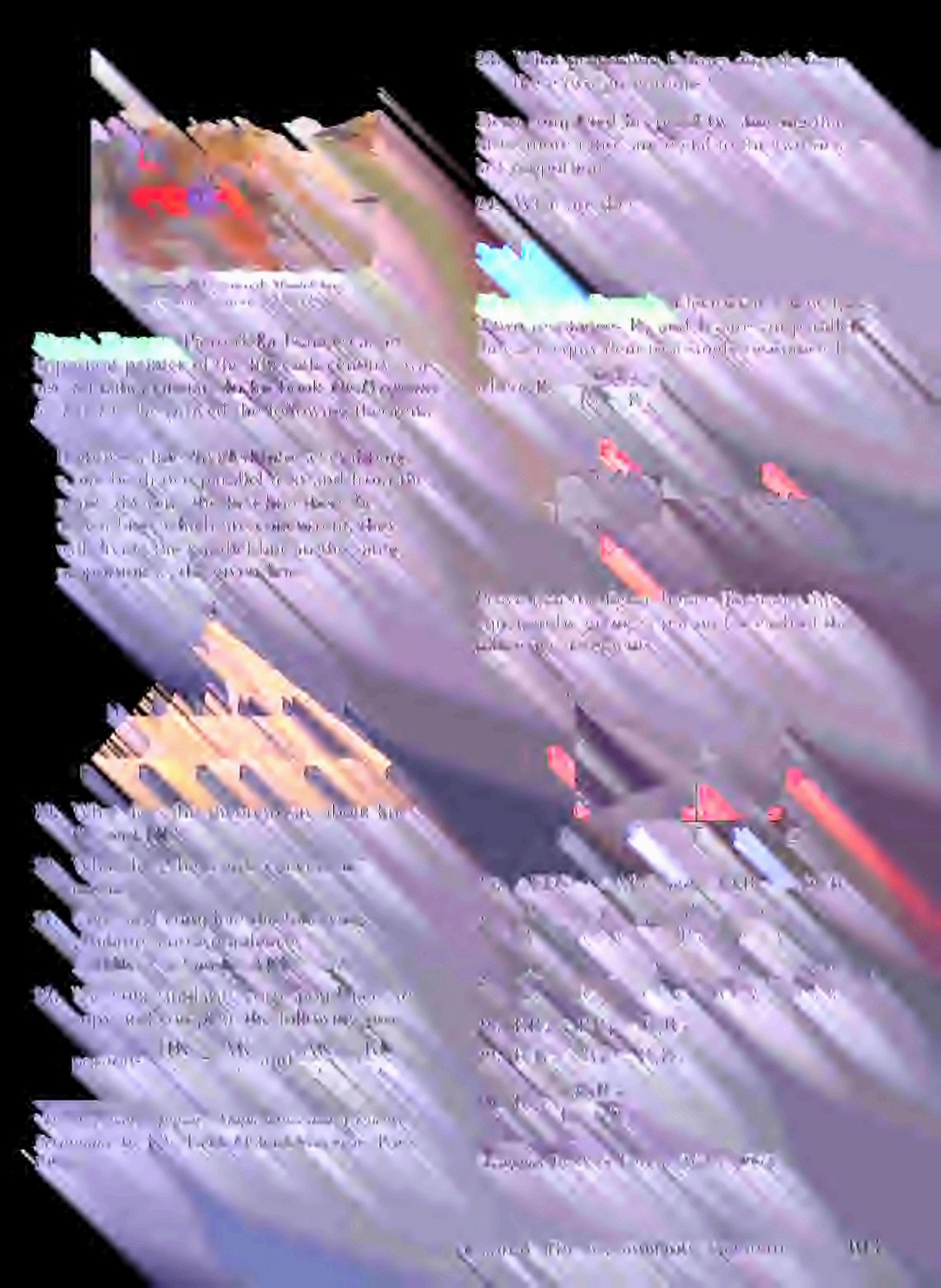


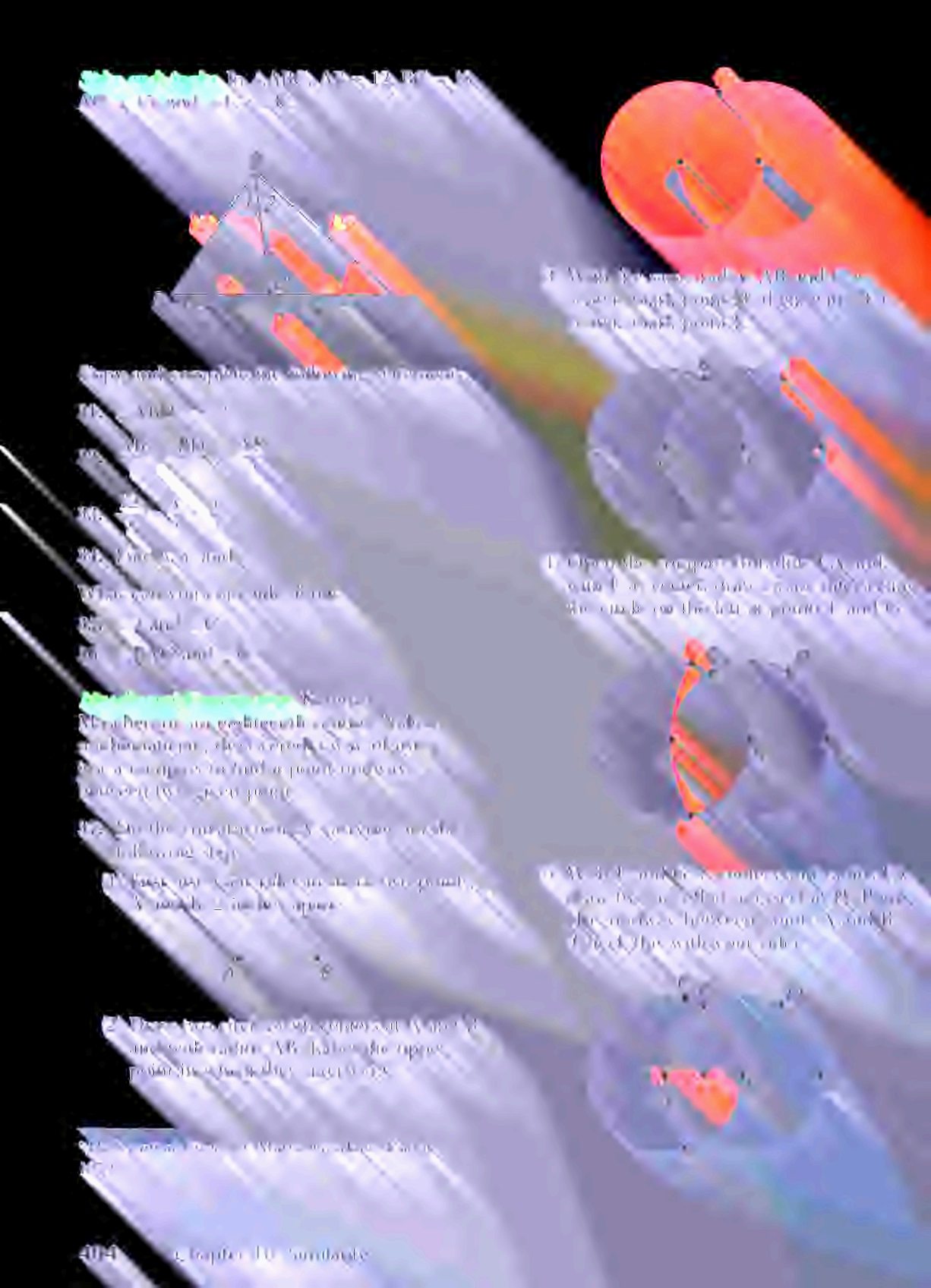


















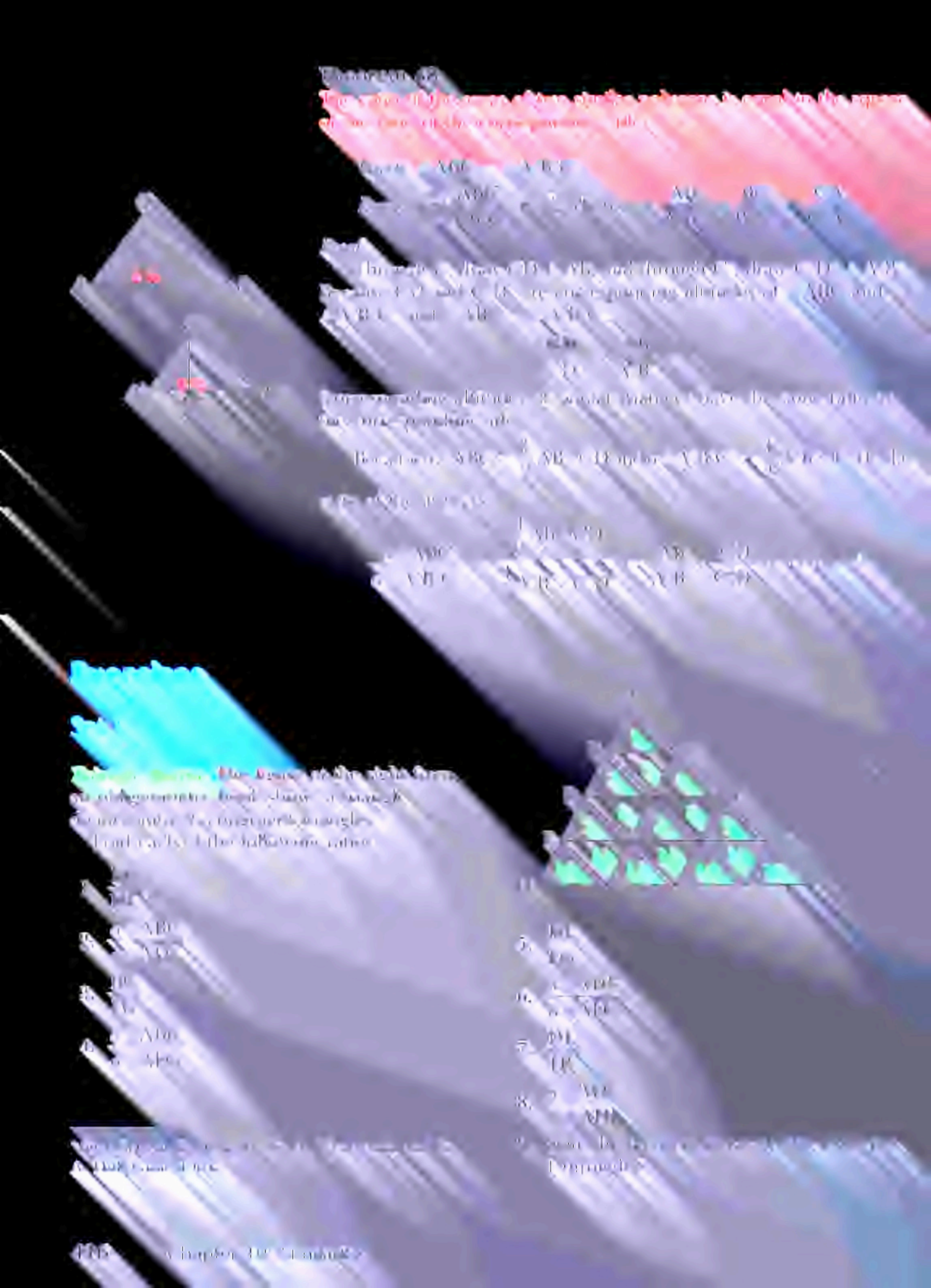


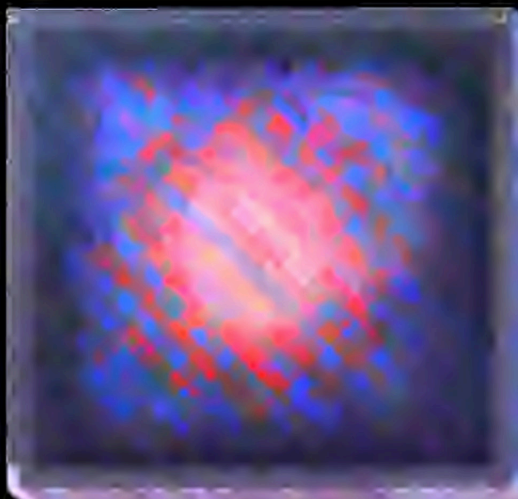


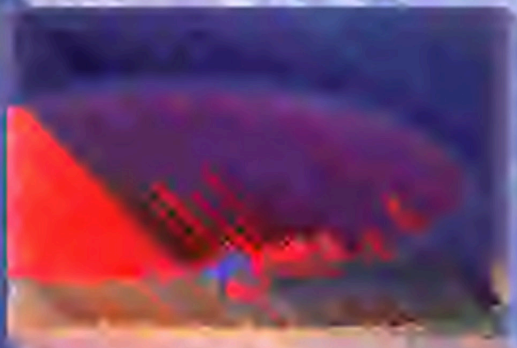


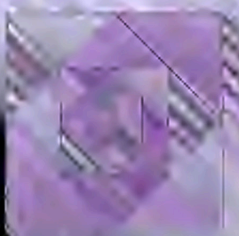








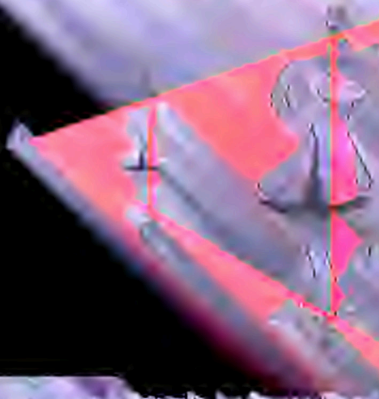




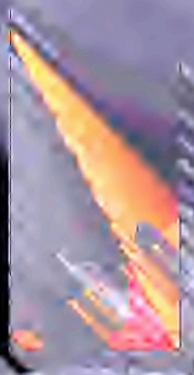


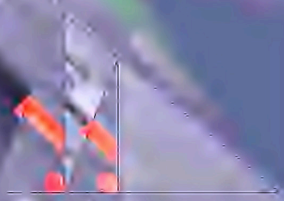


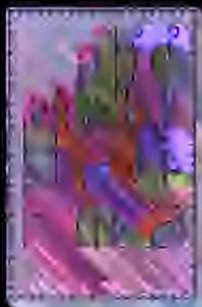












Solving Formulas

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Exercises

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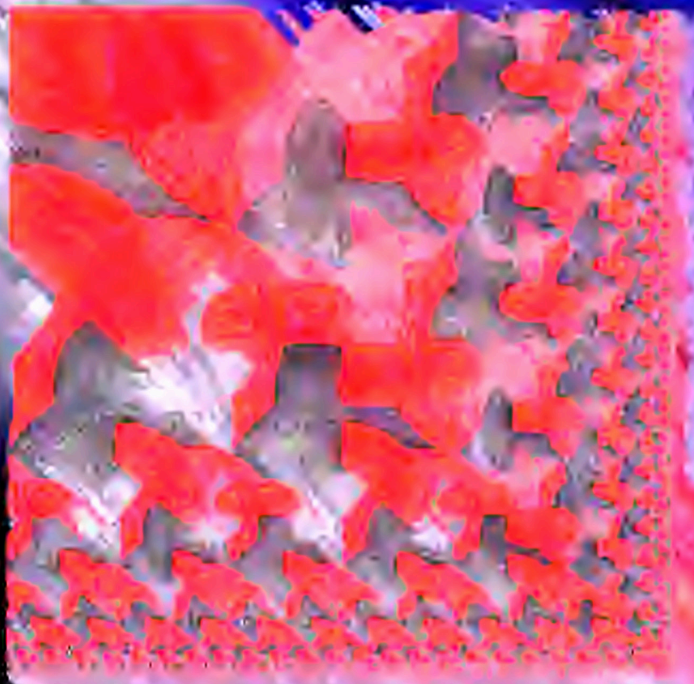
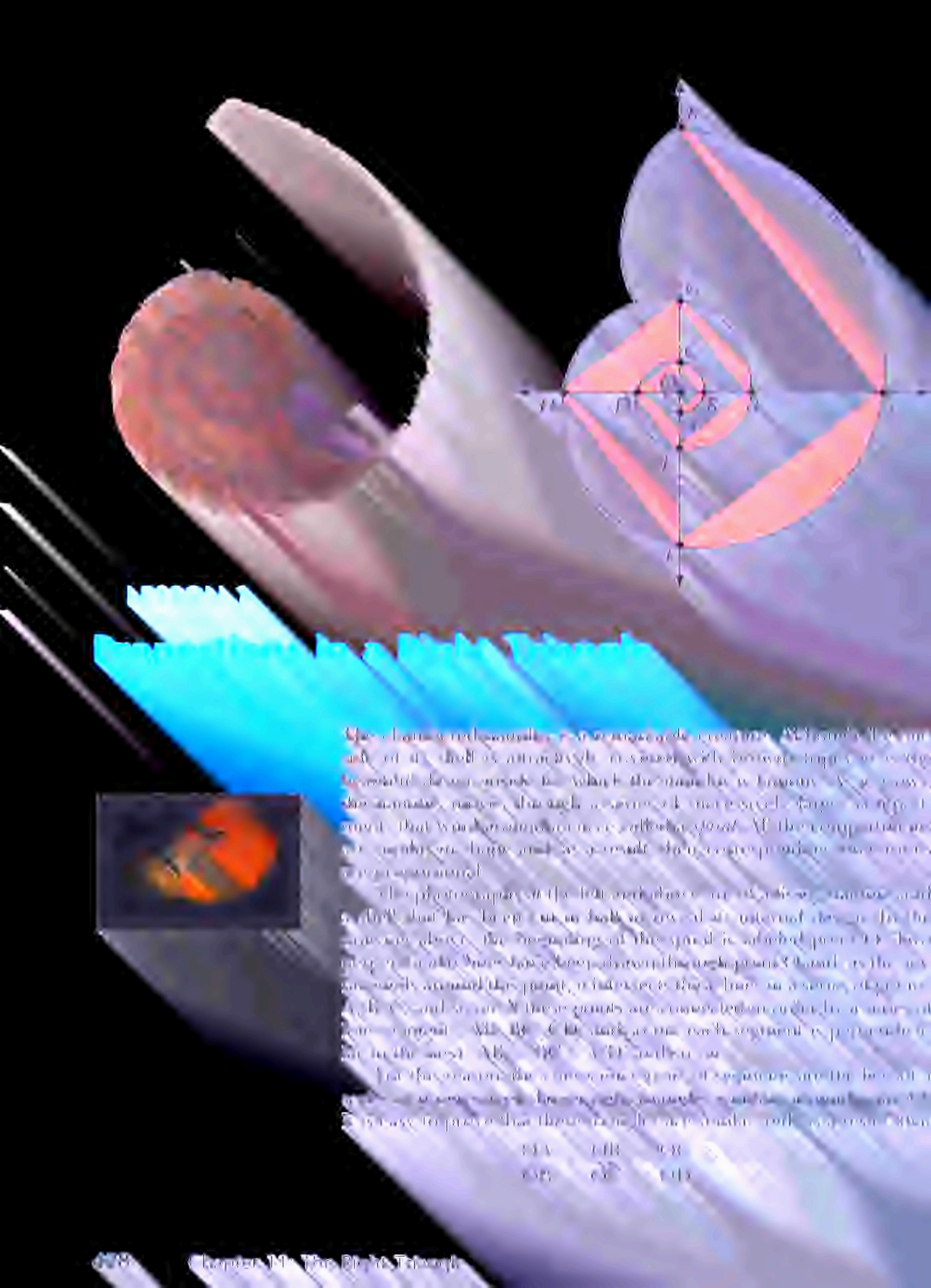
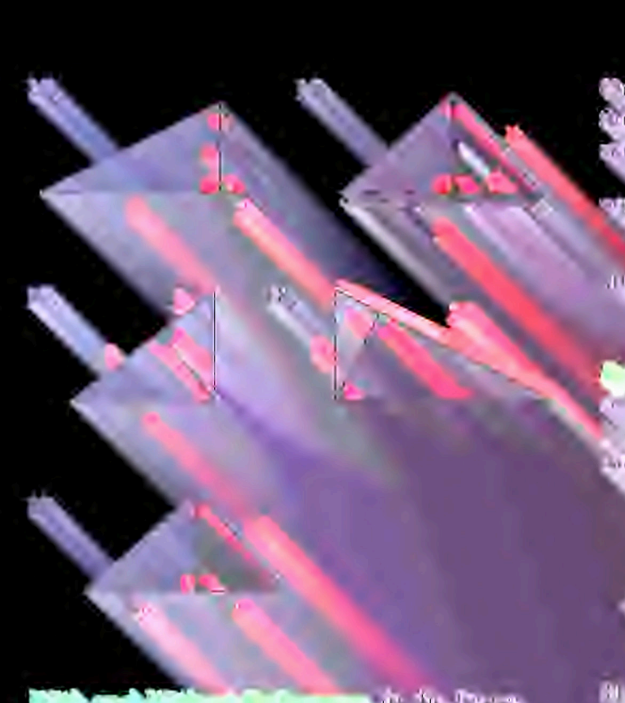
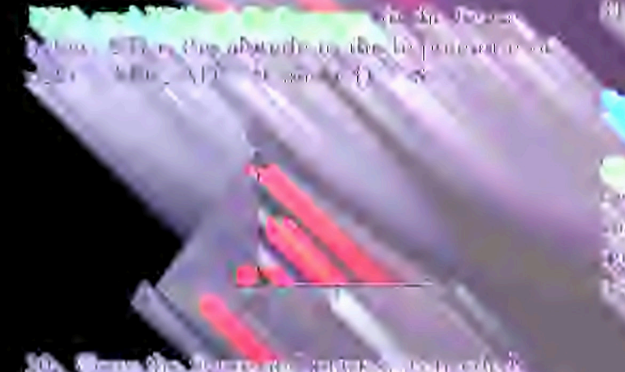
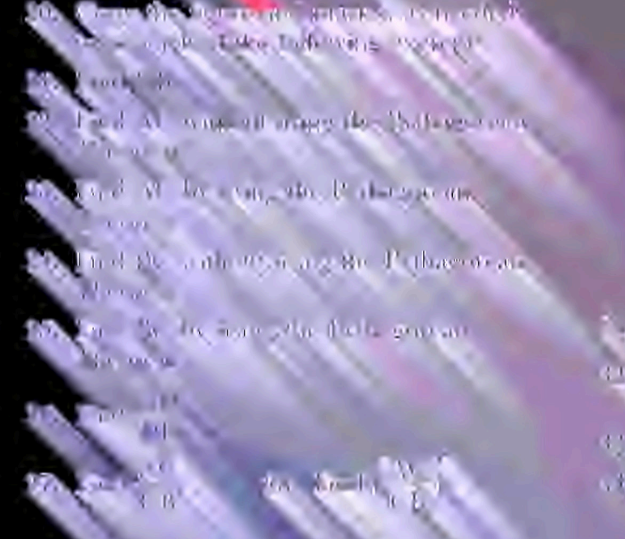




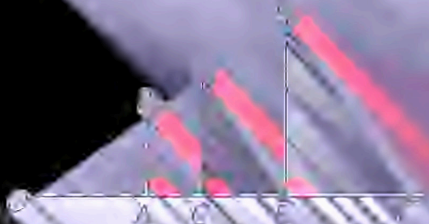




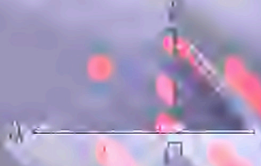




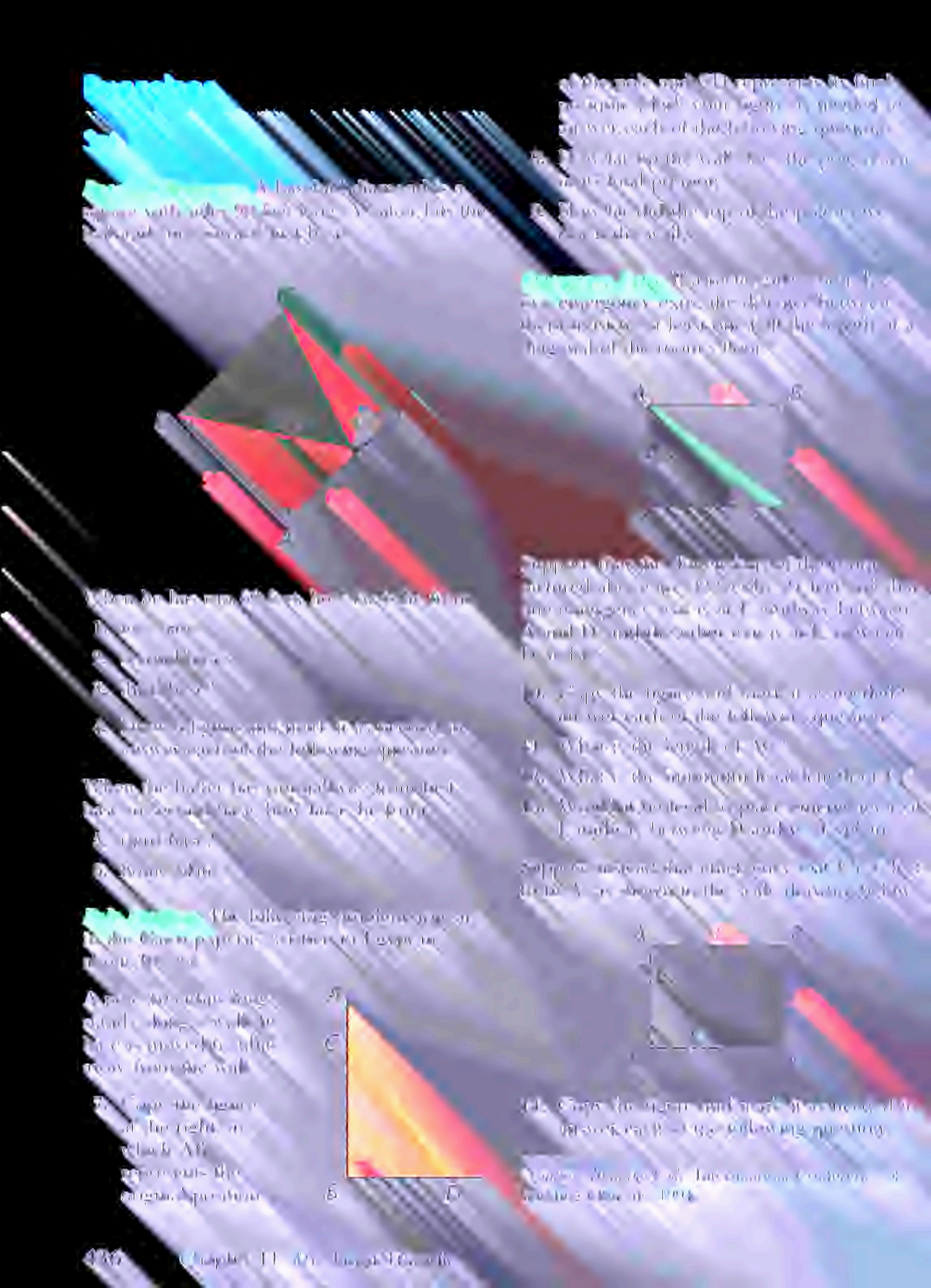


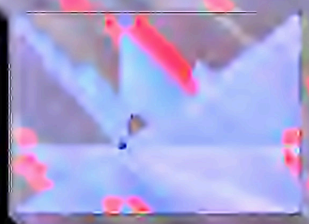




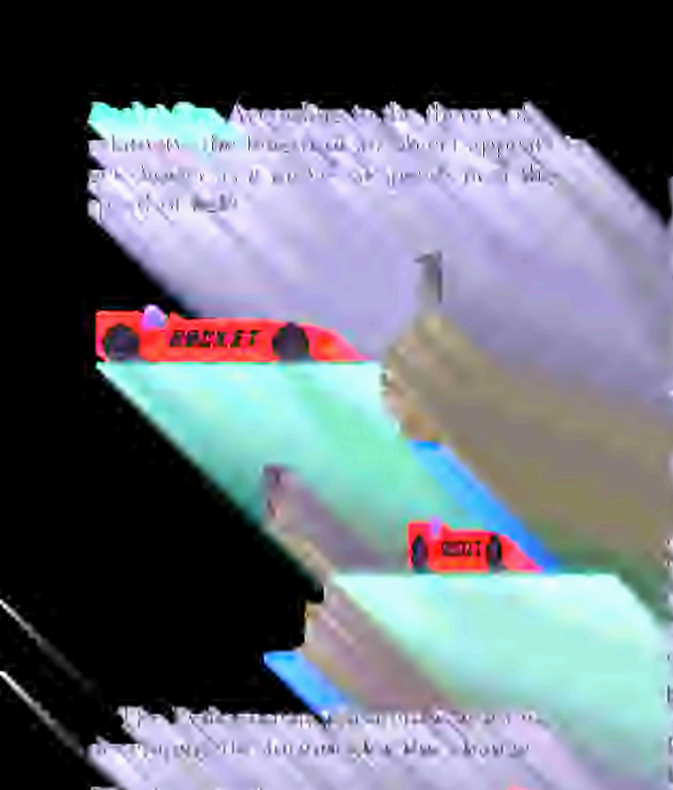


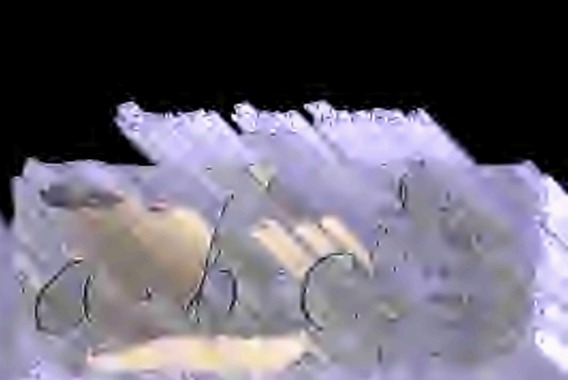









ROCKET







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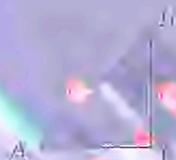



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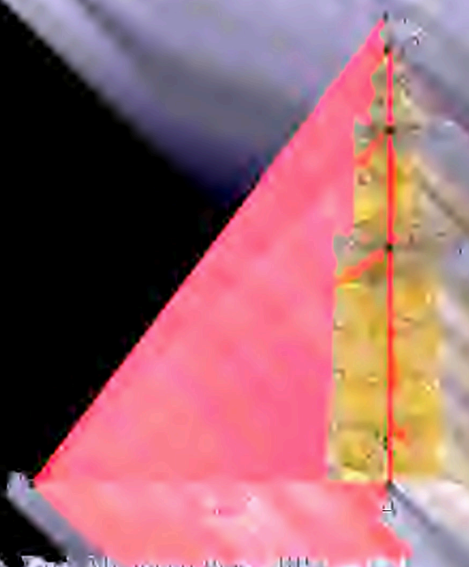

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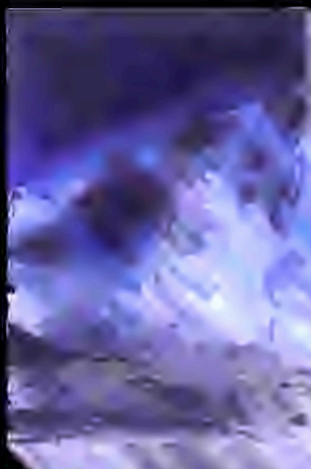
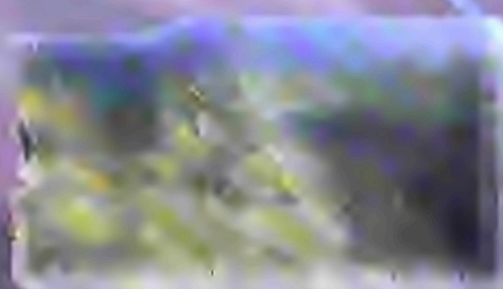
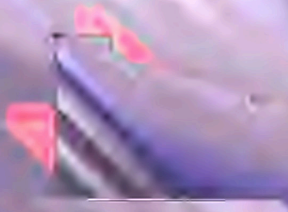
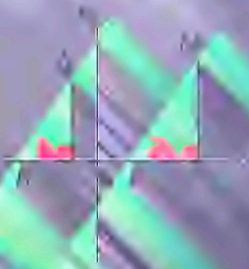






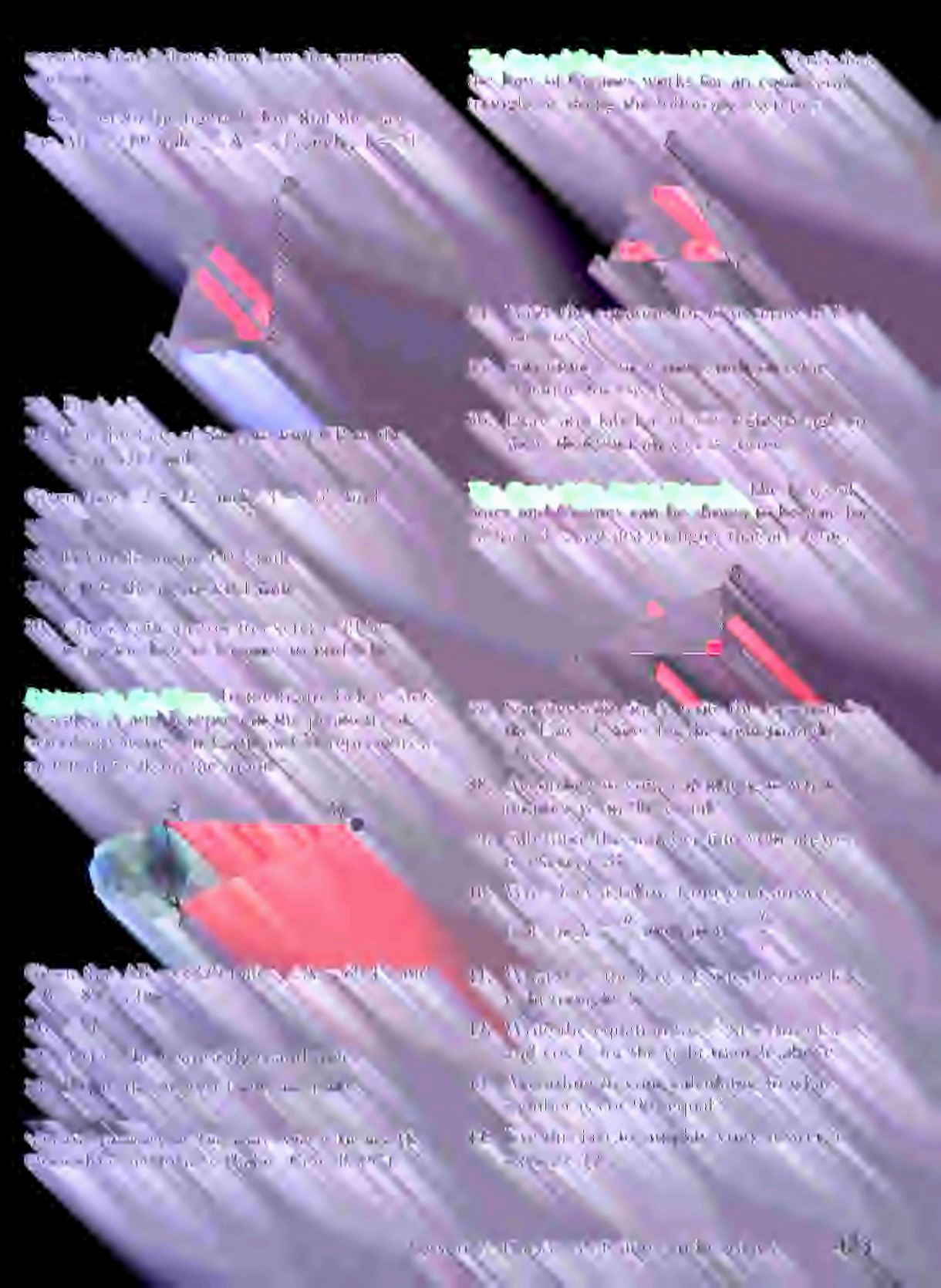




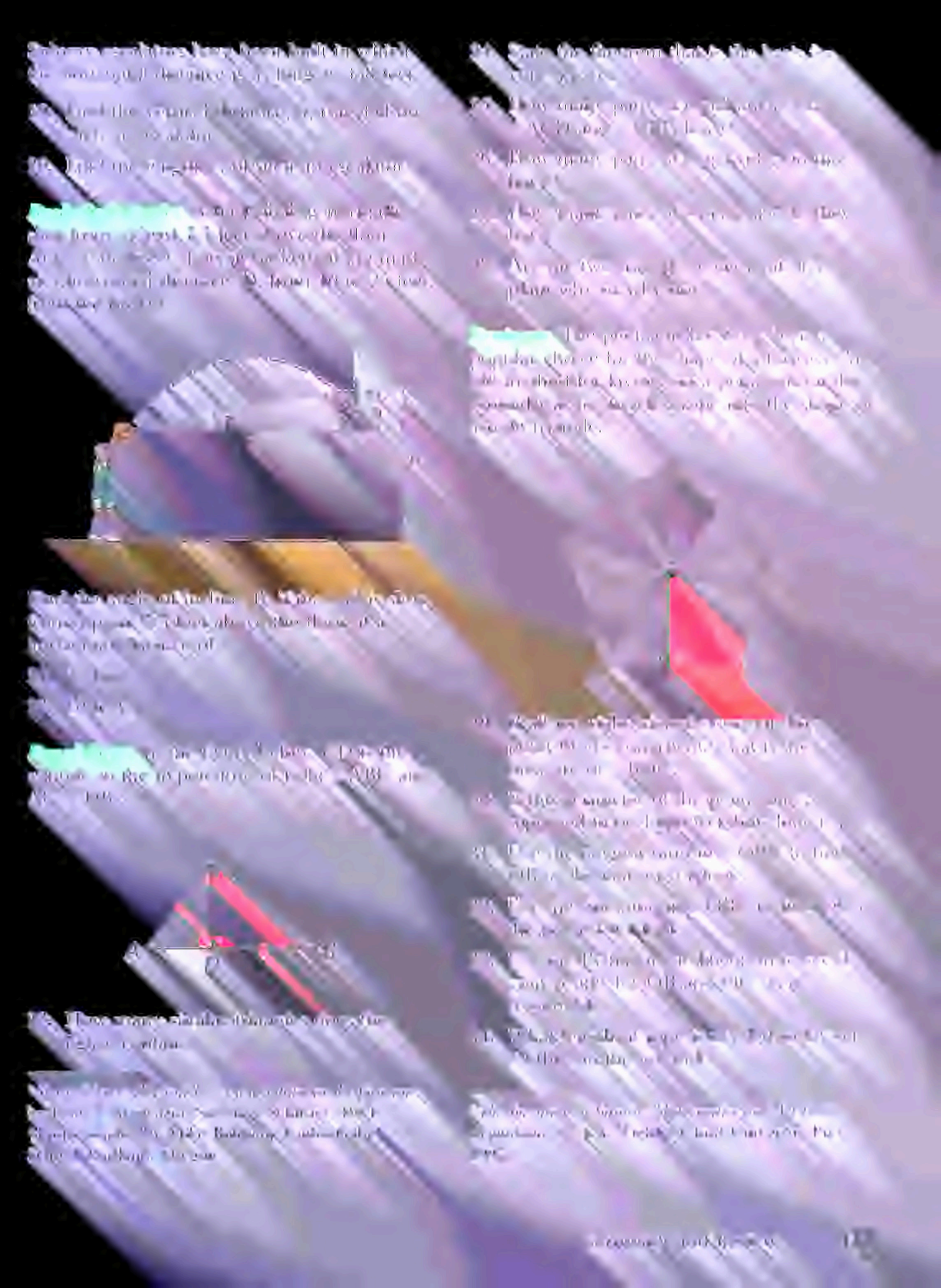














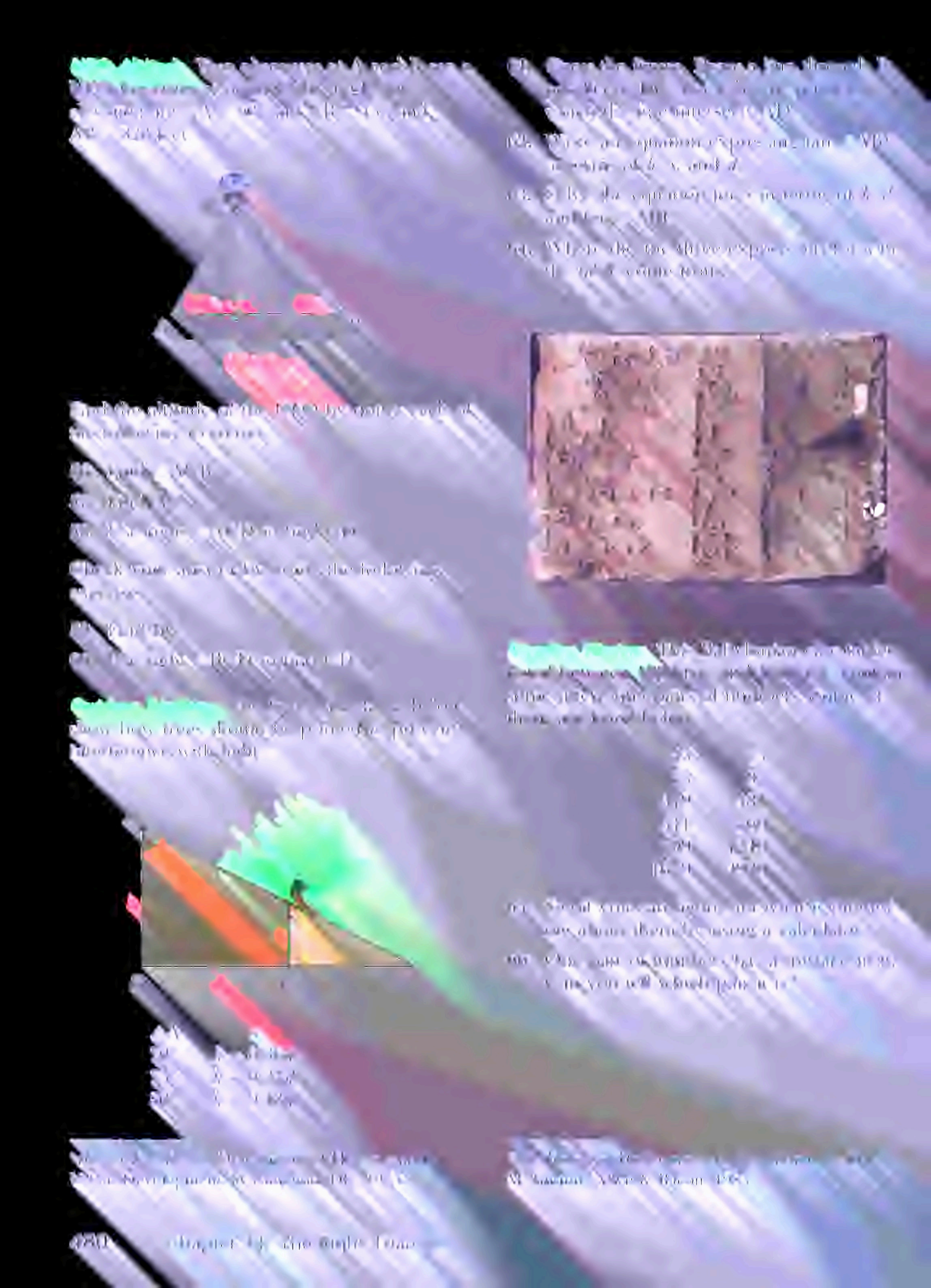






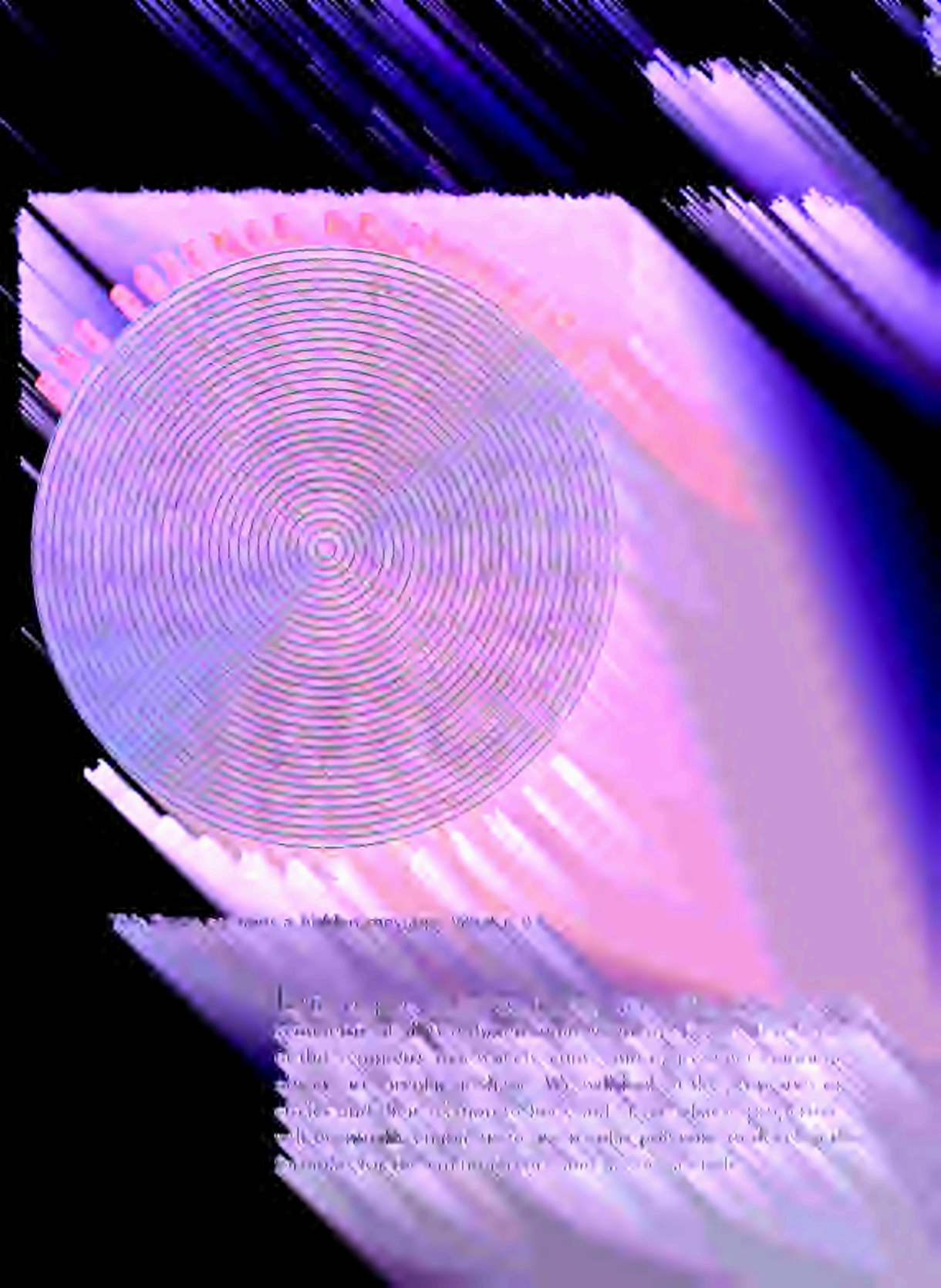






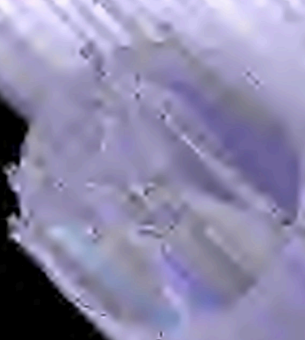


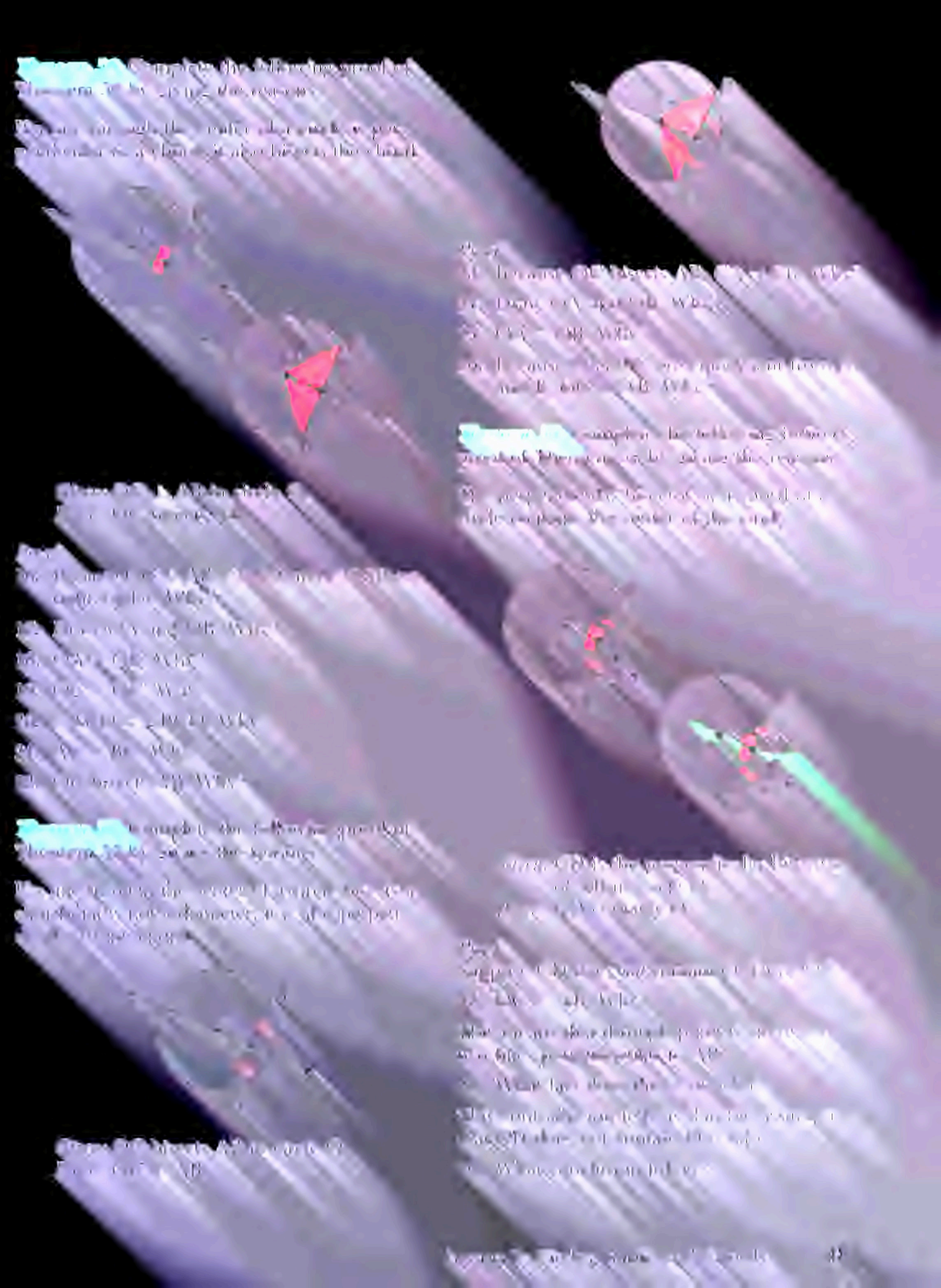


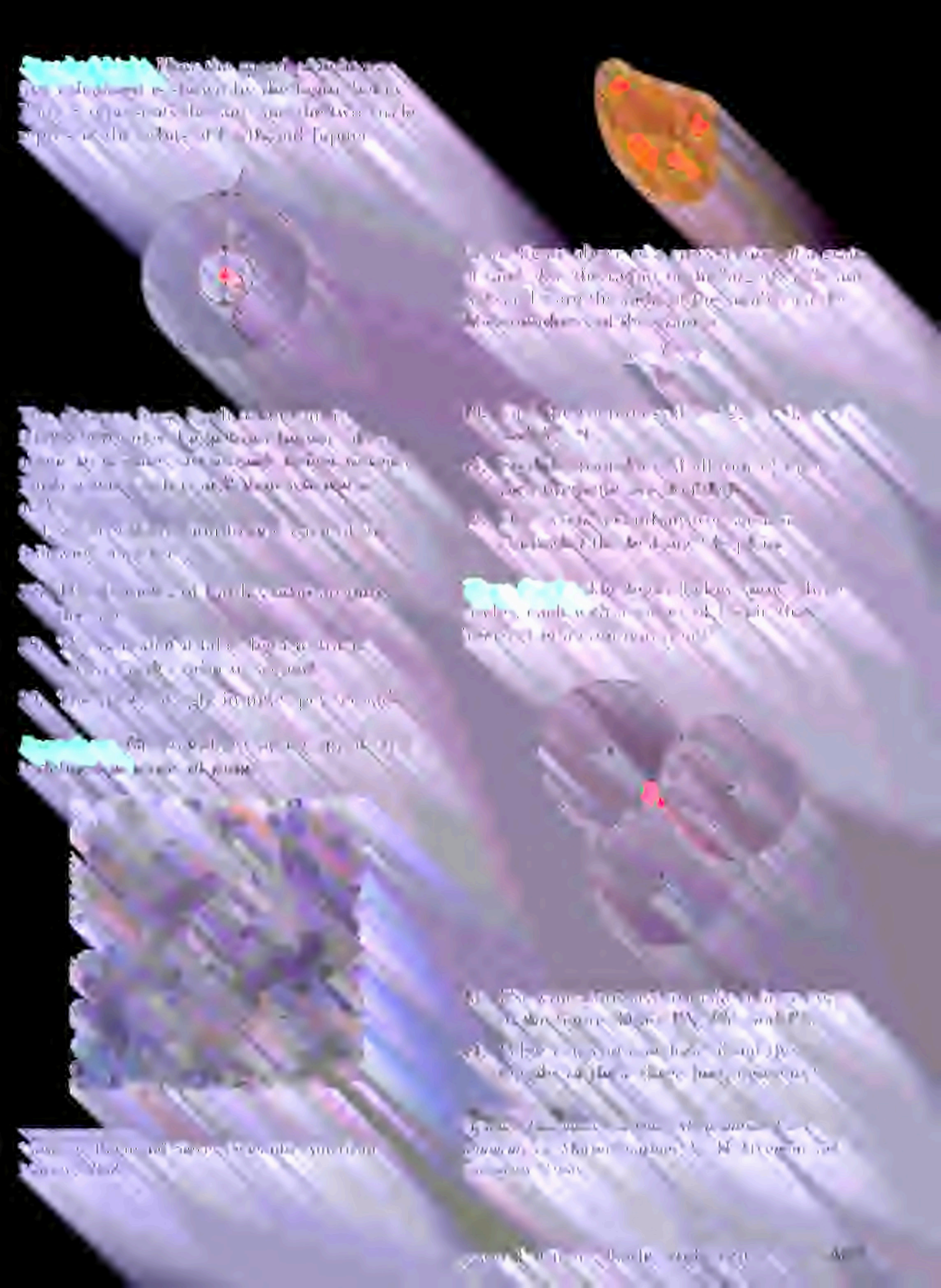


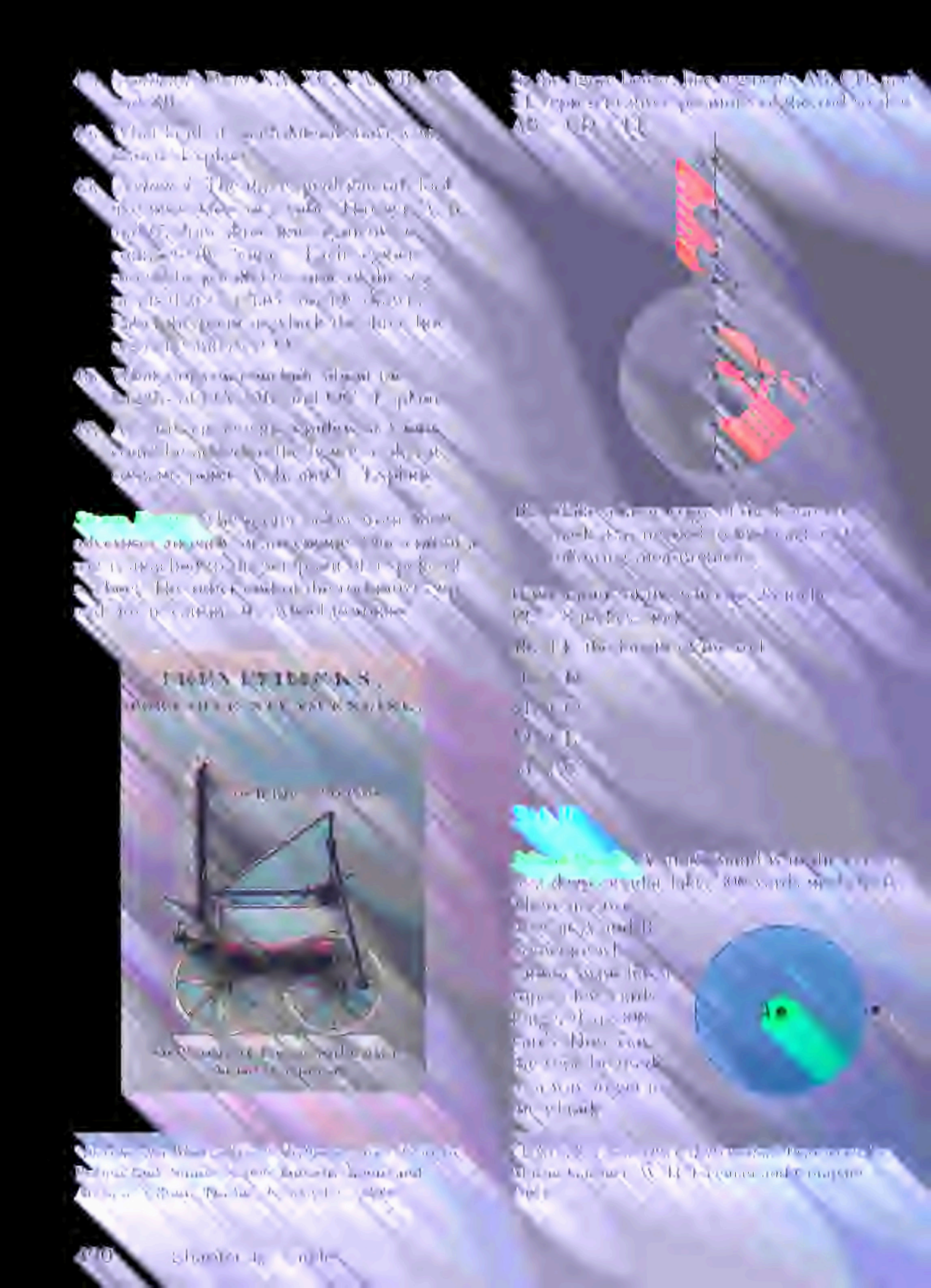






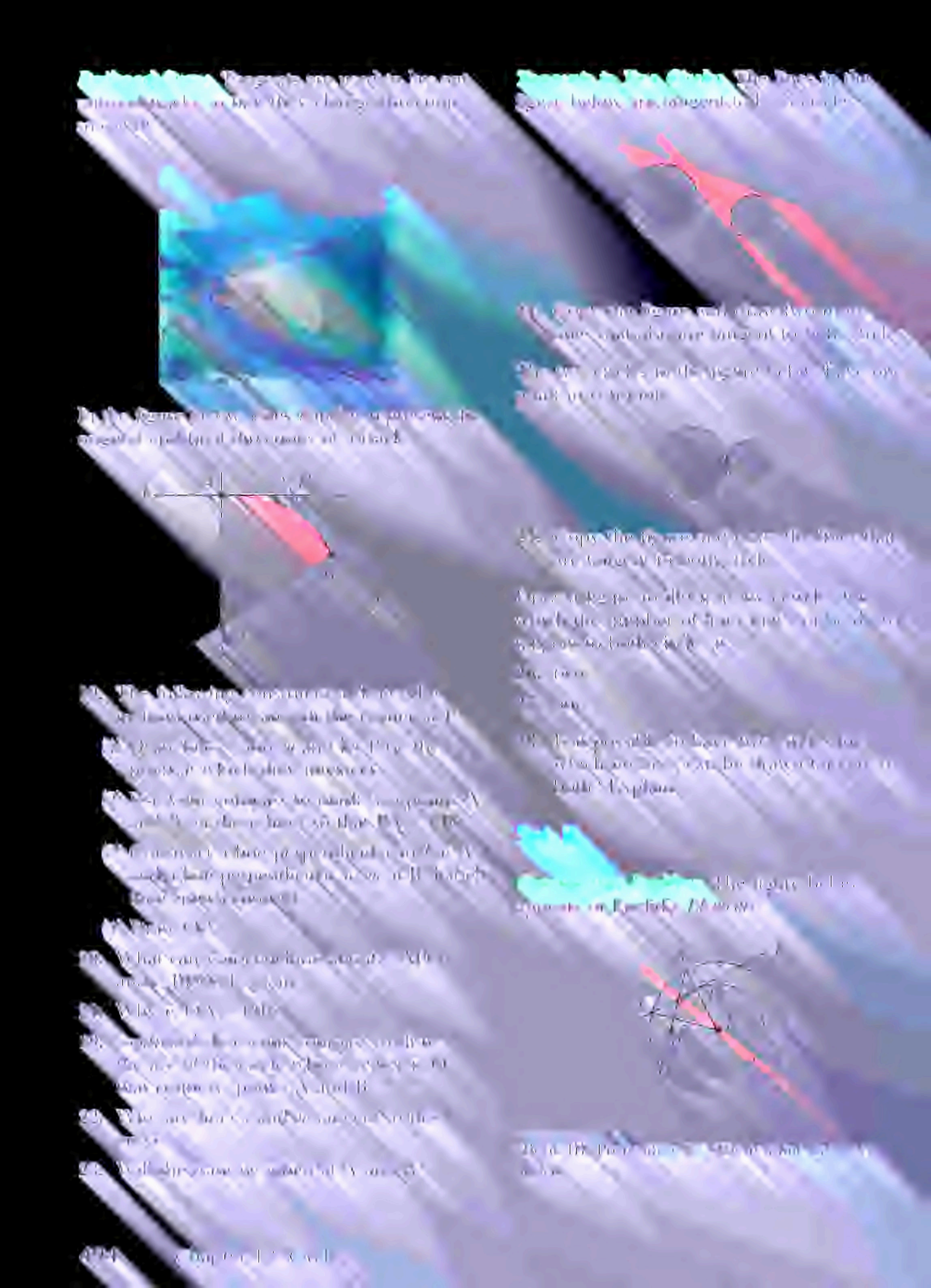


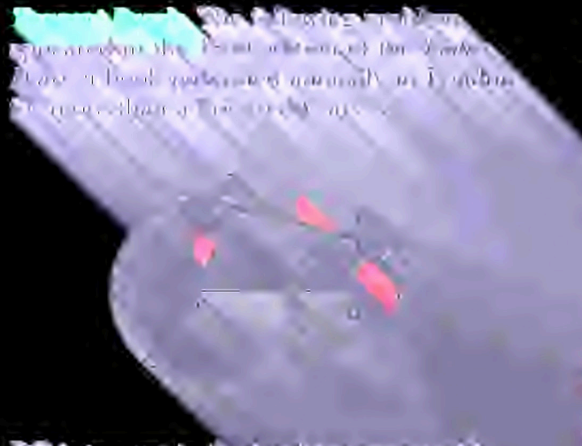


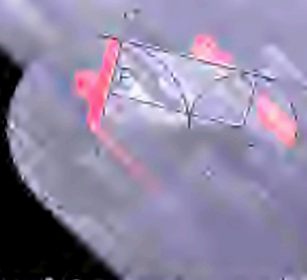


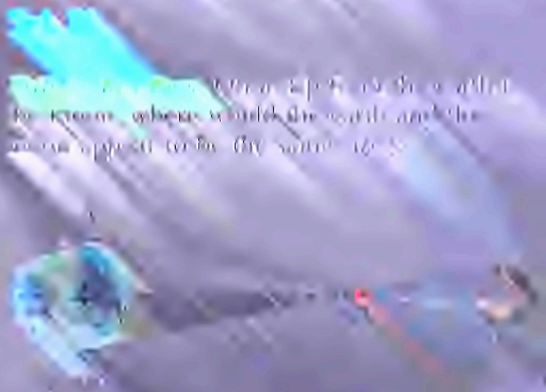








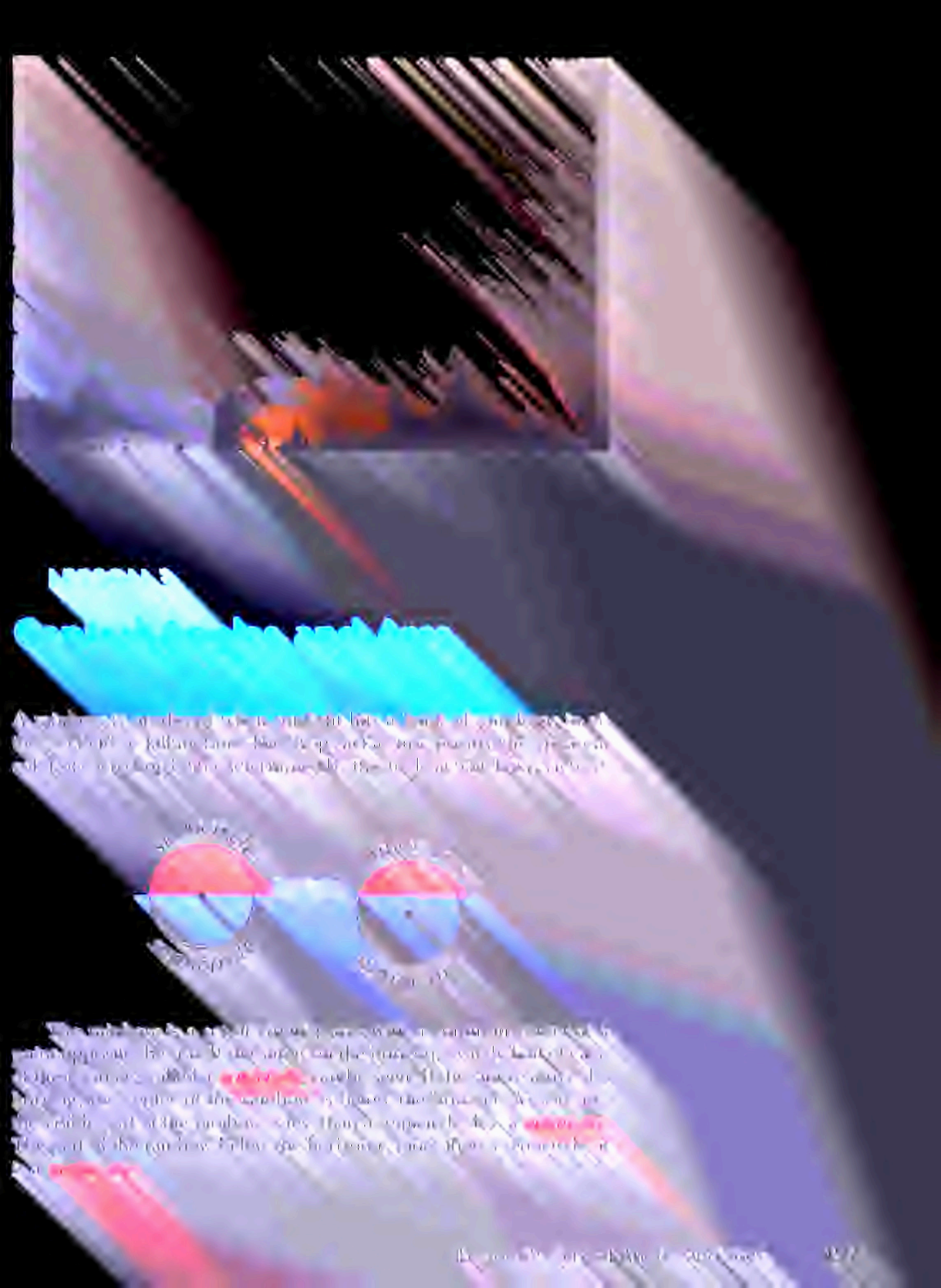


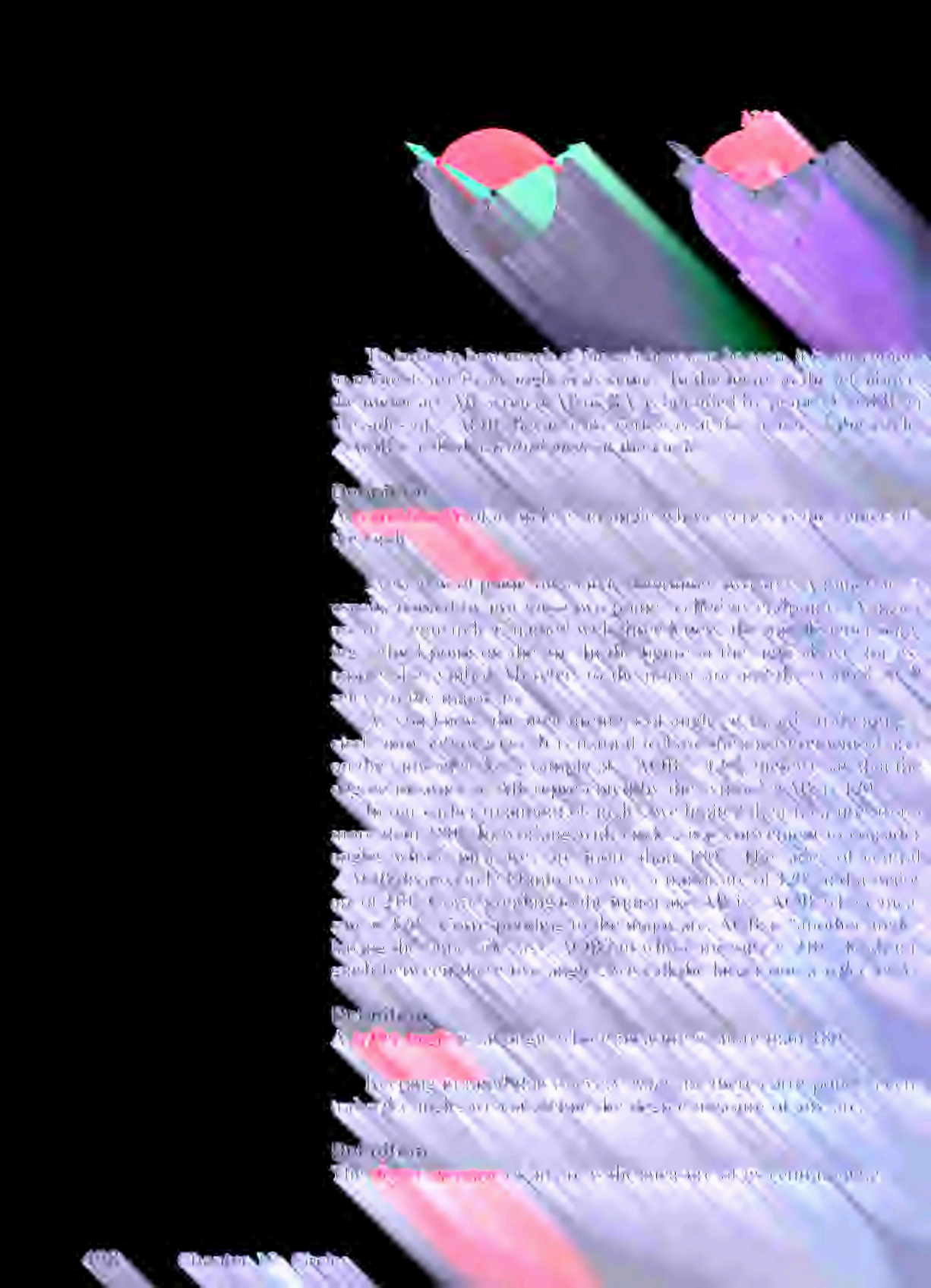












In the figure to the right, points A, B, and C lie on circle O. Find the measure of $\angle ABC$.

A. $\frac{1}{2}(360^\circ - x^\circ)$

B. $\frac{1}{2}x^\circ$

C. x°

D. $360^\circ - x^\circ$

E. $\frac{1}{2}(x^\circ - 360^\circ)$

Answer: A

Solution: Let $\angle AOC$ be the reflex angle with measure x°. Then $\angle AOC$ and $\angle ABC$ are inscribed angles that intercept the same arc. Therefore, $\angle ABC = \frac{1}{2}(360^\circ - x^\circ)$.

Problem 10: The circumference of a circle is 12π inches. What is the area of the circle?

A. 6π square inches

B. 12π square inches

C. 24π square inches

D. 36π square inches

E. 144π square inches

Answer: D

Solution: Let r be the radius of the circle. Then $2\pi r = 12\pi$, so $r = 6$. The area of the circle is $\pi r^2 = \pi(6)^2 = 36\pi$ square inches.

Problem 11: Which of the following is the equation of a line that is perpendicular to the line $y = 3x + 2$?

A. $y = 3x - 2$

B. $y = -3x + 2$

C. $y = \frac{1}{3}x + 2$

D. $y = -\frac{1}{3}x + 2$

E. $y = -\frac{1}{3}x - 2$

Answer: D

Solution: The slope of the line $y = 3x + 2$ is 3. The slope of a line perpendicular to this line is the negative reciprocal of 3, which is $-\frac{1}{3}$. Therefore, the equation of a line perpendicular to $y = 3x + 2$ is $y = -\frac{1}{3}x + 2$.

Problem 12: The sum of the interior angles of a polygon is 1080°. How many sides does the polygon have?

A. 6

B. 7

C. 8

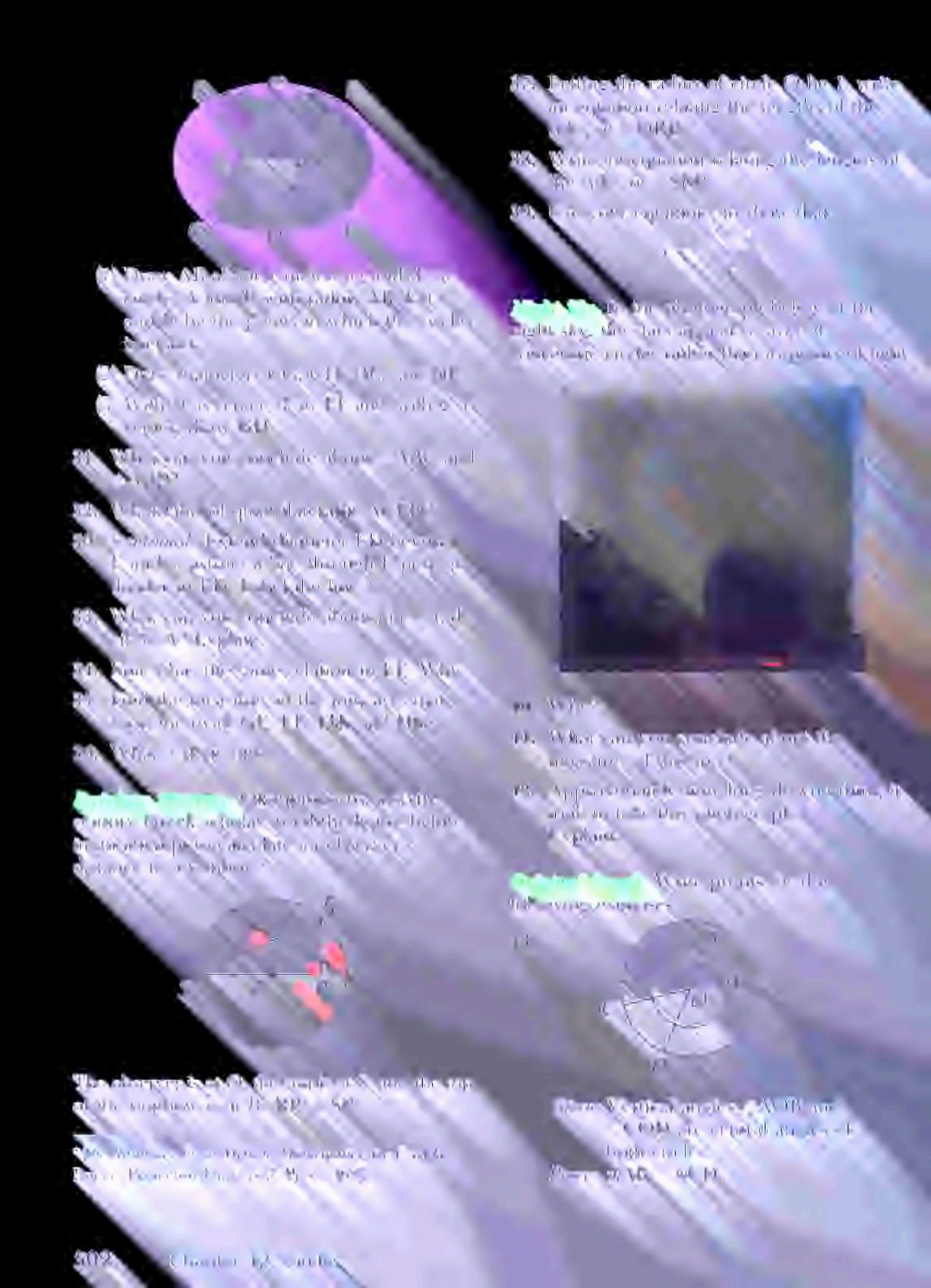
D. 9

E. 10

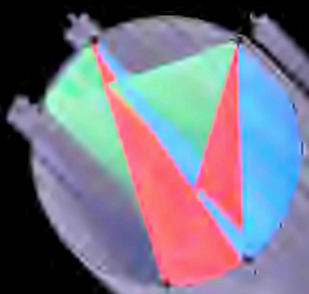
Answer: C

Solution: Let n be the number of sides of the polygon. Then $(n - 2) \cdot 180^\circ = 1080^\circ$, so $n - 2 = 6$, and $n = 8$. Therefore, the polygon has 8 sides.

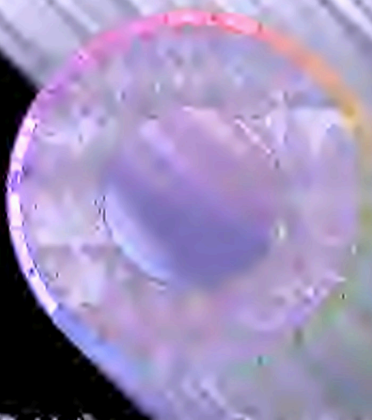





EDISON'S

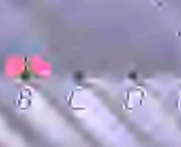










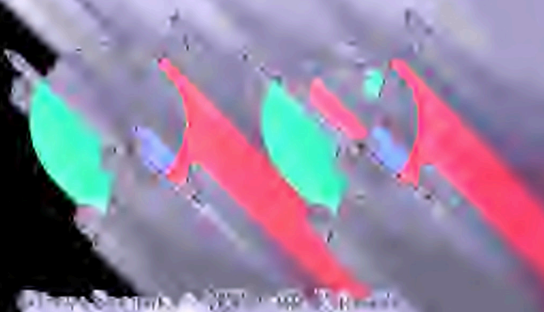


Complete the following proof of Theorem 6.7 by giving the reasons.

Theorem 6.7: A segment joining the midpoints of two sides of a triangle is parallel to the third side and is half as long.

Given: $\overline{DE}$ joins the midpoints of $\overline{AB}$ and $\overline{AC}$.

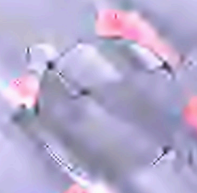
Prove: $\overline{DE} \parallel \overline{BC}$, $DE = \frac{1}{2}BC$



Proof:

Statements	Reasons
1. $\overline{DE}$ joins the midpoints of $\overline{AB}$ and $\overline{AC}$.	1. Given
2. $AD = \frac{1}{2}AB$, $AE = \frac{1}{2}AC$	2. Definition of midpoint
3. $m\angle ADE = m\angle ABC$, $m\angle AED = m\angle ACB$	3. Corresponding angles theorem
4. $\triangle ADE \sim \triangle ABC$	4. AA similarity
5. $\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$	5. Definition of similar triangles
6. $\frac{1}{2}AB = AD$, $\frac{1}{2}AC = AE$	6. Multiplication property of equality
7. $\frac{1}{2}BC = DE$	7. Substitution property of equality
8. $DE = \frac{1}{2}BC$	8. Symmetric property of equality
9. $\overline{DE} \parallel \overline{BC}$	9. Converse of the corresponding angles theorem

In the figure to the right, $\overline{AB}$ and $\overline{AC}$ are tangent to $\odot O$ at B and C. Prove that $\overline{AO}$ is the perpendicular bisector of $\overline{BC}$.

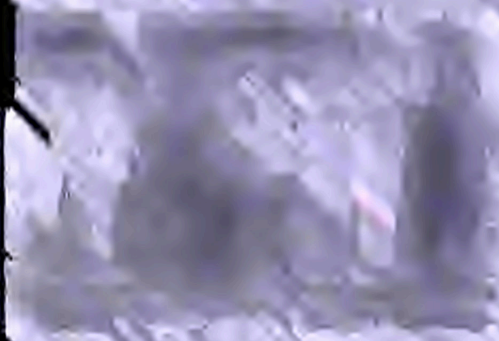


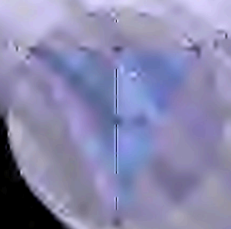
Given: $\overline{AB}$ and $\overline{AC}$ are tangent to $\odot O$ at B and C.

Prove: $\overline{AO}$ is the perpendicular bisector of $\overline{BC}$.

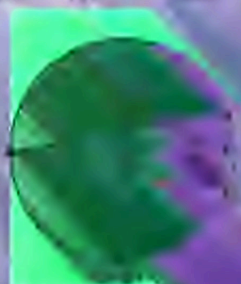
Proof:

Statements	Reasons
1. $\overline{AB}$ and $\overline{AC}$ are tangent to $\odot O$ at B and C.	1. Given
2. $\overline{AB} \perp \overline{OB}$, $\overline{AC} \perp \overline{OC}$	2. A tangent is perpendicular to the radius drawn to the point of tangency.
3. $\triangle ABO$ and $\triangle ACO$ are right triangles.	3. Definition of right triangle
4. $\overline{OB} \cong \overline{OC}$	4. All radii of a circle are congruent.
5. $\overline{AO} \cong \overline{AO}$	5. Reflexive property of congruence
6. $\triangle ABO \cong \triangle ACO$	6. HL congruence
7. $\overline{AB} \cong \overline{AC}$	7. CPCTC
8. A is on the perpendicular bisector of $\overline{BC}$.	8. If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.
9. $\overline{AO}$ is the perpendicular bisector of $\overline{BC}$.	9. Two points determine a line.

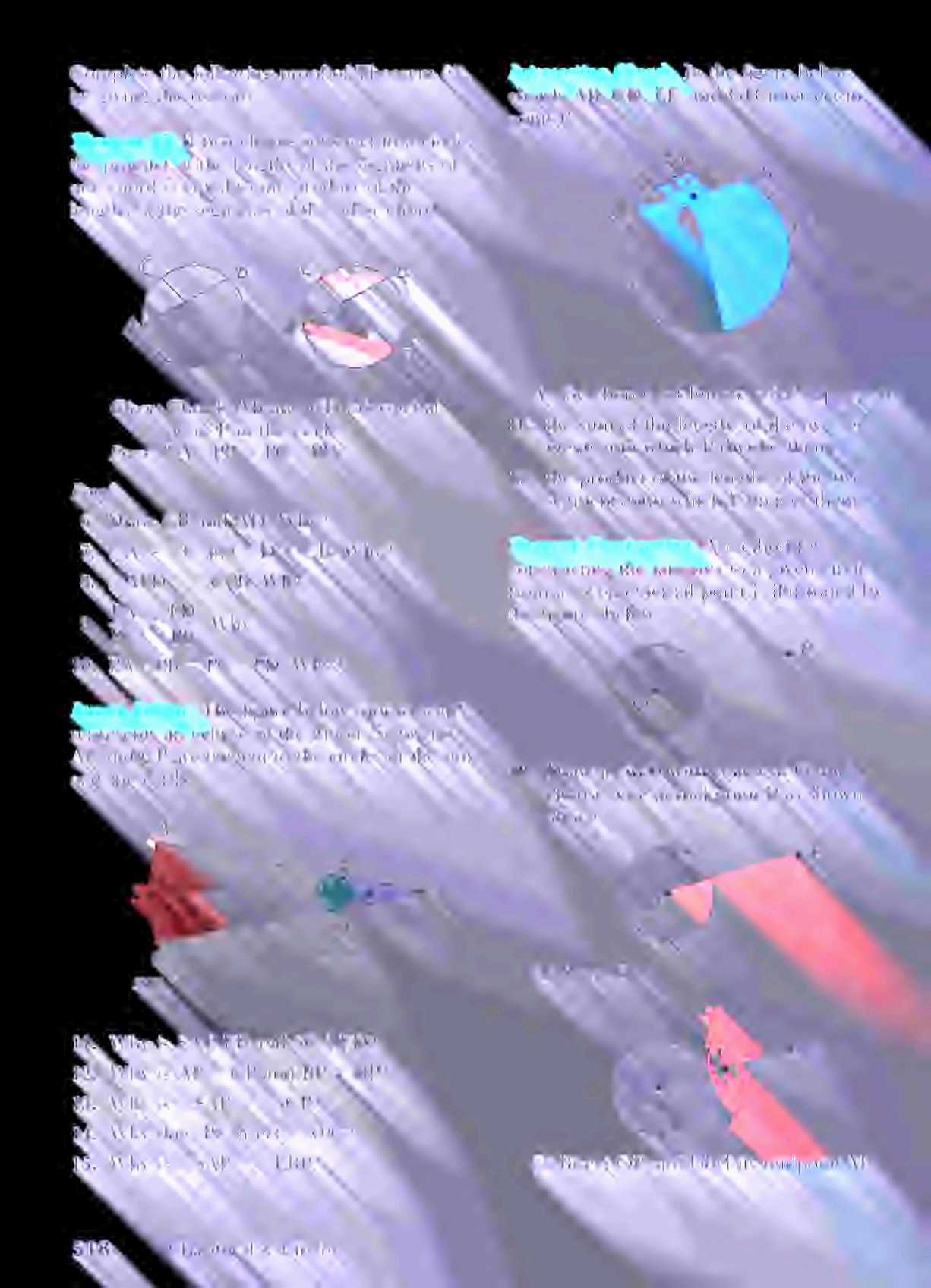


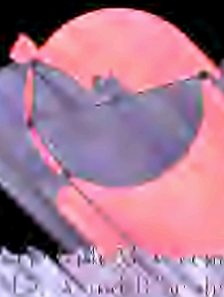










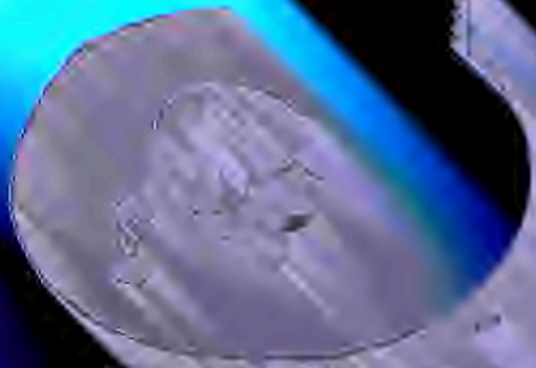






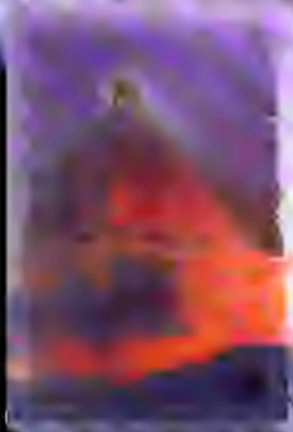






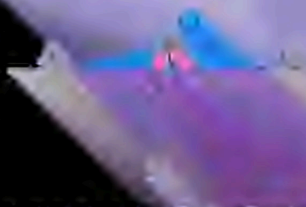


















Systems of Equations



Exercises

Set 1

Problem The figure below shows a [illegible] when taken [illegible] in a [illegible]



1. What are the [illegible] respect to the [illegible]?
2. What are the [illegible] respect to the [illegible]?
3. What would be [illegible] which [illegible] to [illegible]?
4. What is [illegible] AB?
5. What relation [illegible] AB [illegible]?
6. What relation [illegible] A, B, and C have [illegible]?

Circumcircle The figure below [illegible] something about the circumcircle of different types of triangles.



7. What [illegible] the [illegible] with [illegible]?

[illegible]

8. What kind of [illegible] the [illegible] of a right triangle [illegible]?
9. What [illegible] the [illegible] of a [illegible] triangle [illegible]?

[illegible]

10. [illegible]
11. [illegible]
12. [illegible]
13. What is the [illegible] of a [illegible]?
14. What [illegible] the [illegible]?

Exercise The [illegible] the [illegible] the [illegible] the [illegible] computer monitor.



[illegible]

15. [illegible] the [illegible]
    1. [illegible]
    2. [illegible]
    3. [illegible]
16. [illegible]
17. What do you notice about these three [illegible]?
18. Why [illegible]?



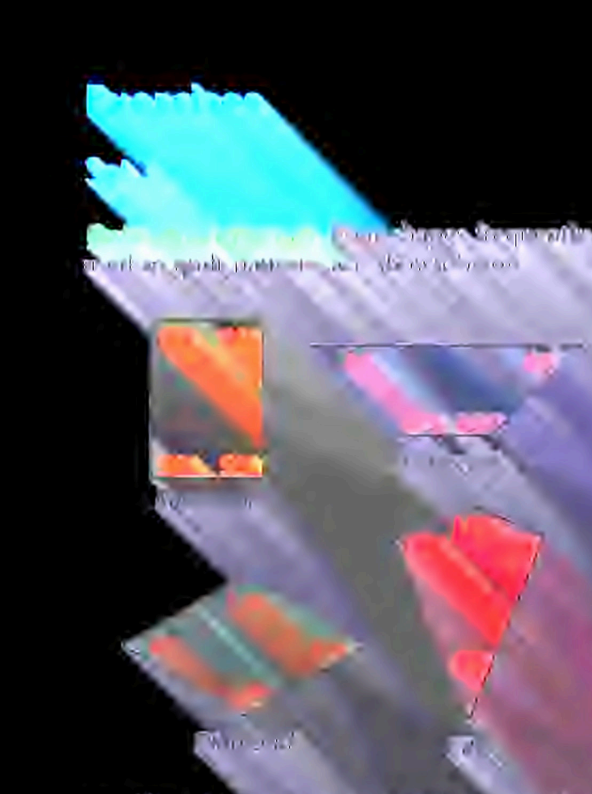


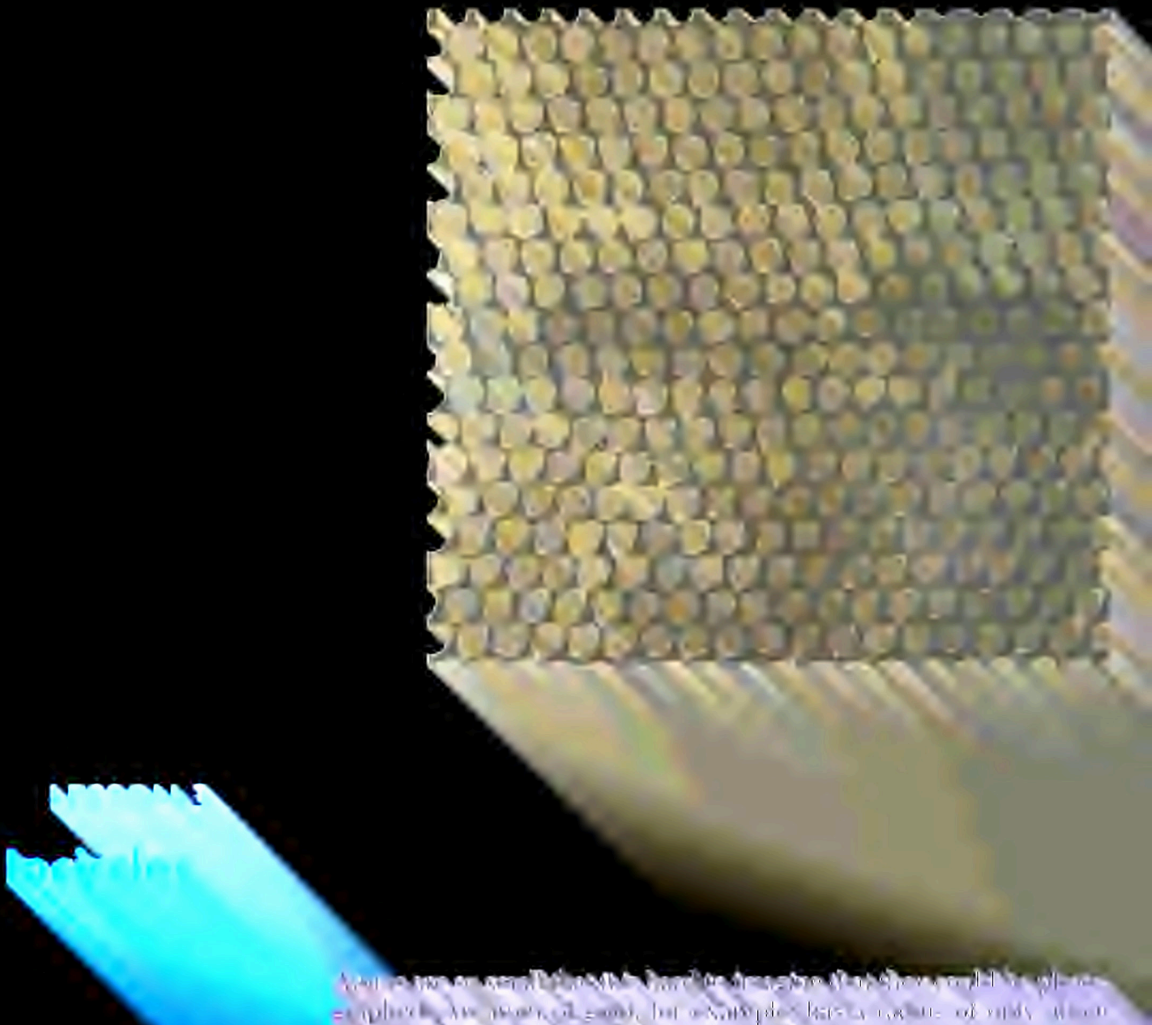








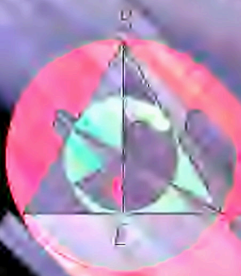








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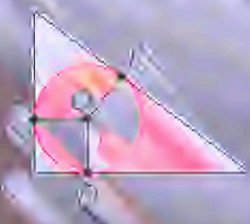
C





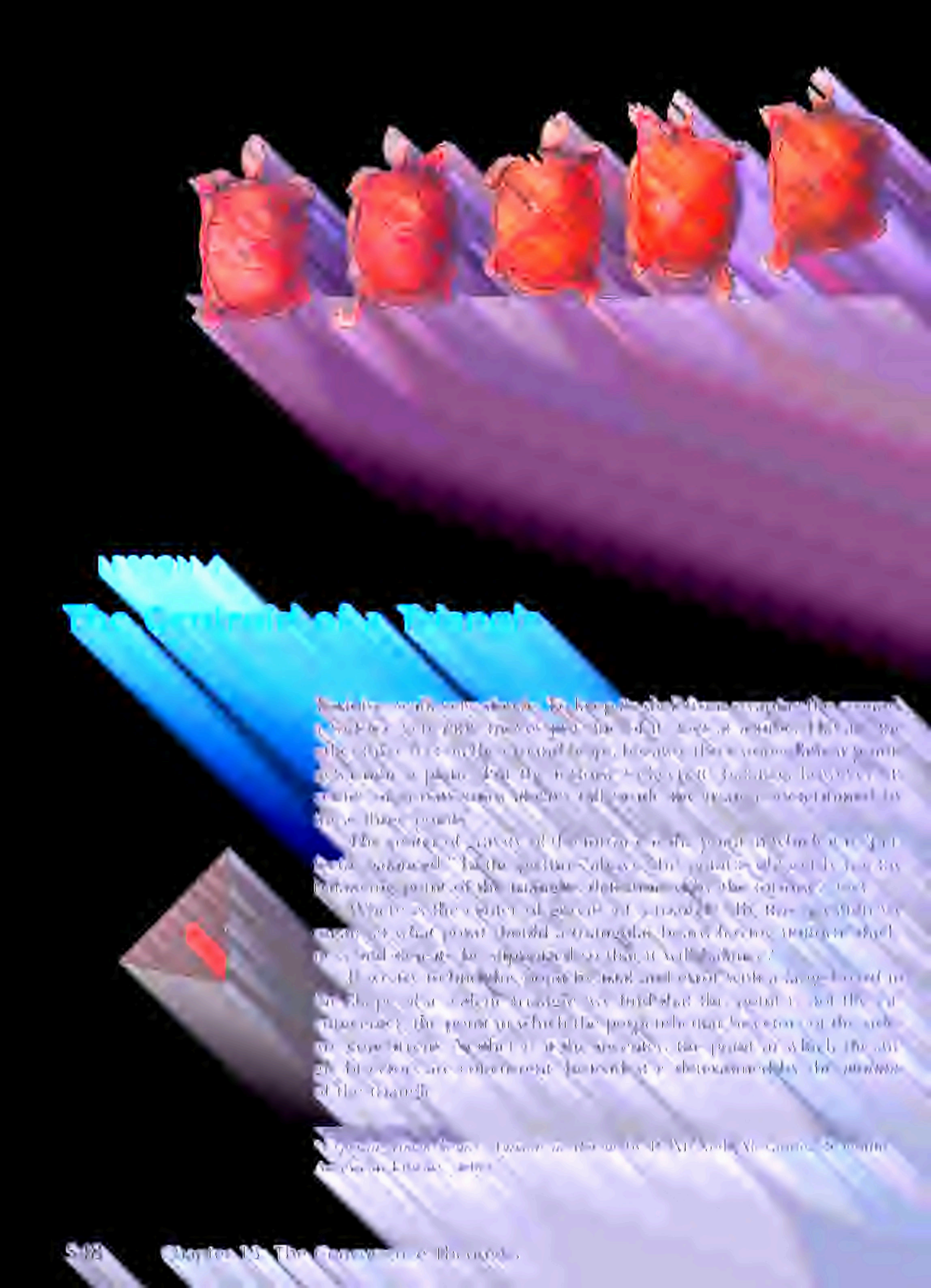


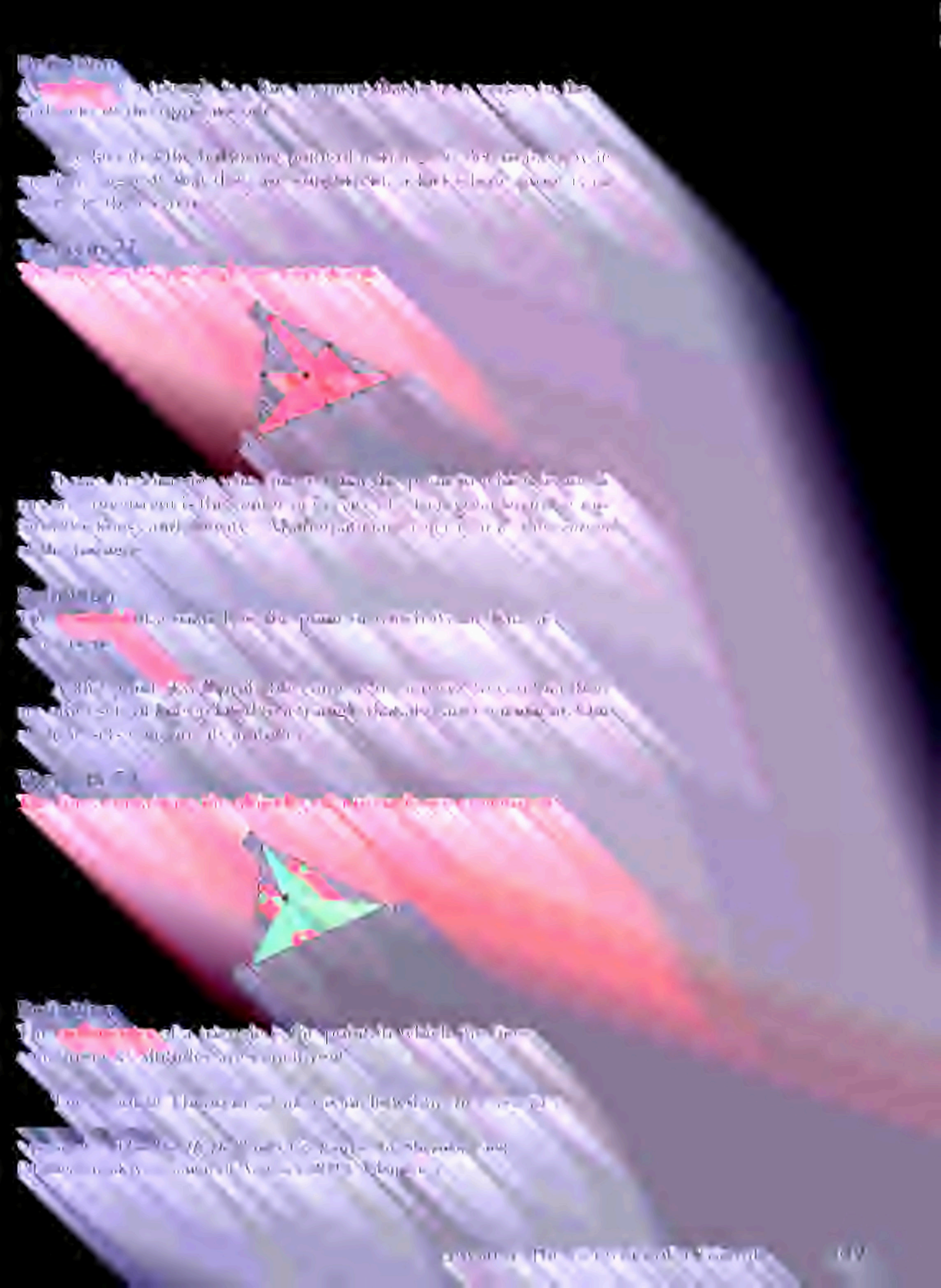


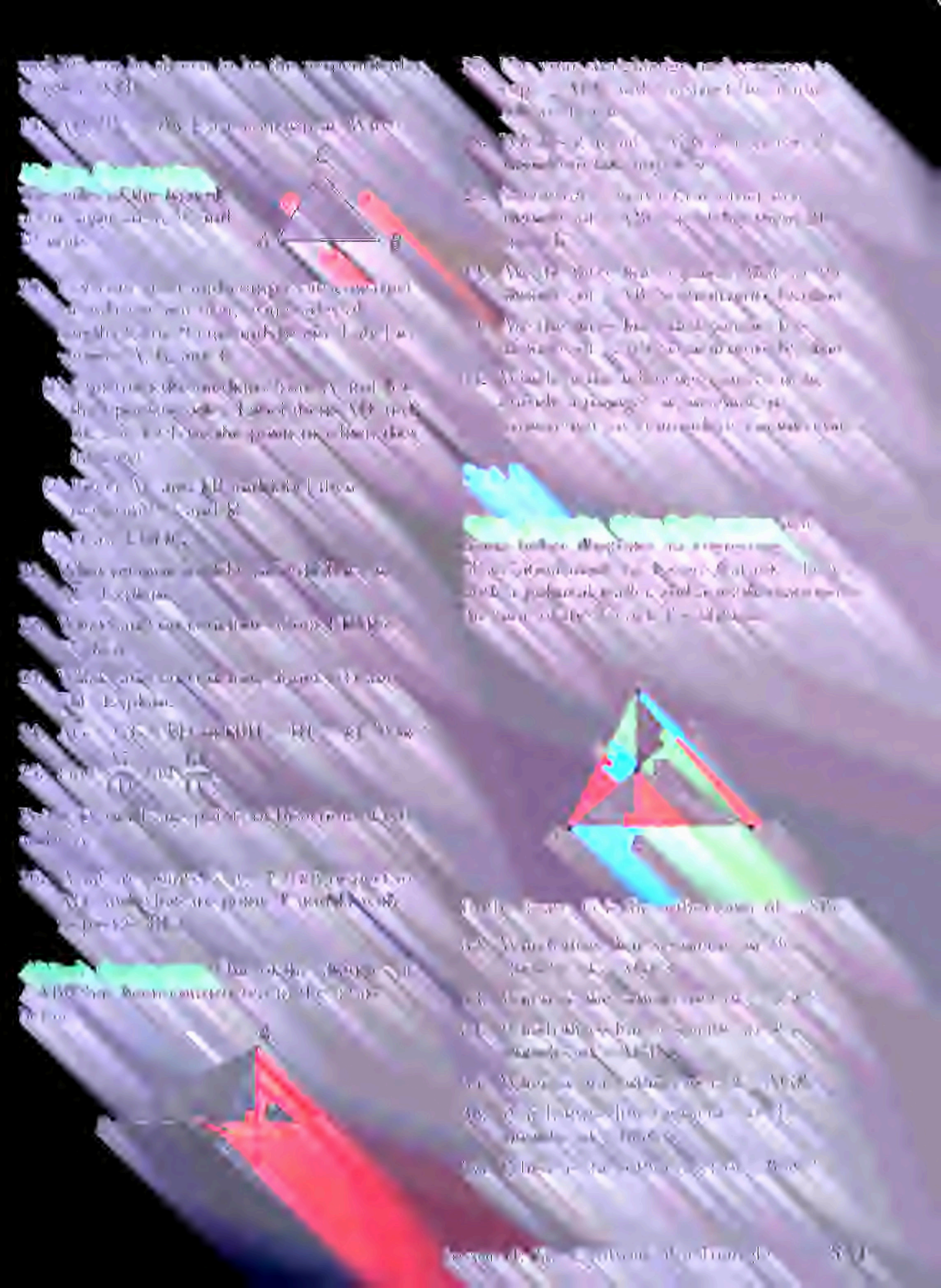


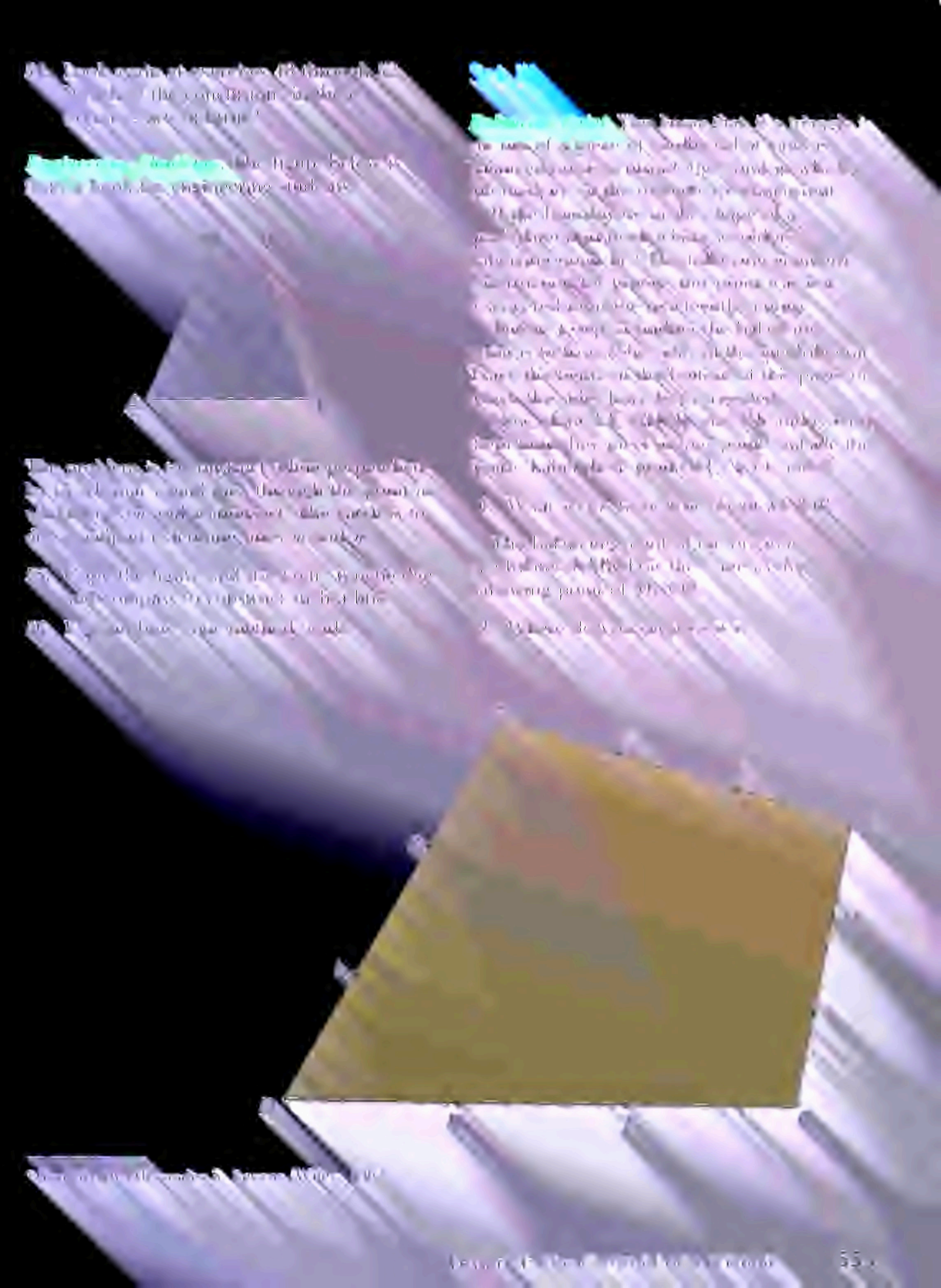


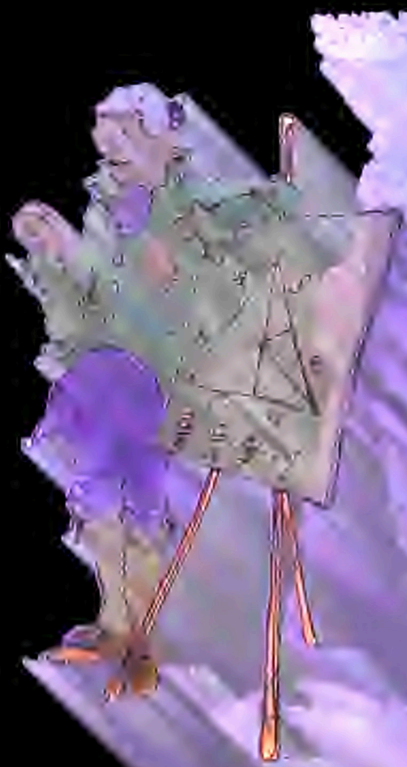








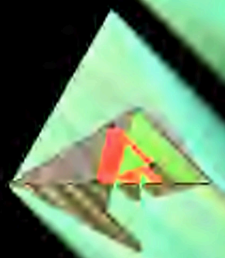












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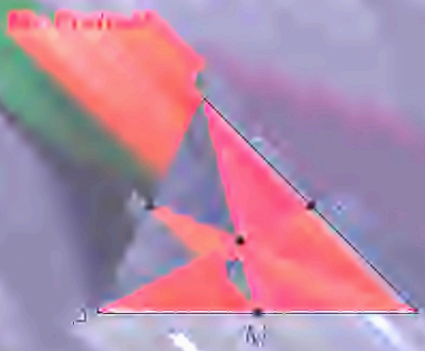

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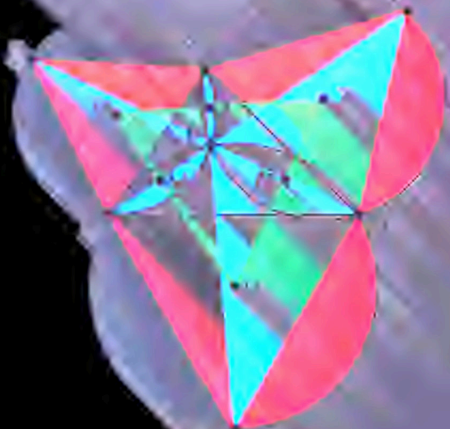
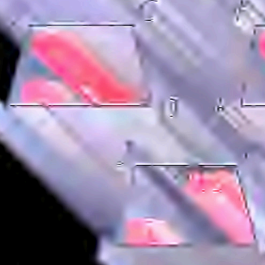
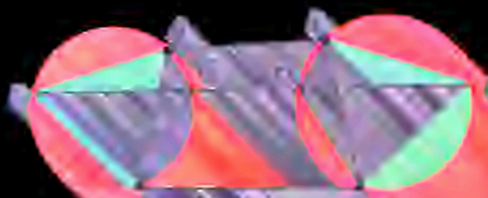
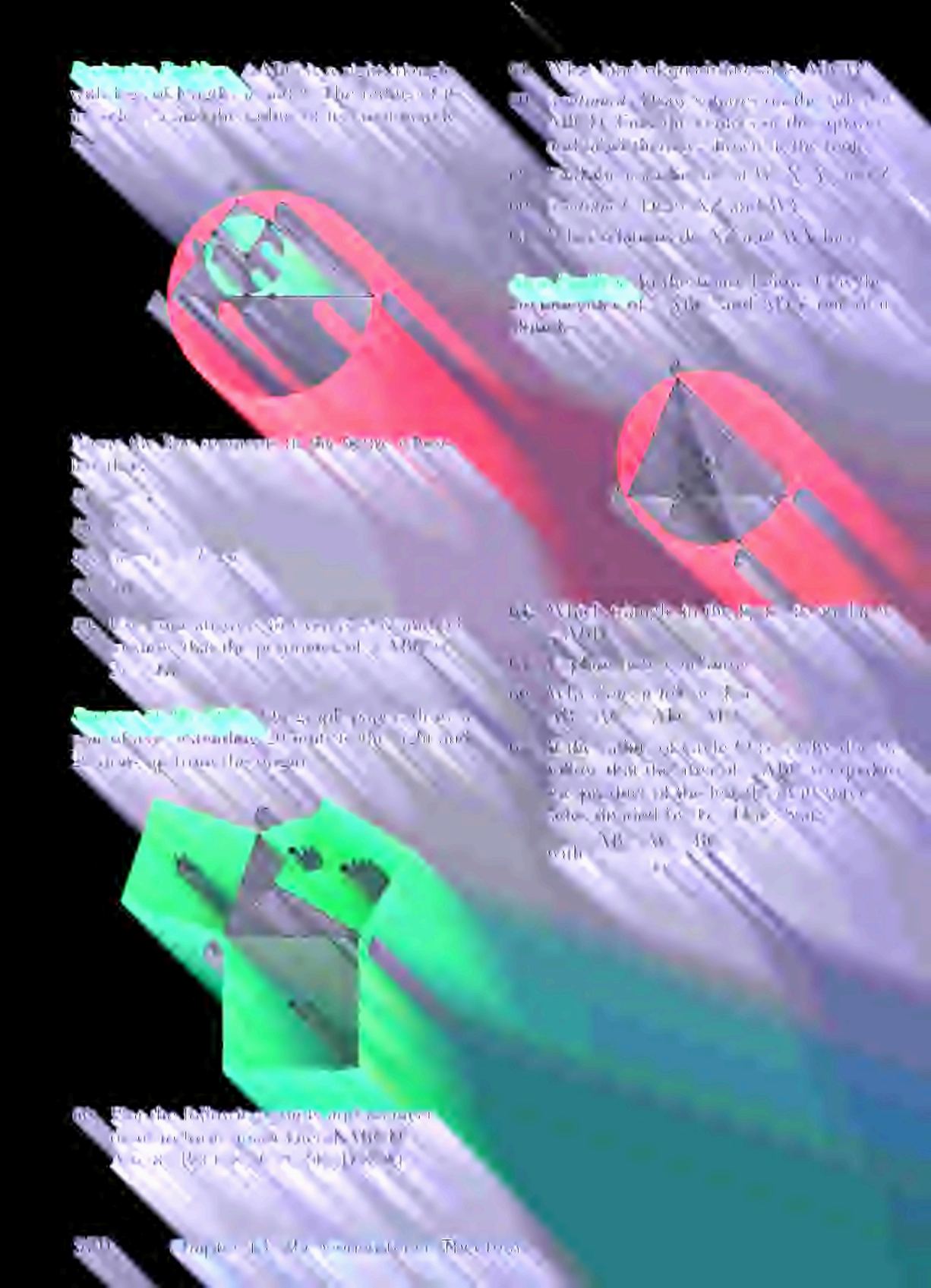




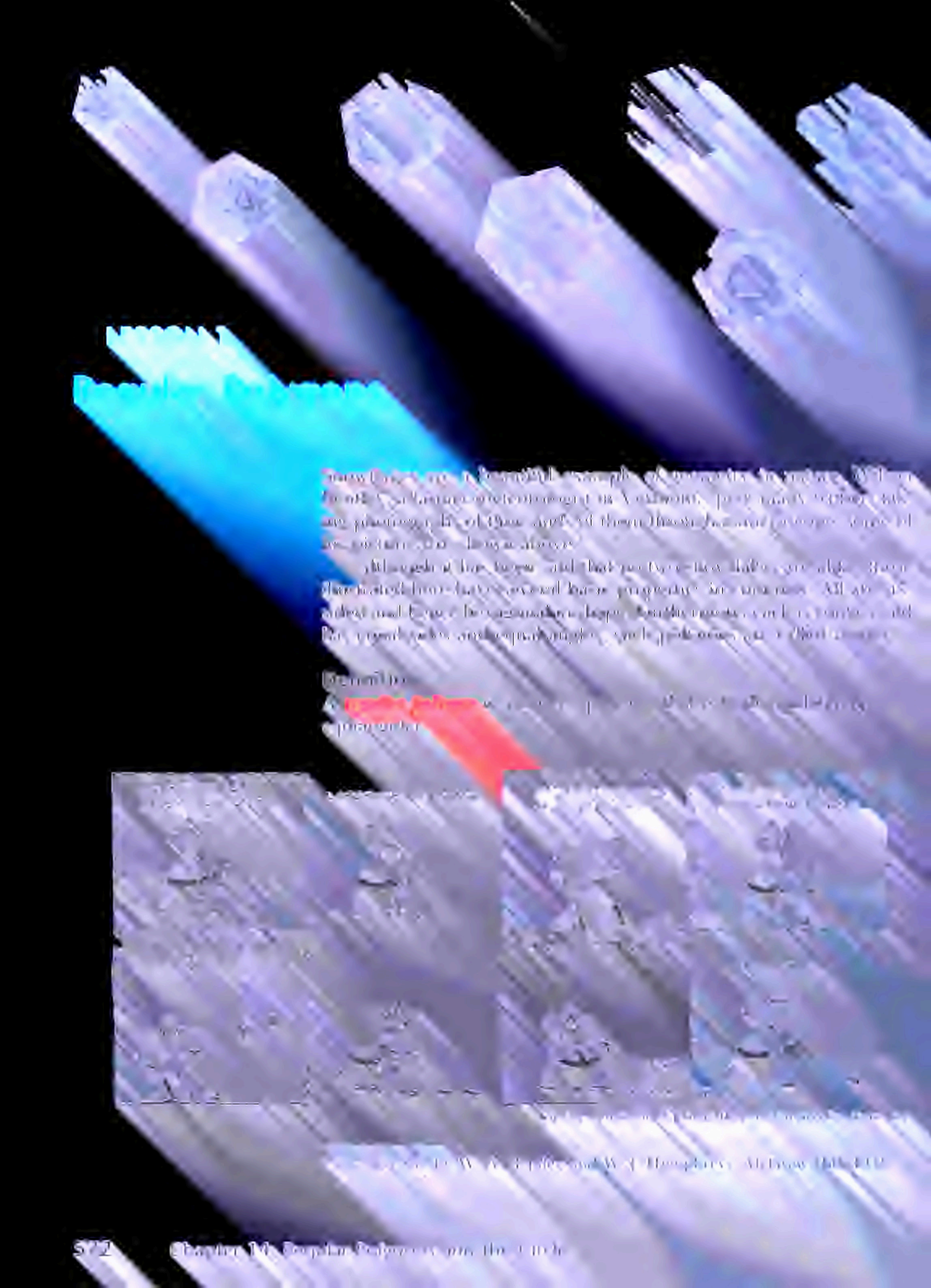


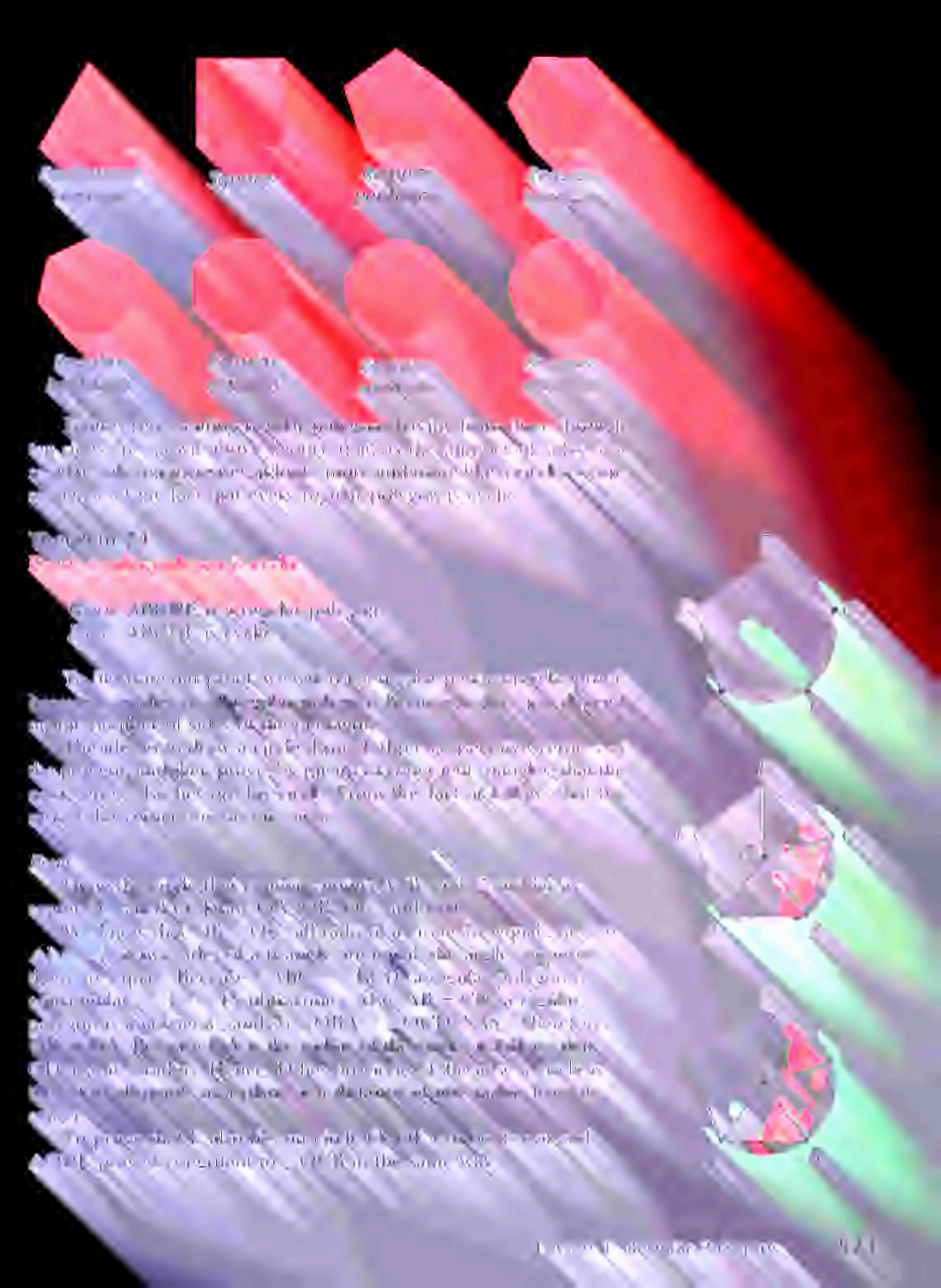












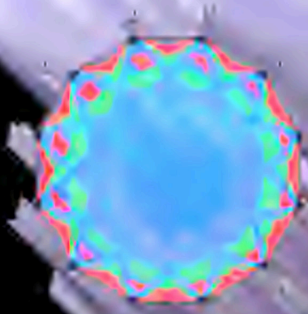




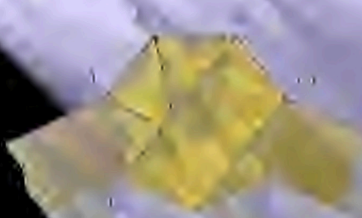






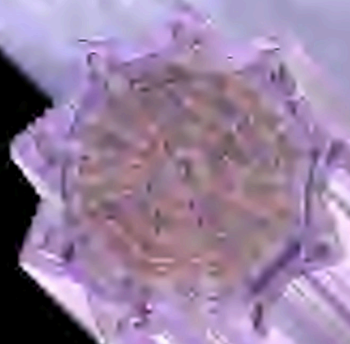


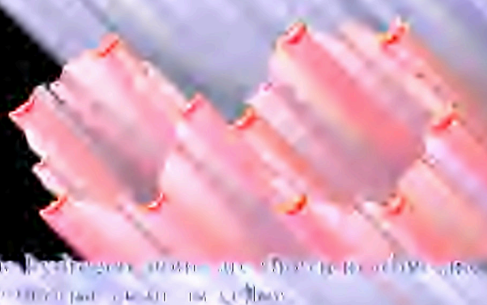


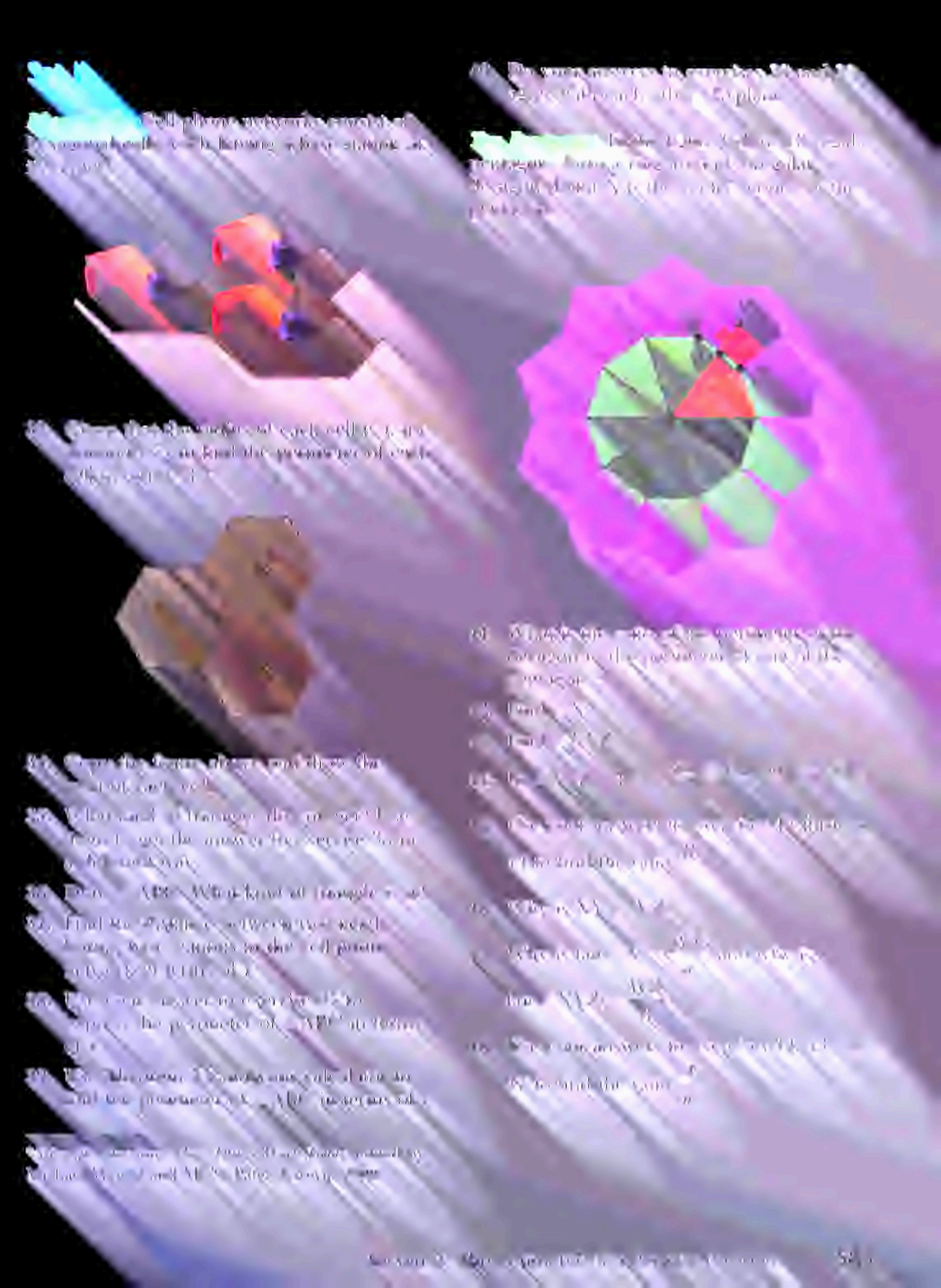


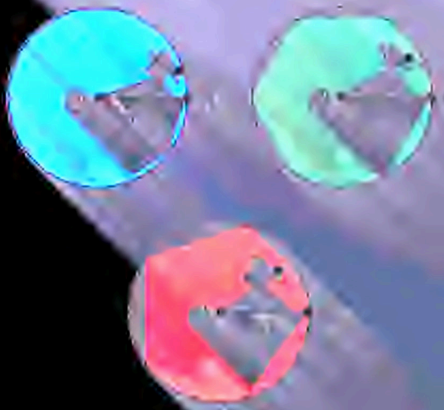




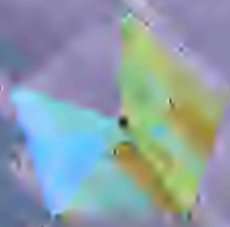


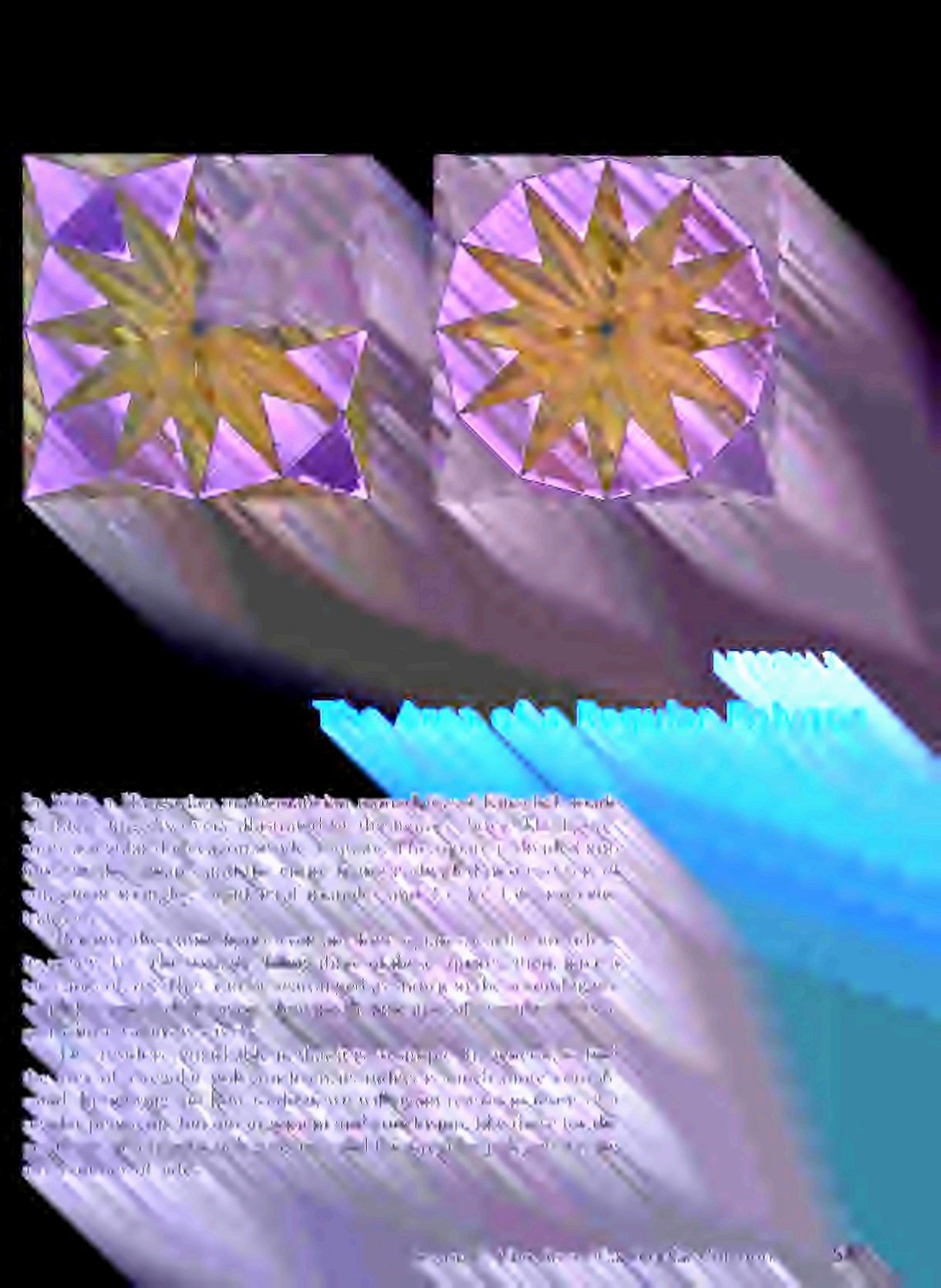








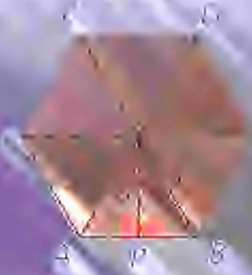


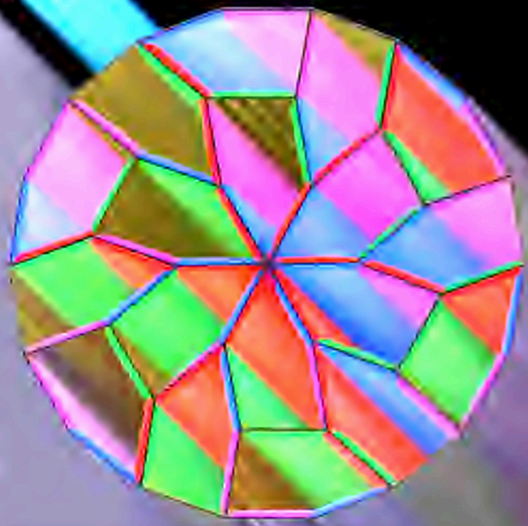




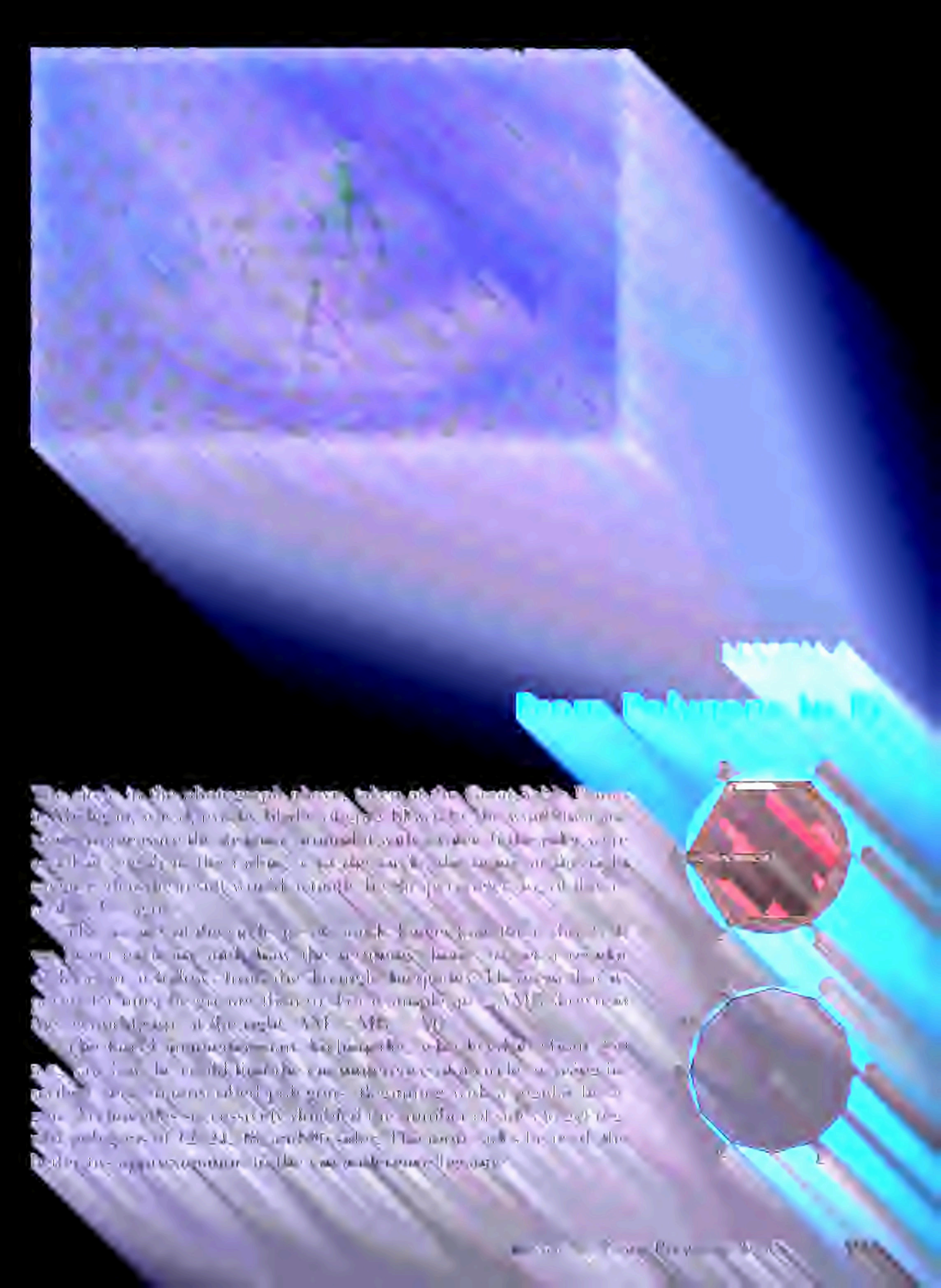


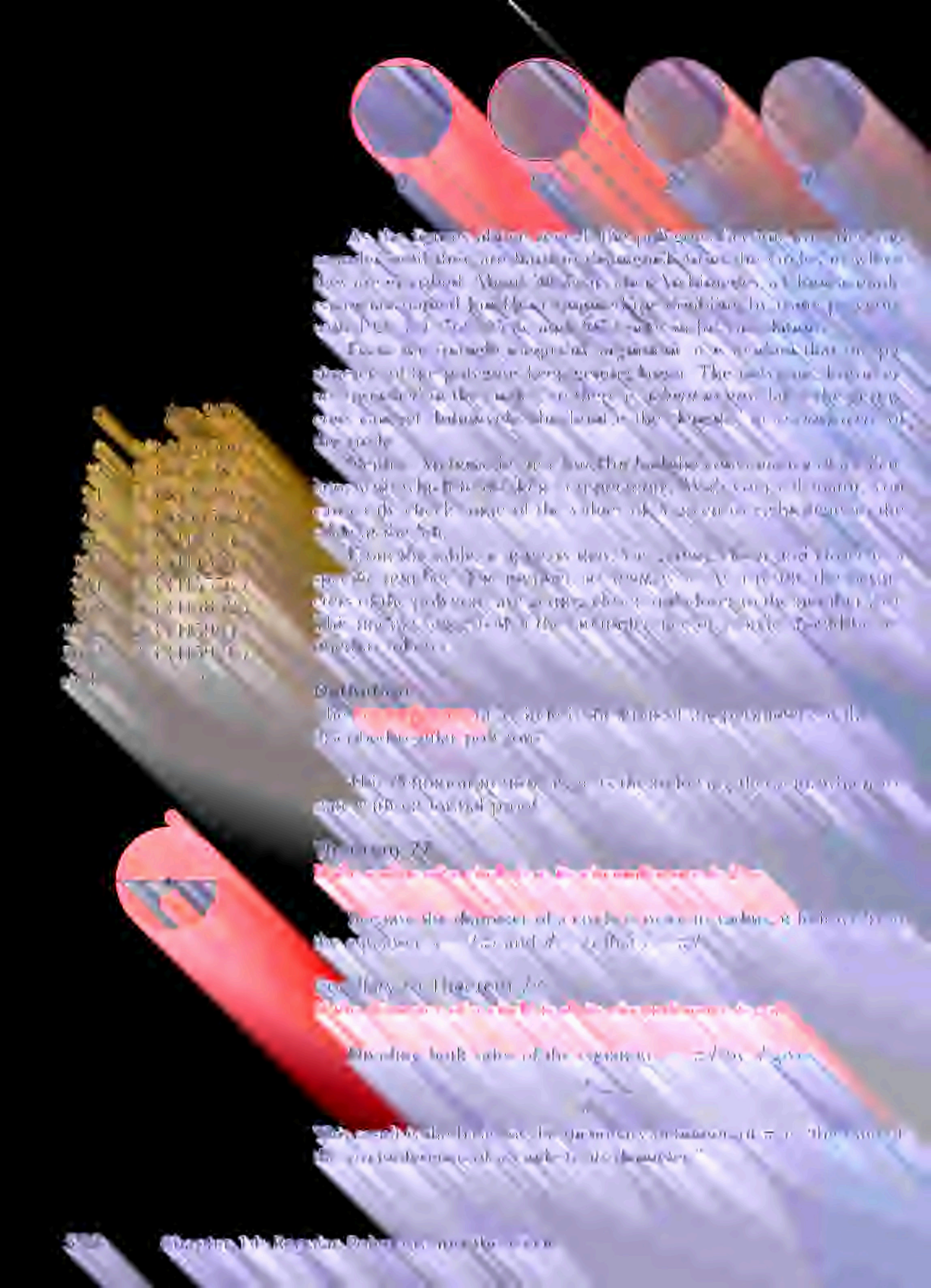




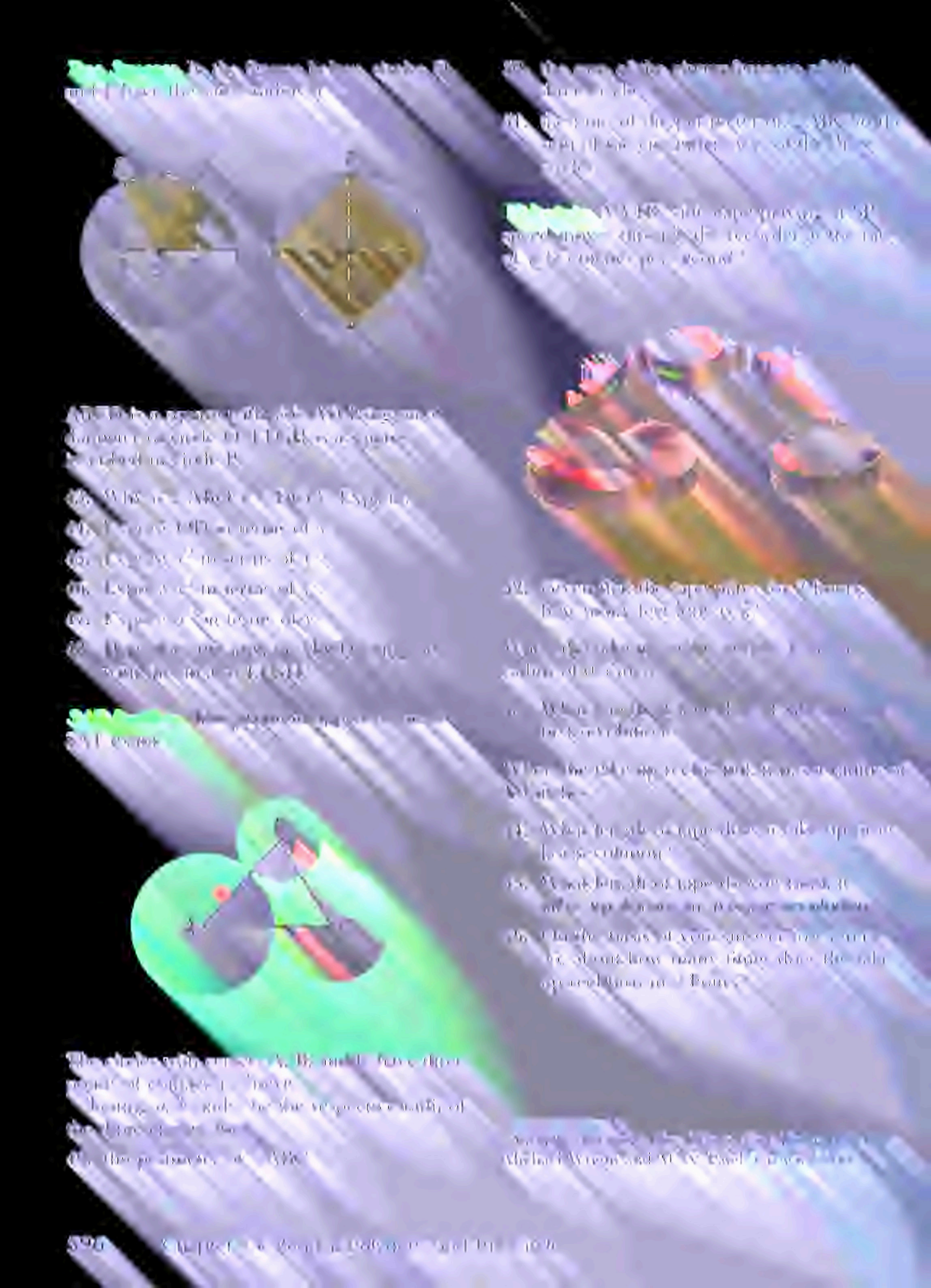














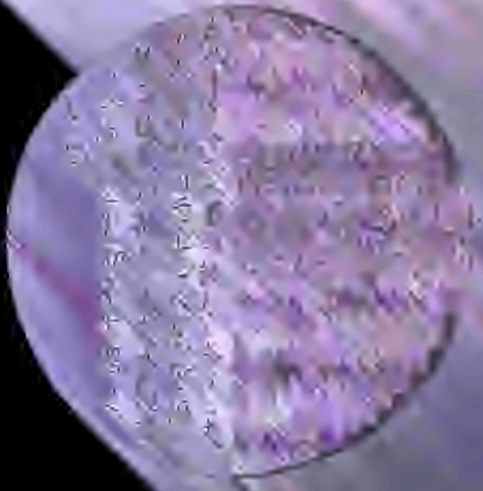


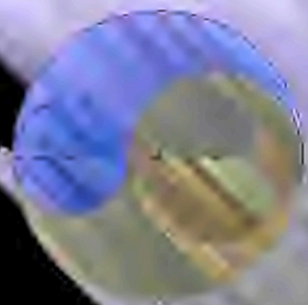


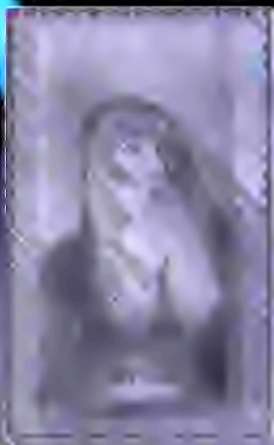














These examples illustrate the general pattern. Suppose that the [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]



[illegible]

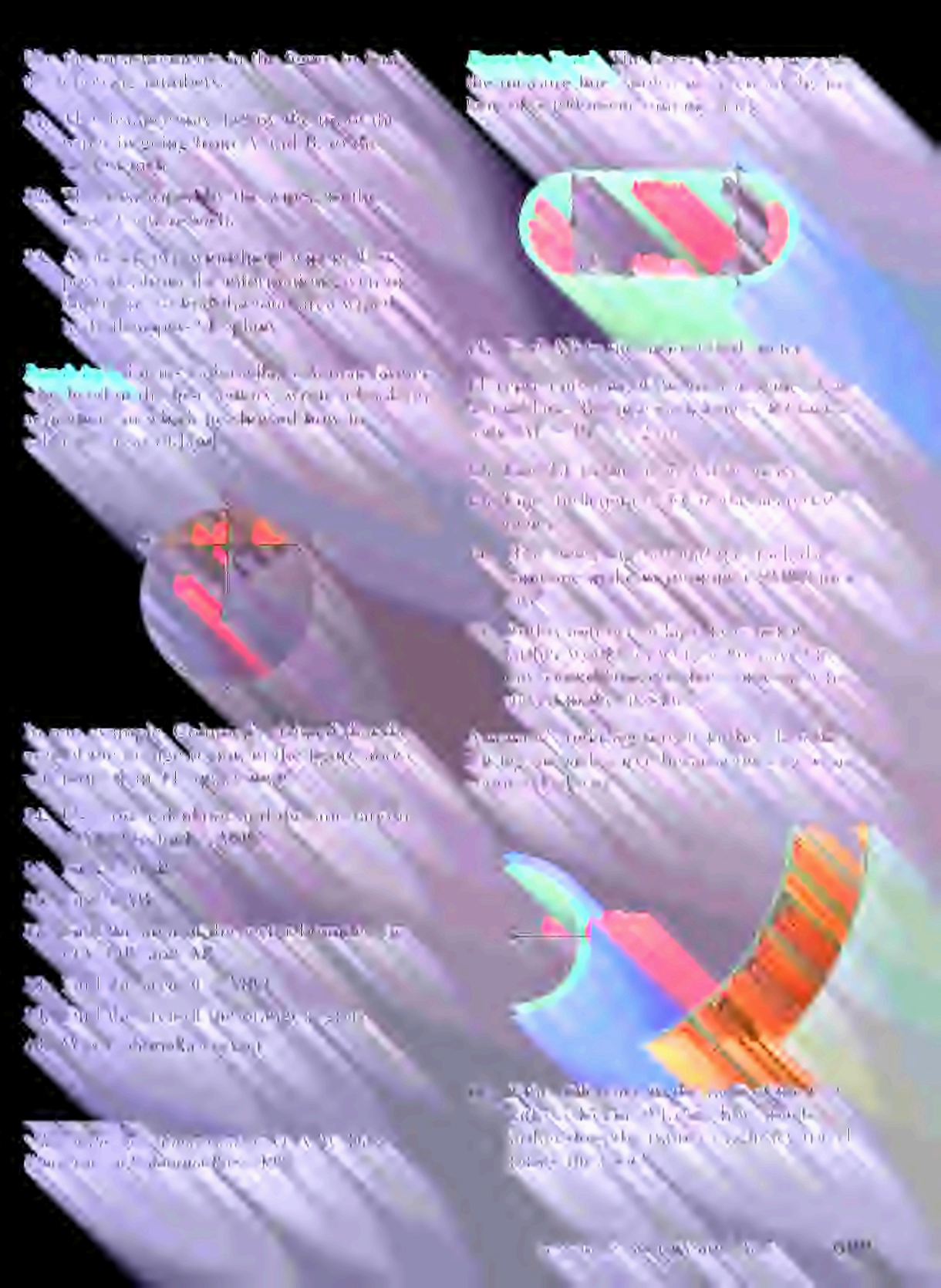
608 Chapter 14 Vector Calculus and Vector Fields

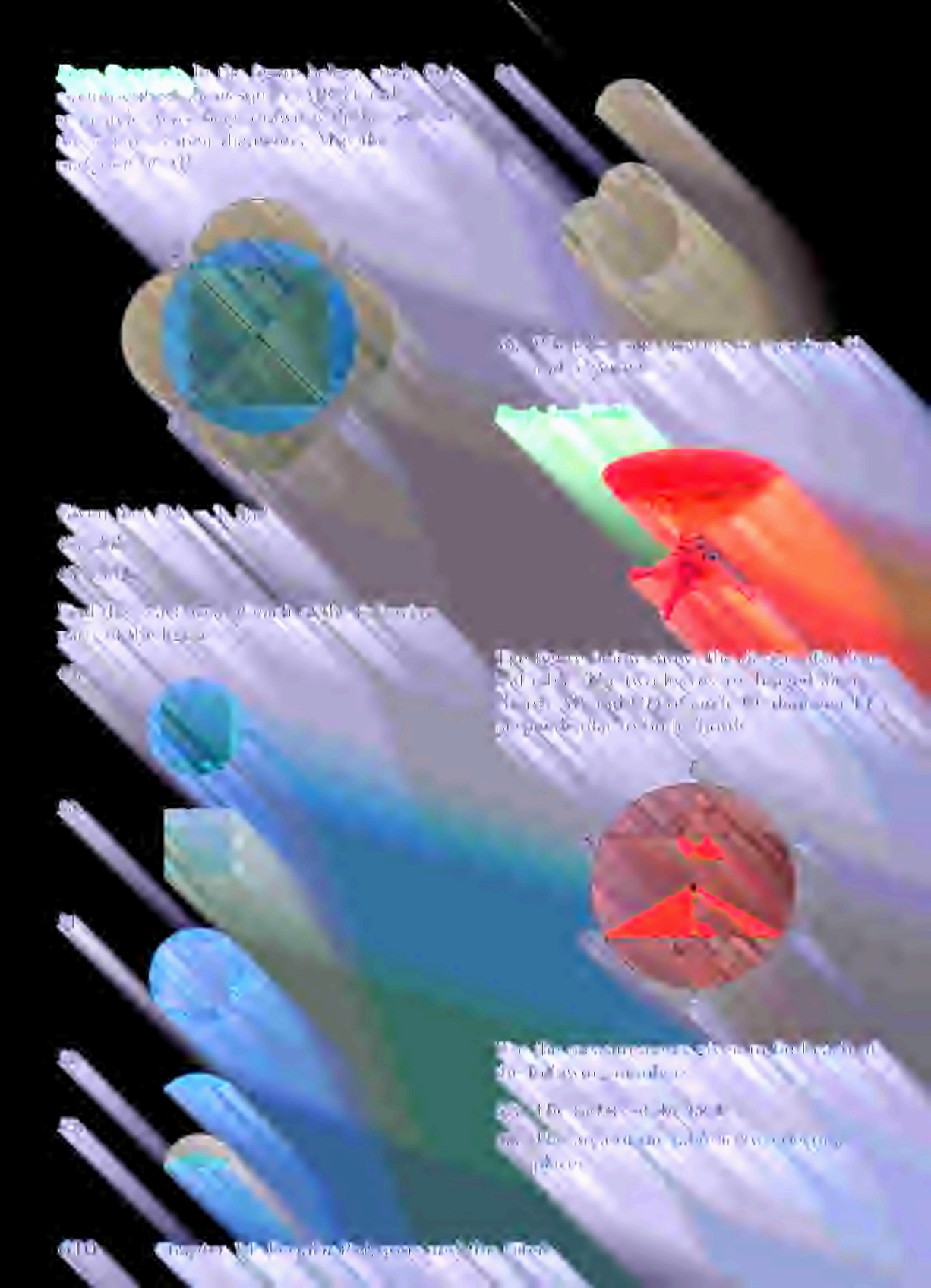


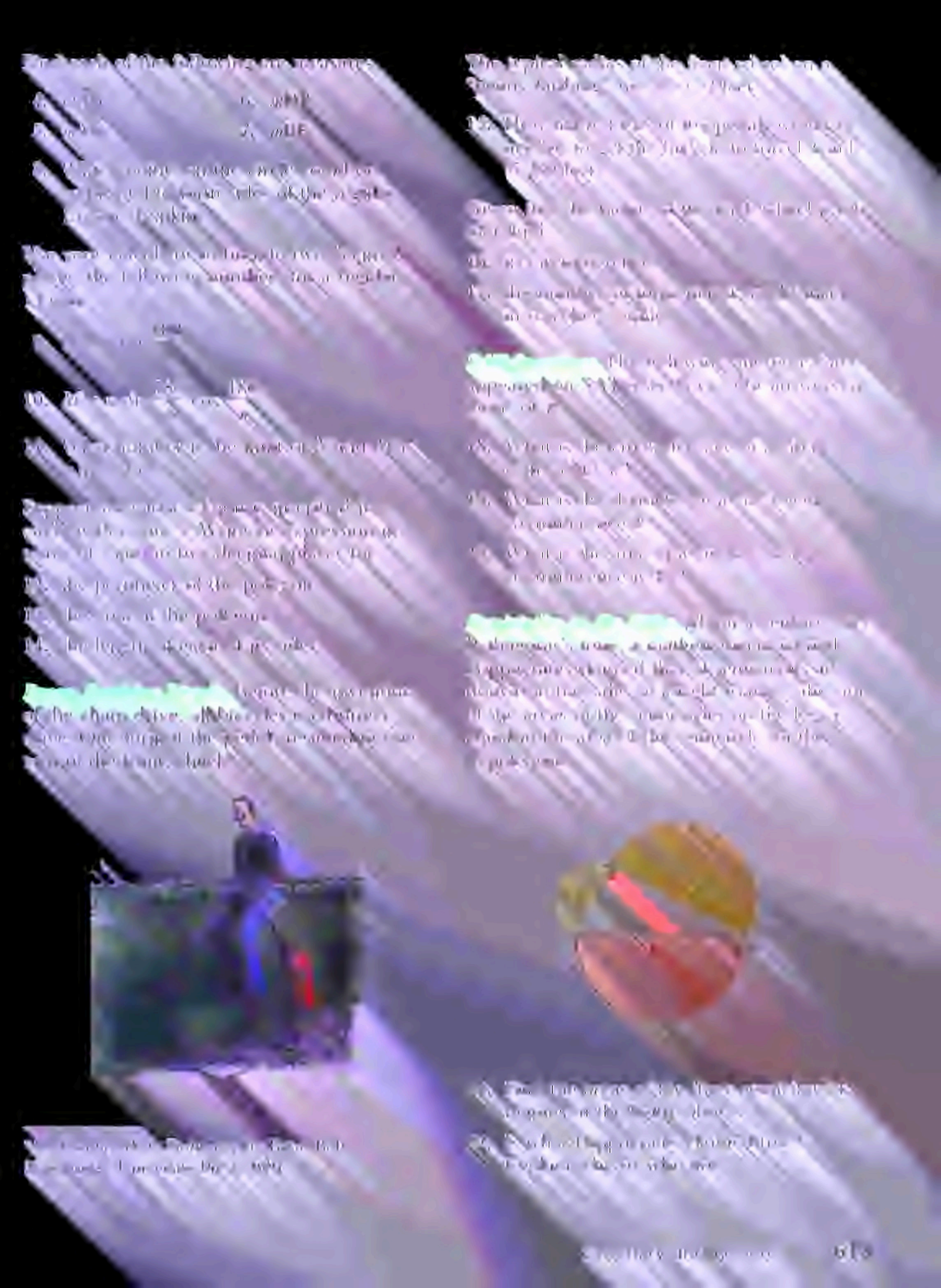












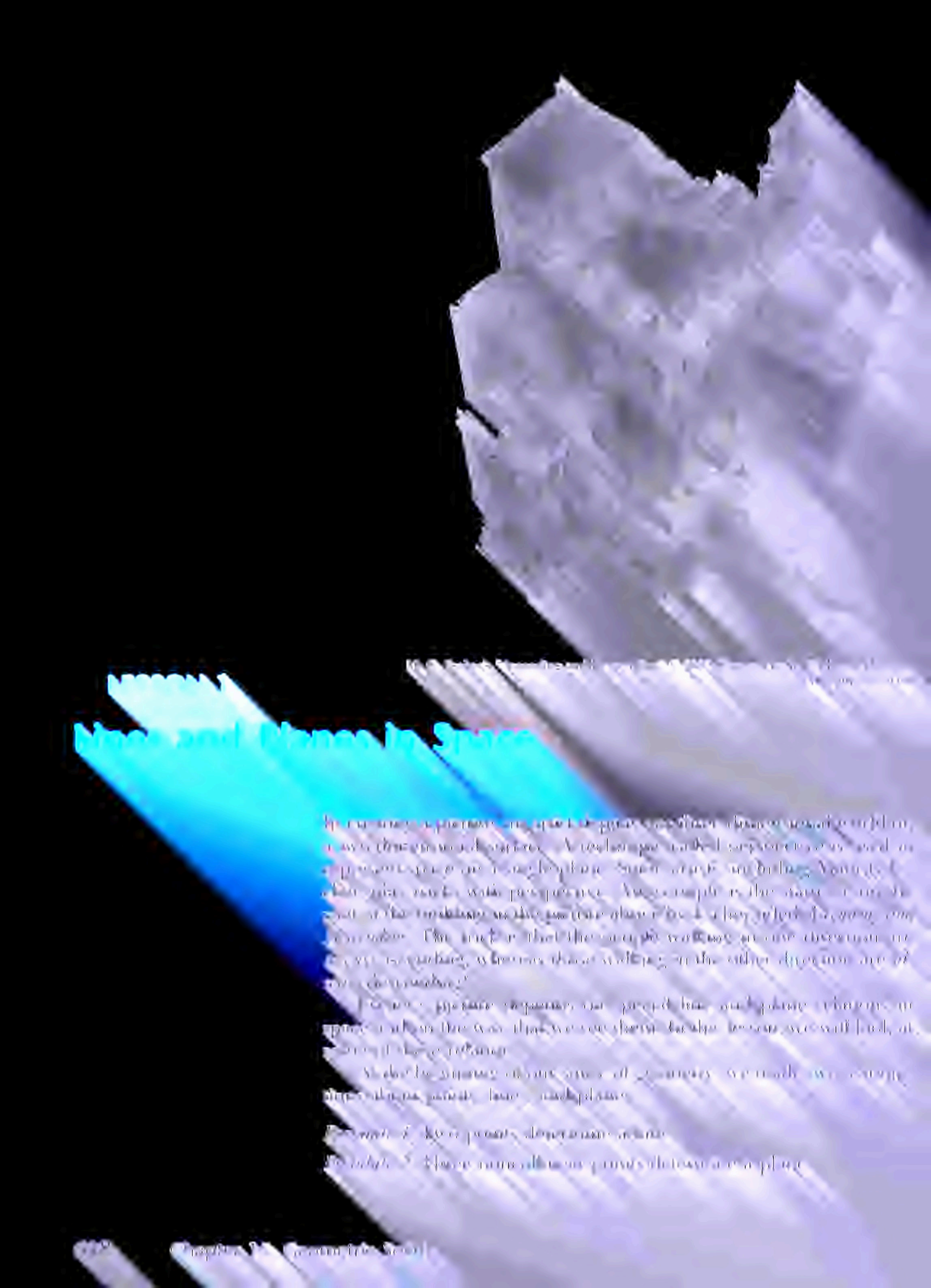






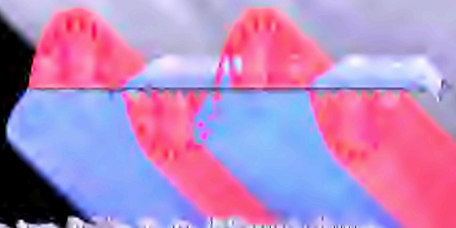






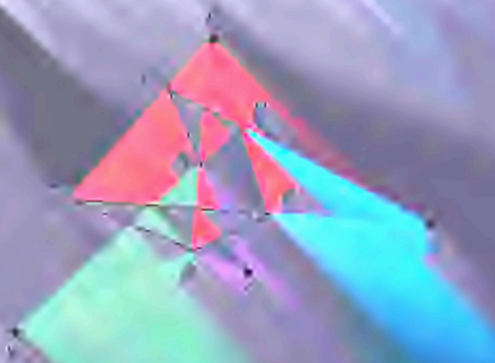






















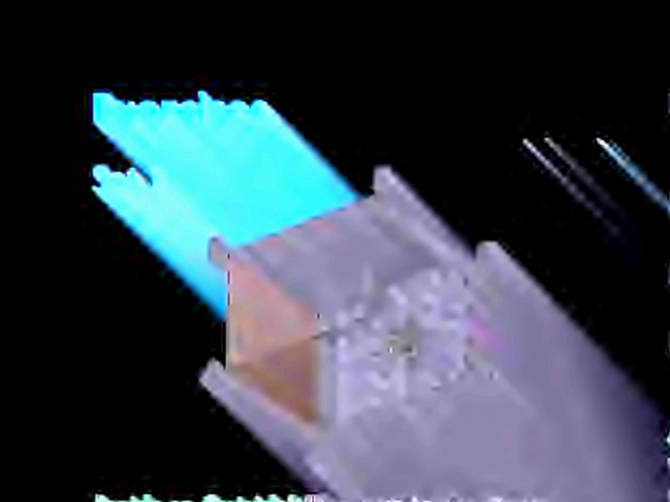

SVERIGE 25
SVERIGE 50
SVERIGE 75

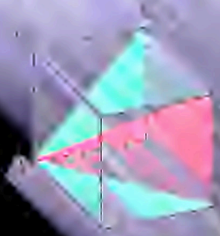






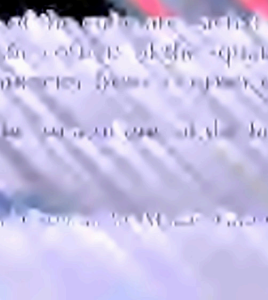






Find the measure of the following angles.











Prisms

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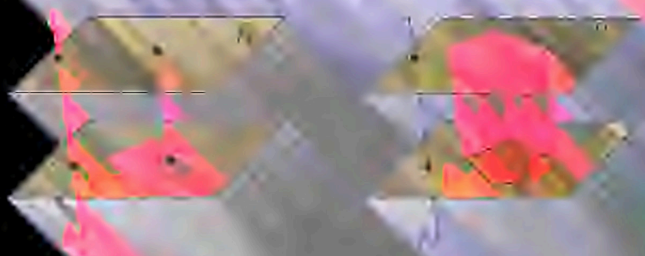
[illegible]

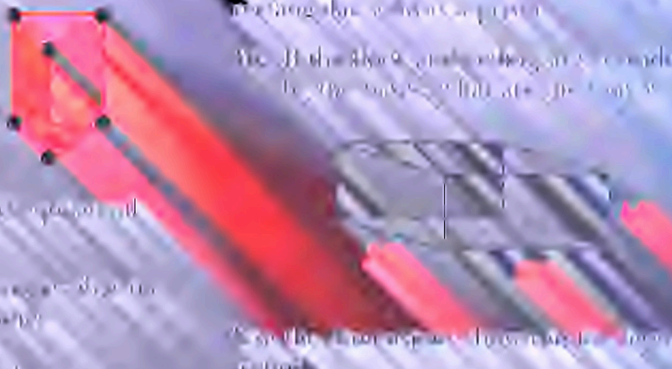
[illegible]

Definition

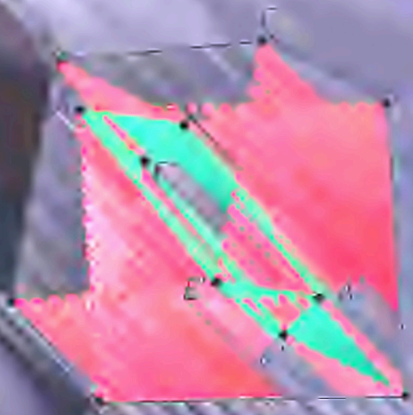
[illegible]







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LEONHARD EULER 1707 - 1783











The Volume of a Pulse

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[illegible]

Definition

[illegible]

[illegible]

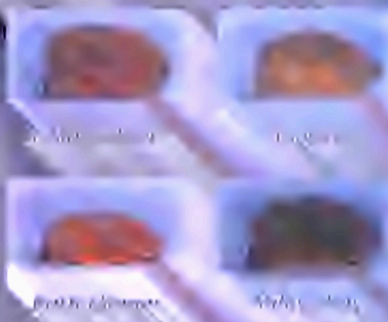
[illegible]



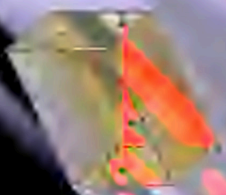


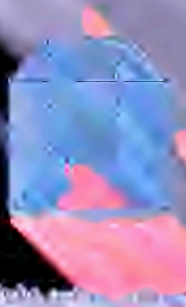








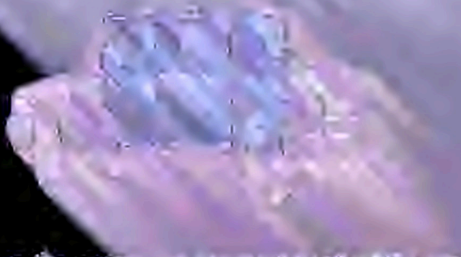


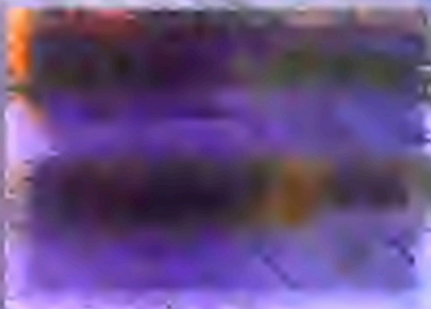


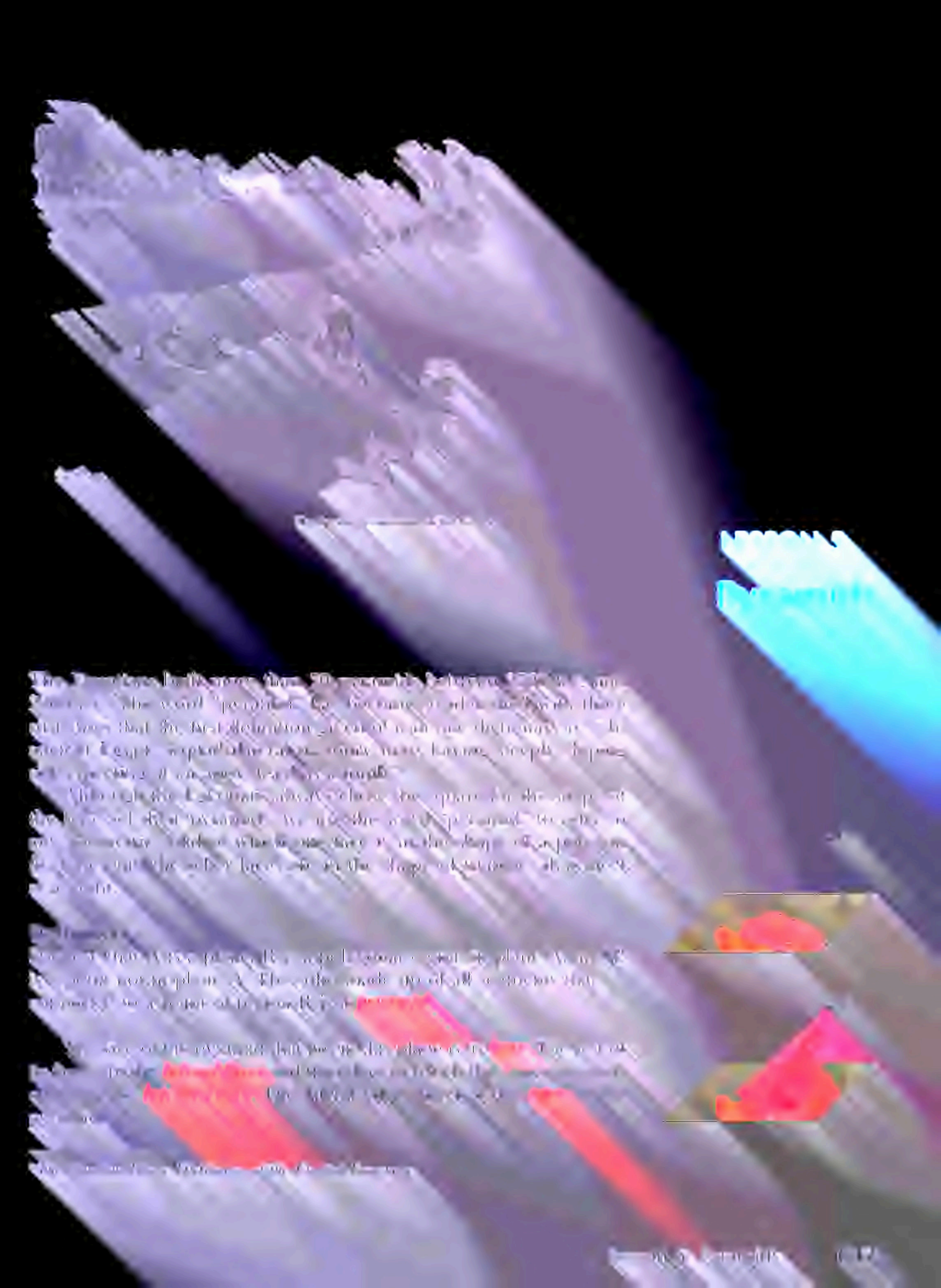


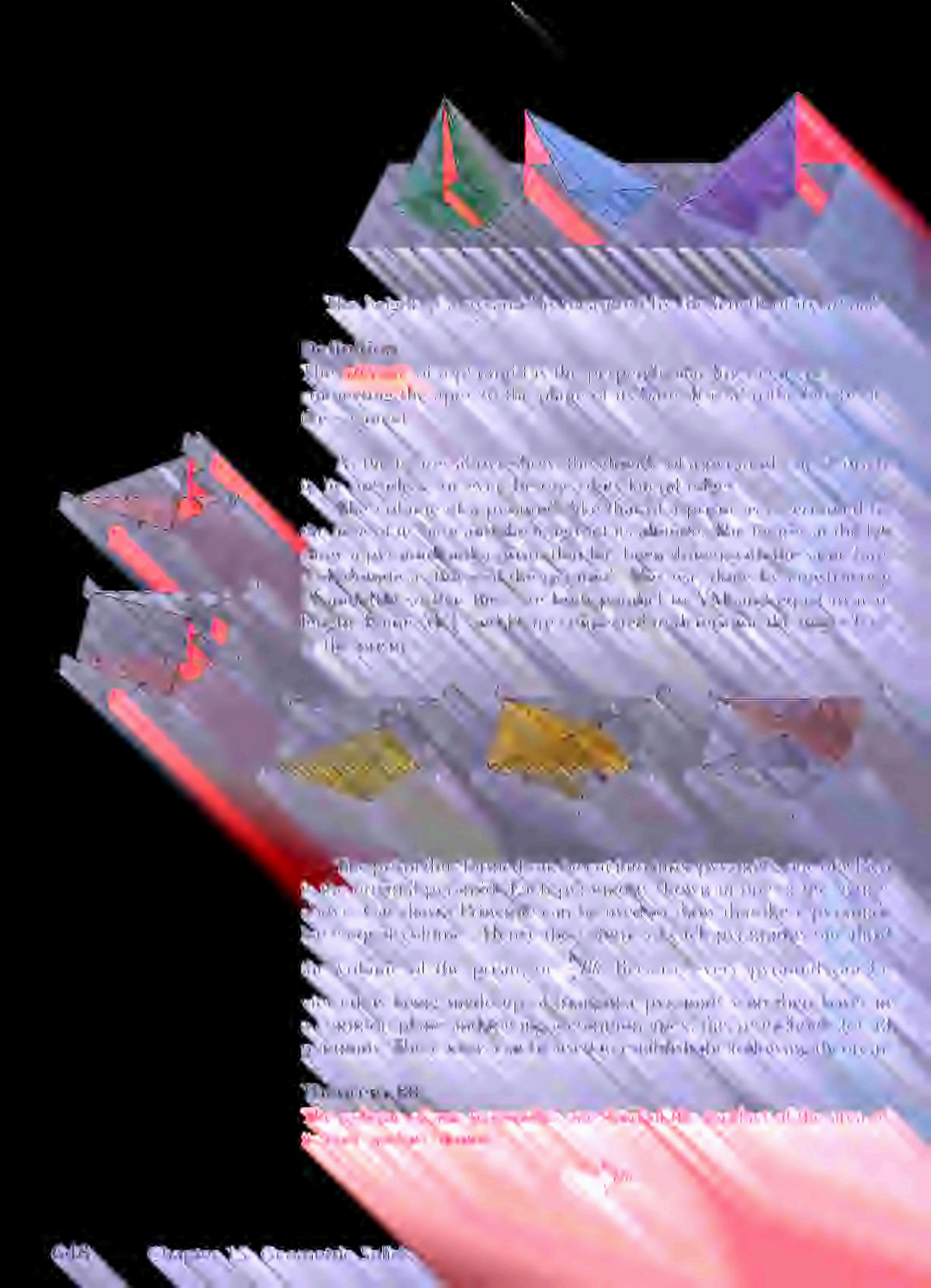




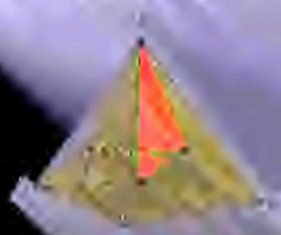


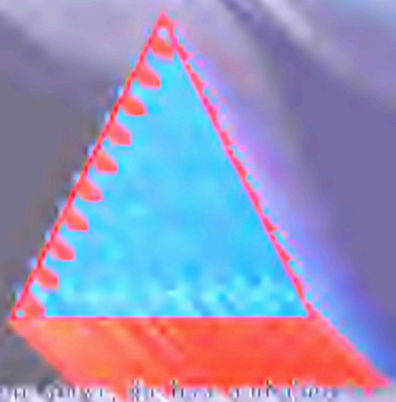




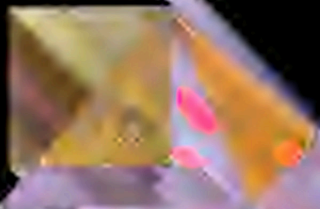




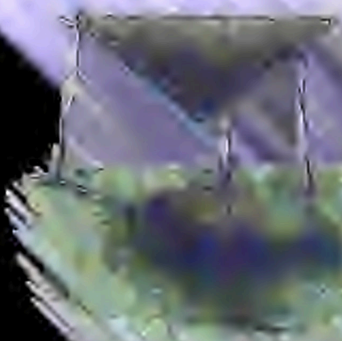


















Cylinders and Cones

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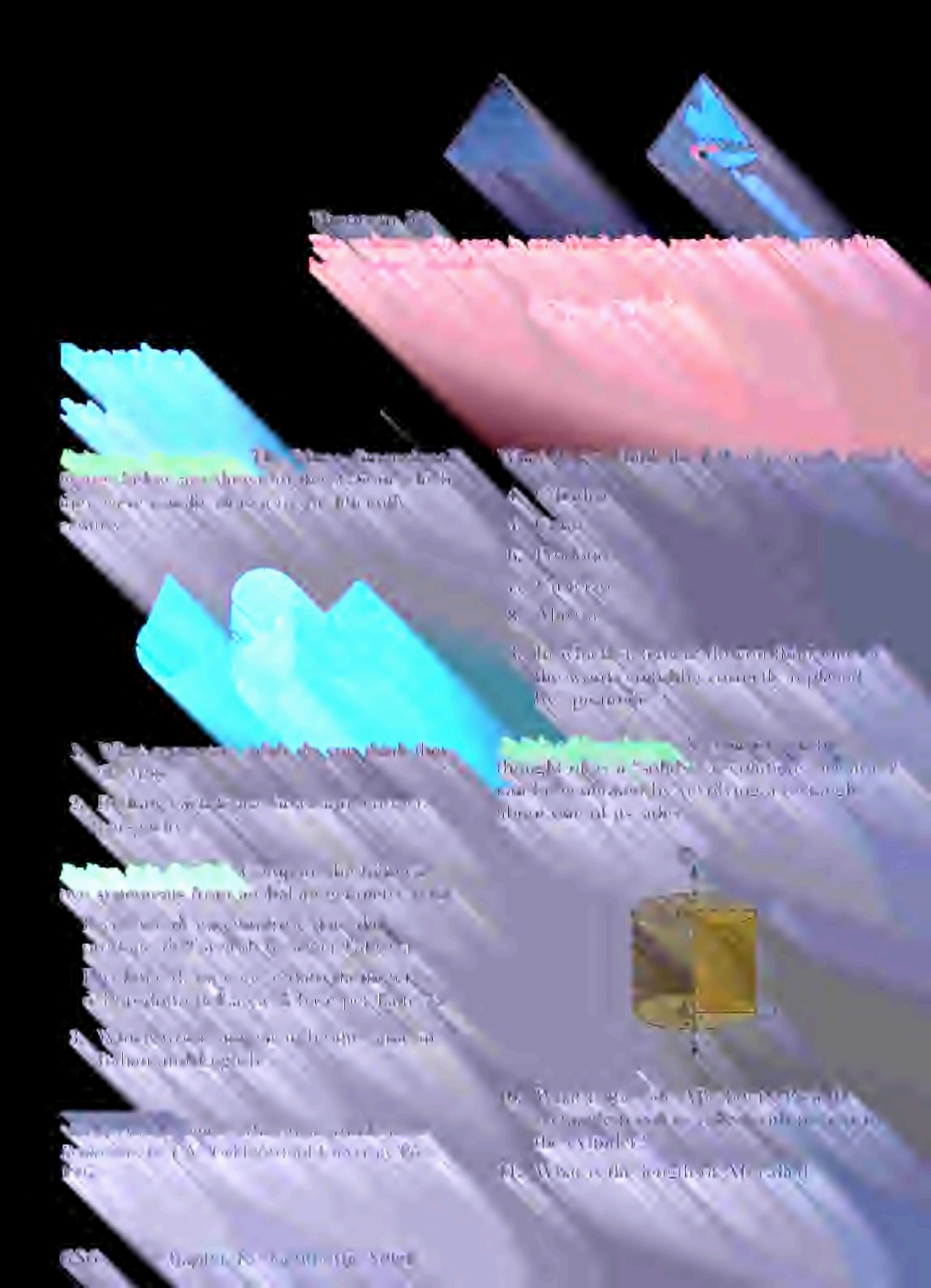
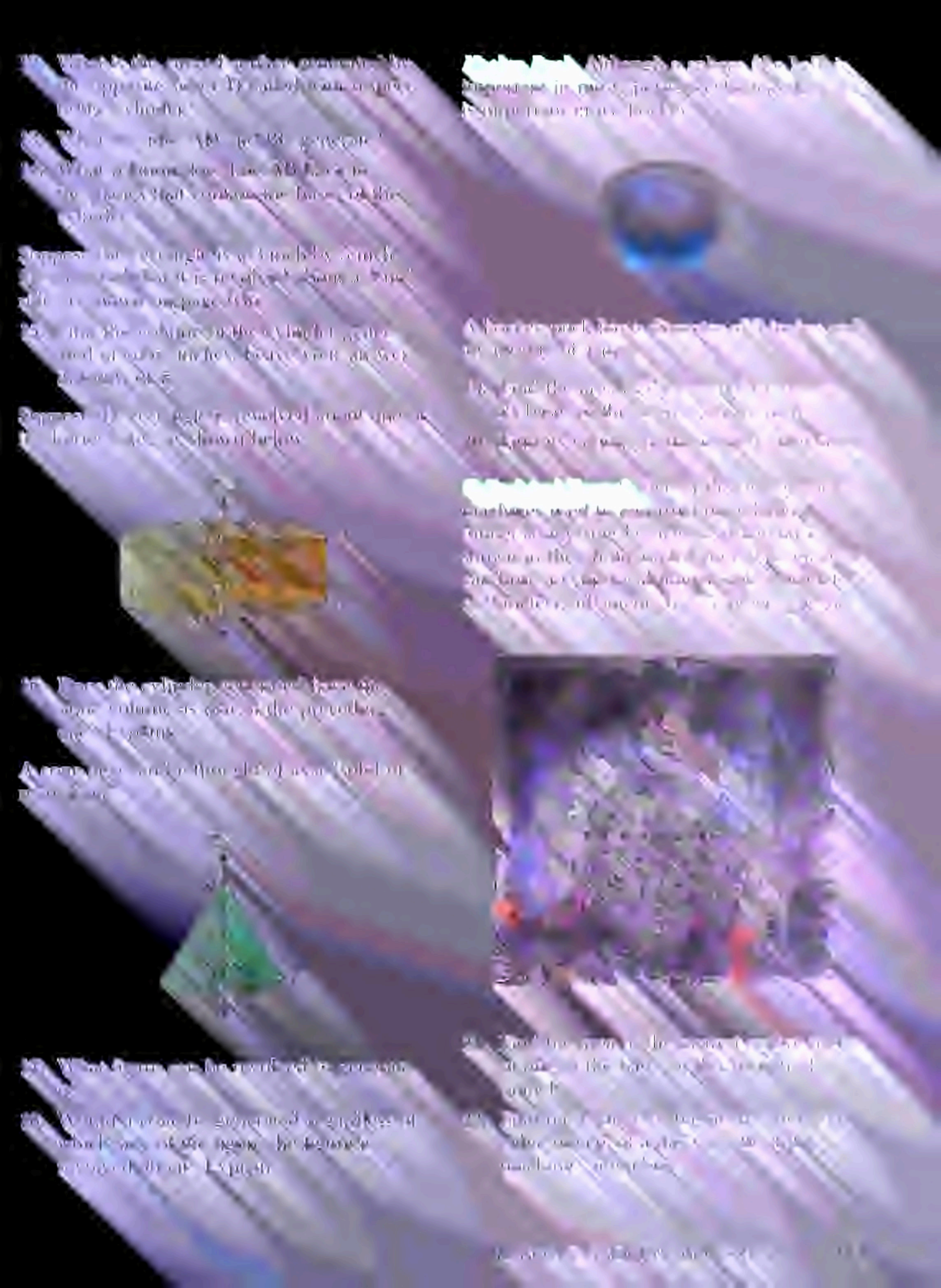
Definition

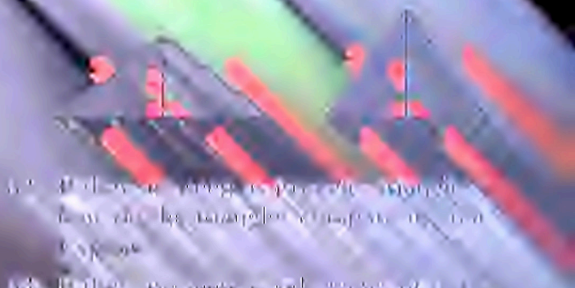
[illegible]













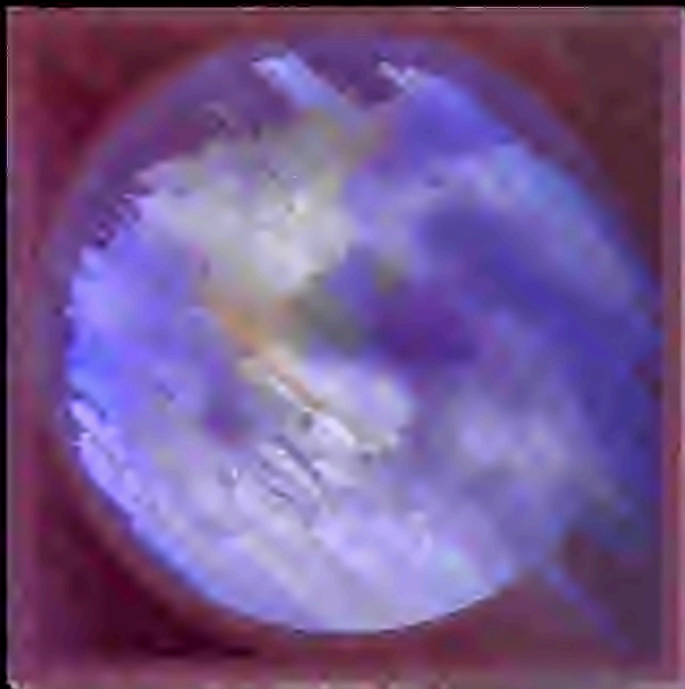










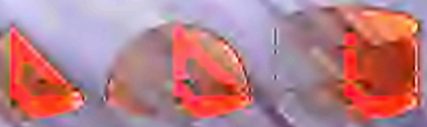


Spheres









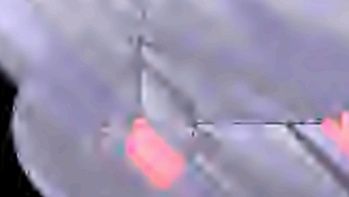






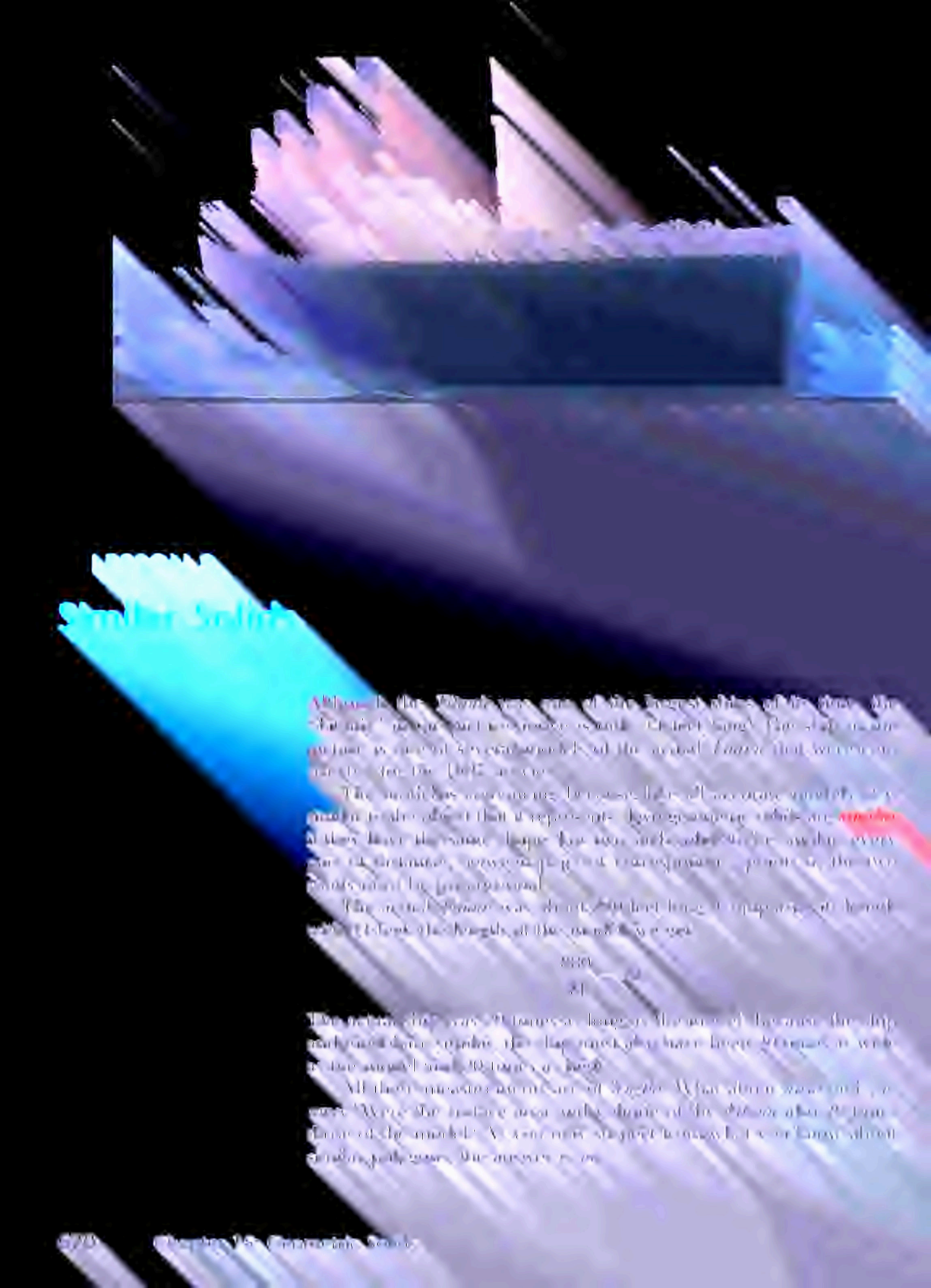












Similar Solids

Although the *Blank* was one of the largest ships of its day, the *Titanic's* [illegible] is still [illegible] feet long. The ship's [illegible] is [illegible] times the [illegible] of the actual *Titanic* that were on [illegible] for the [illegible] service.

The [illegible] because [illegible] is [illegible] to [illegible] that it represents. Two geometric solids are **similar** if they have the same shape but not necessarily the same size. In [illegible] every [illegible] of distance, [illegible] corresponding points of the two [illegible] are proportional.

The actual *Titanic* was about 882 feet long. If you [illegible] to break [illegible] the length of the model, we get

$$\frac{880}{21}$$

The actual *Titanic* was 42 times as long as the [illegible] of the [illegible] the ship [illegible] 42 times as wide as the model and 42 times as high.

All these measurements are of *lengths*. What about *areas* and *volumes*? Were the surface area [illegible] of the *Titanic* also 42 times [illegible] of the model? As you may suspect [illegible] about [illegible] the answer is no.







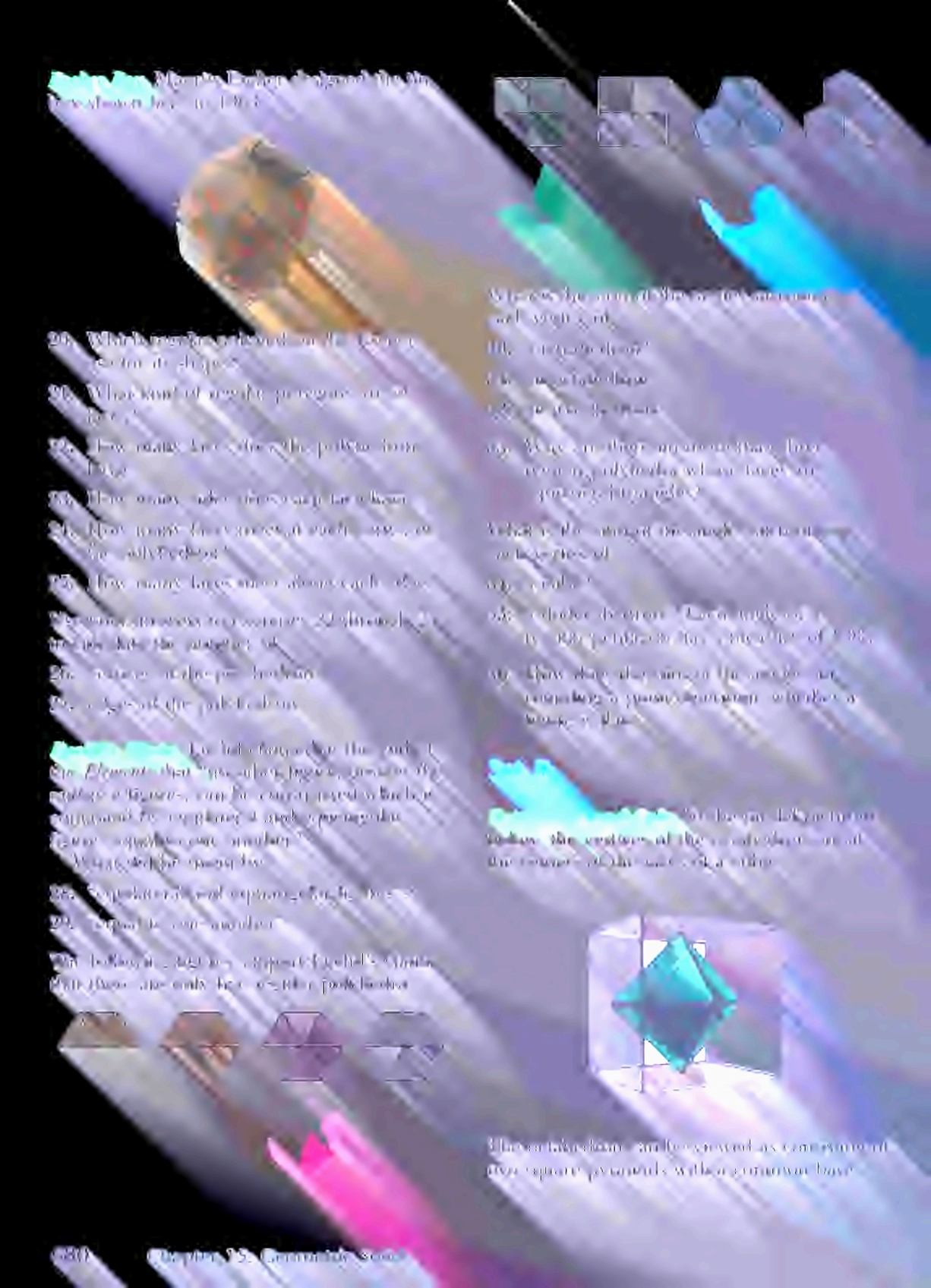




The Regular Polyhedra







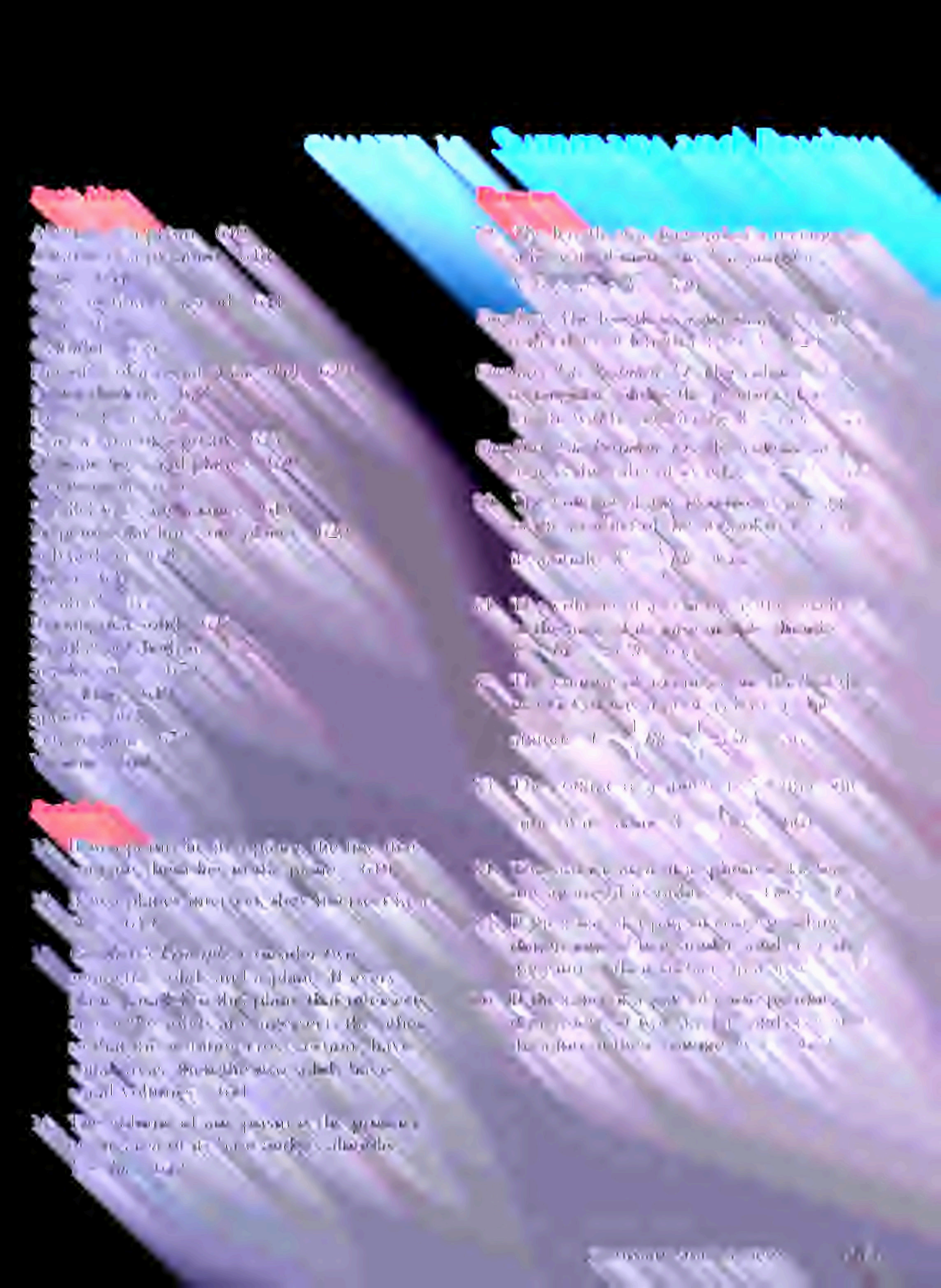




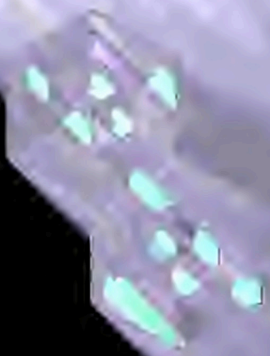








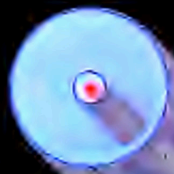


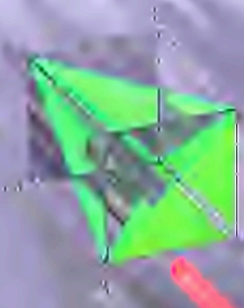






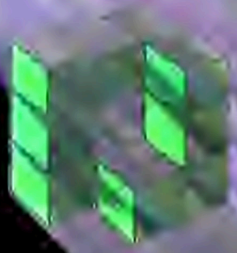






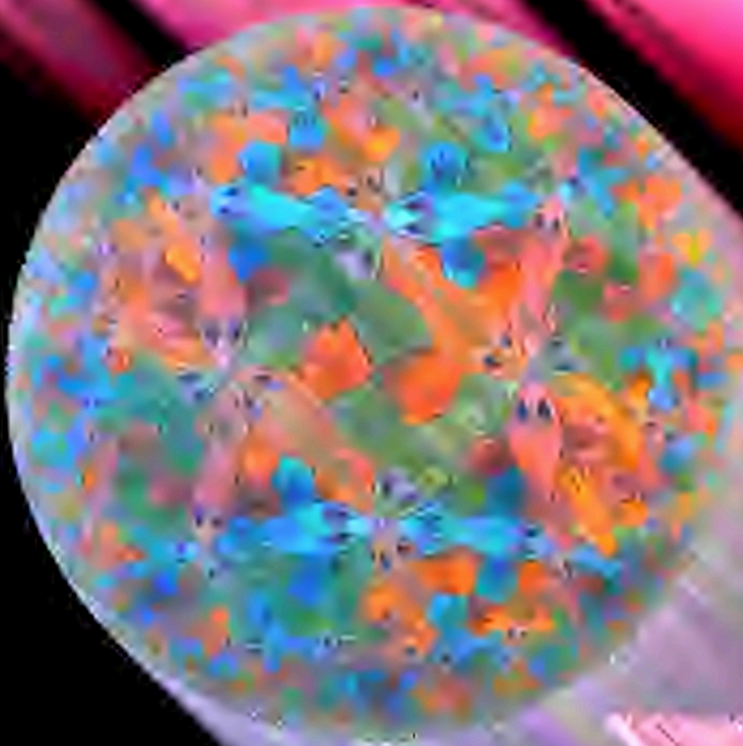


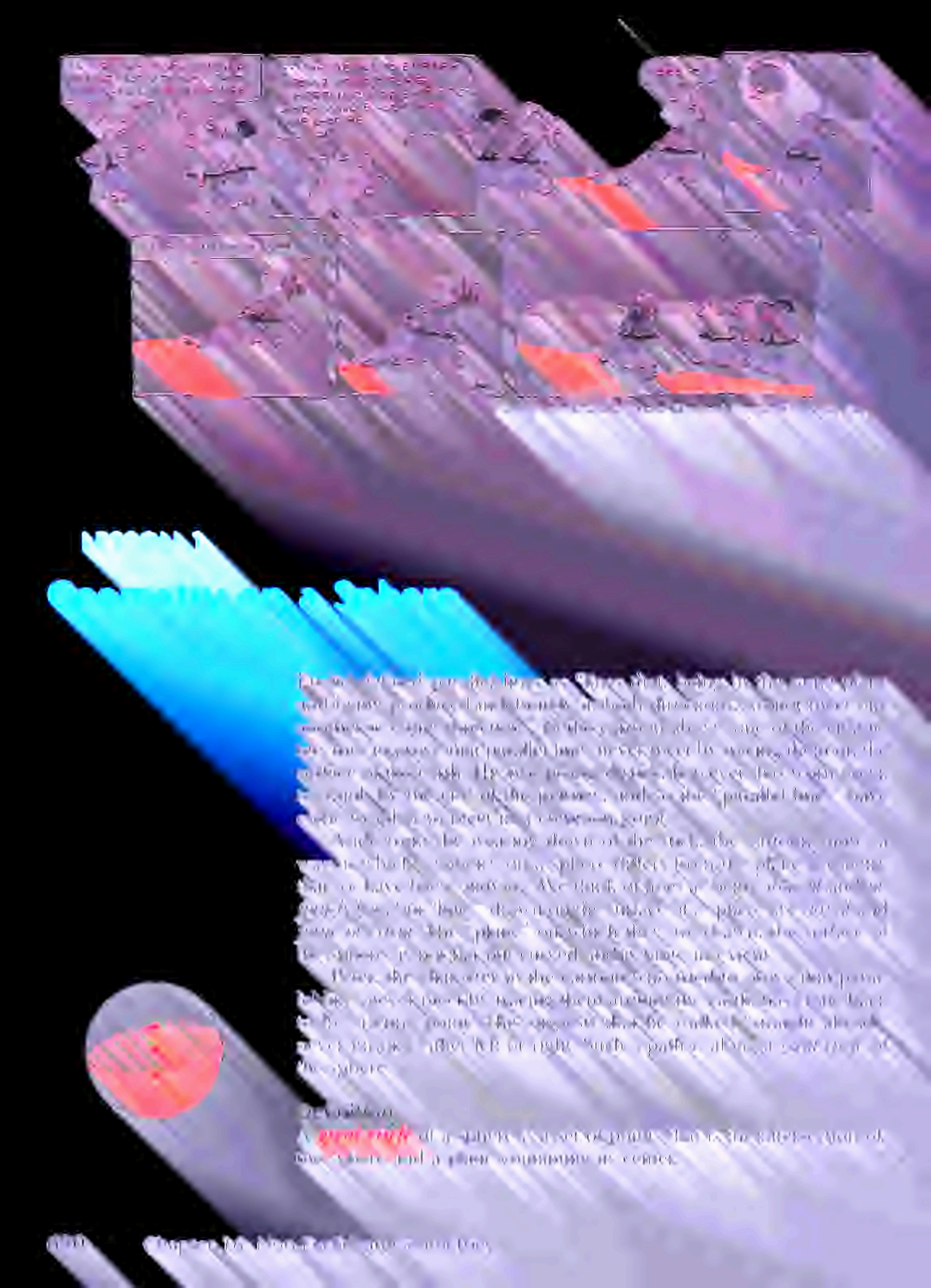


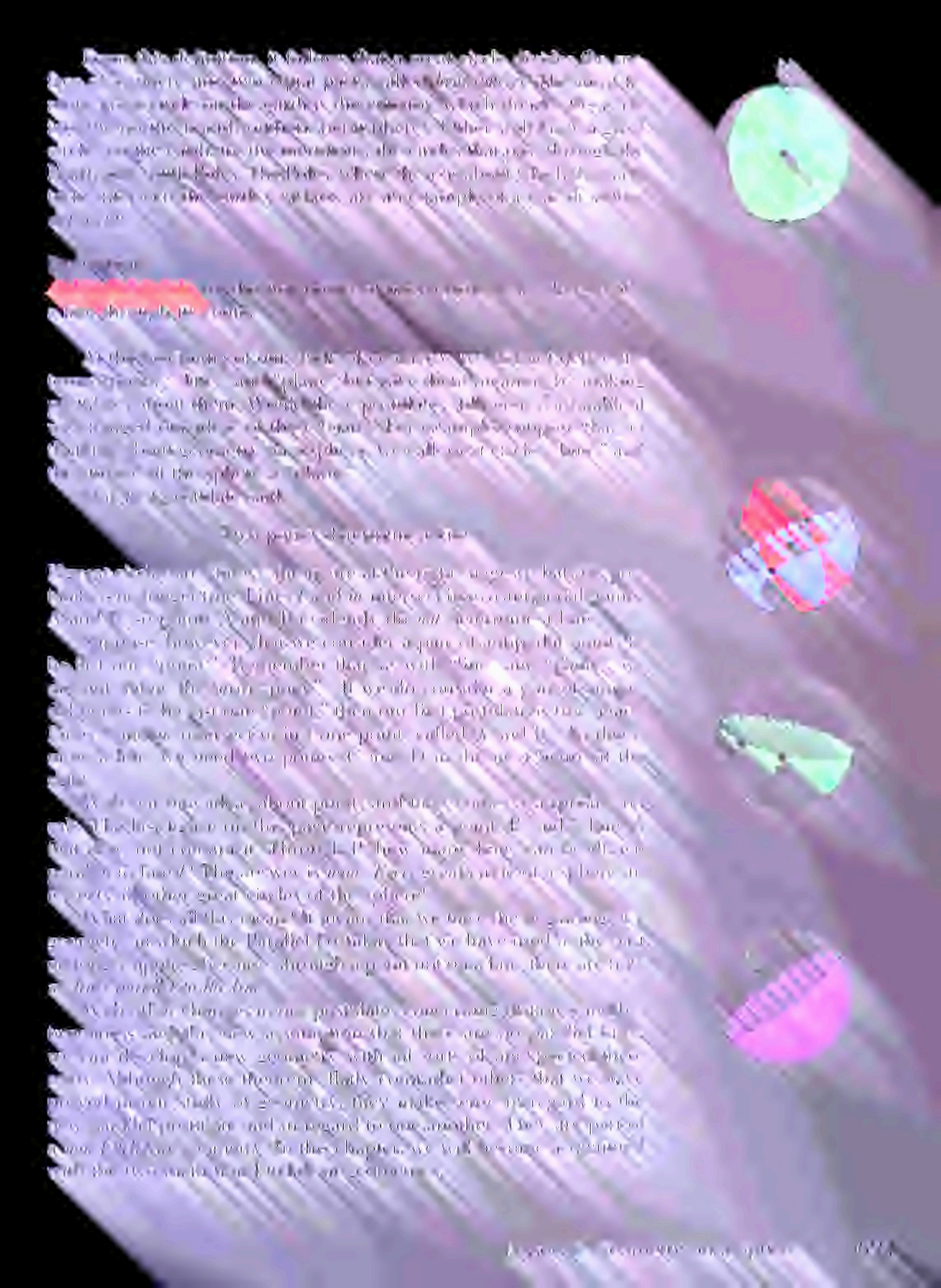




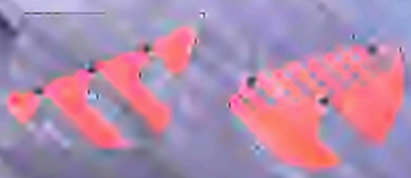


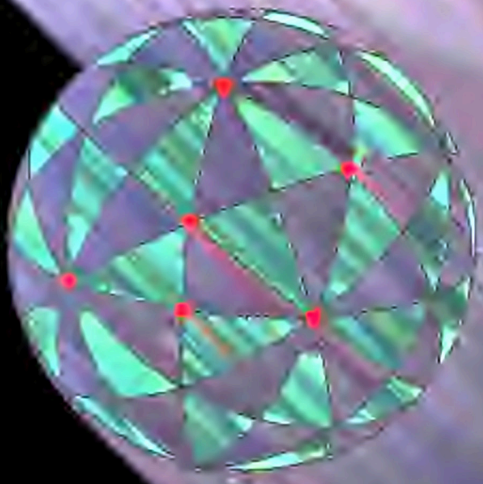








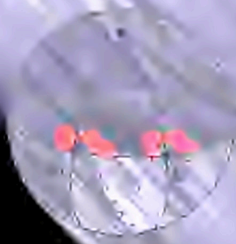




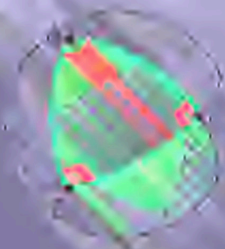














EUCLIDES

AB OMNI NAEVO VINDICATUS

CONATUS GEOMETRICUS

QUO STABILIUNTUR

Principia totius Geometriae

AUCTORE

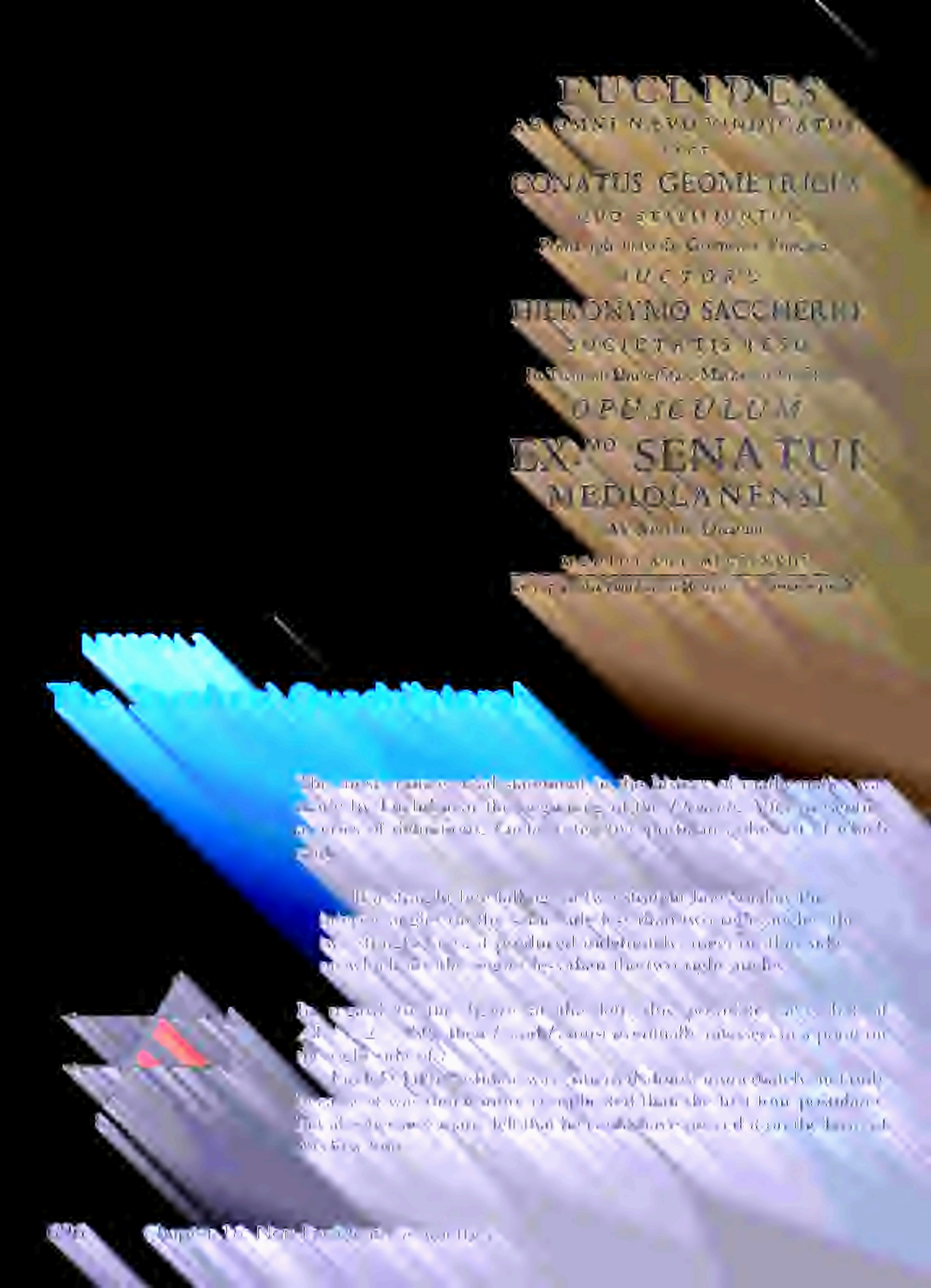
HIERONYMO SACCHERIO

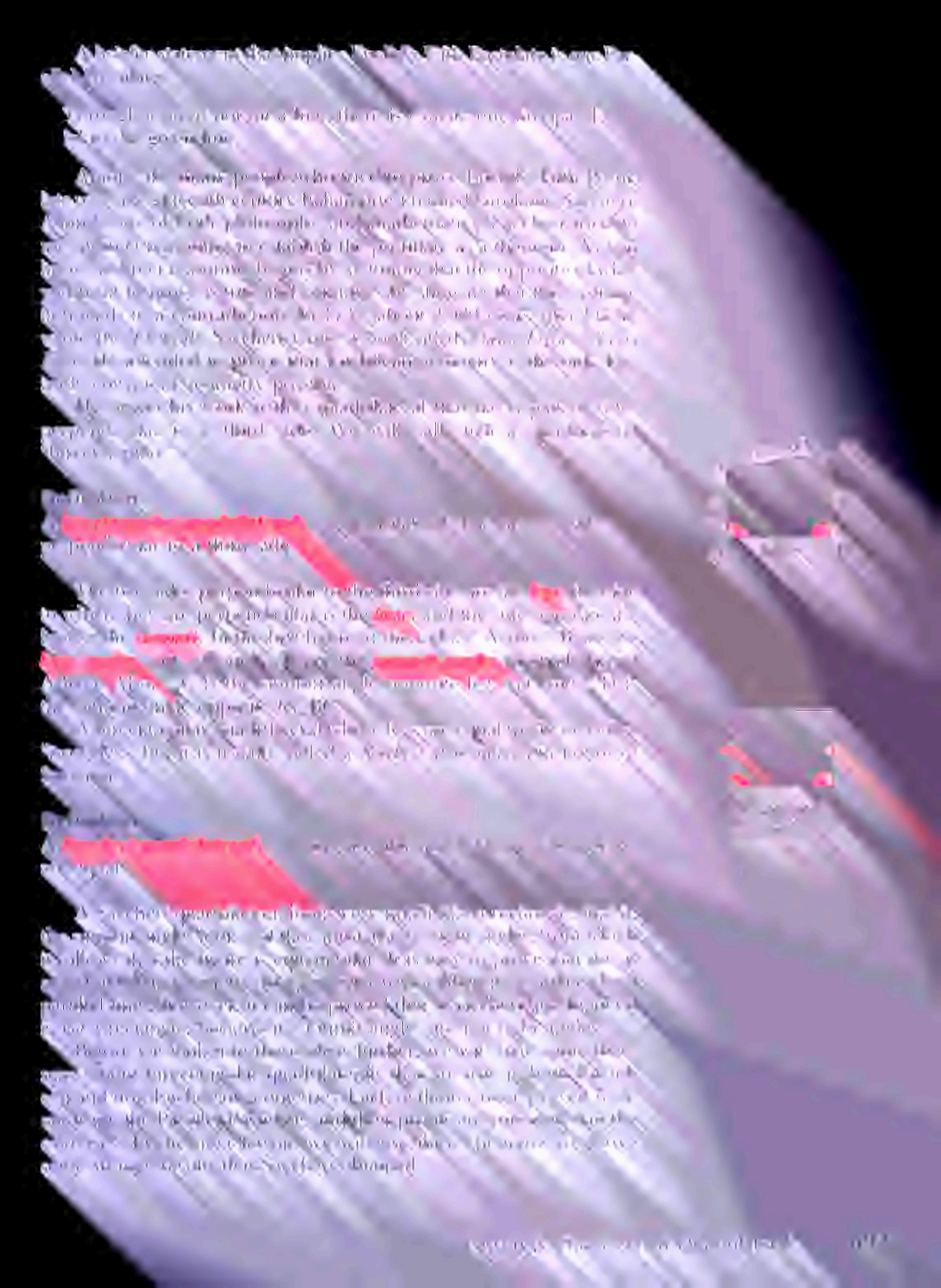
SOCIETATIS IESU

OPUSCULUM

EX.MO SENATUI

MEDIOLANENSI



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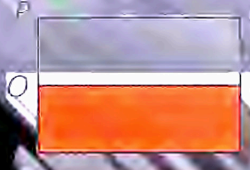

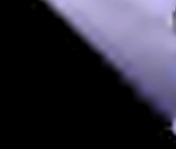

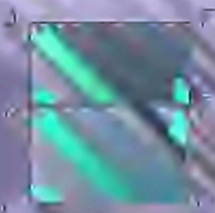

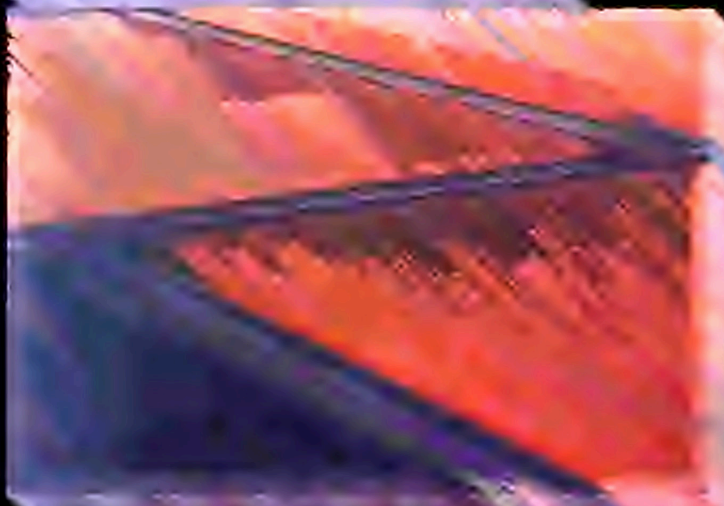





Exercises

1. Use the figure to find the following.

   a. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at point ____.

   b. $\overleftrightarrow{AB}$ and $\overleftrightarrow{EF}$ intersect at point ____.

   c. $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ intersect at point ____.

   d. Name a point that is not on $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$, or $\overleftrightarrow{EF}$.

   e. Name a point that is on $\overleftrightarrow{AB}$ but not on $\overleftrightarrow{CD}$ or $\overleftrightarrow{EF}$.

   f. Name a point that is on $\overleftrightarrow{CD}$ but not on $\overleftrightarrow{AB}$ or $\overleftrightarrow{EF}$.

   g. Name a point that is on $\overleftrightarrow{EF}$ but not on $\overleftrightarrow{AB}$ or $\overleftrightarrow{CD}$.

2. Use the figure to find the following.

   a. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at point ____.

   b. $\overleftrightarrow{AB}$ and $\overleftrightarrow{EF}$ intersect at point ____.

   c. $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ intersect at point ____.

   d. Name a point that is not on $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$, or $\overleftrightarrow{EF}$.

   e. Name a point that is on $\overleftrightarrow{AB}$ but not on $\overleftrightarrow{CD}$ or $\overleftrightarrow{EF}$.

   f. Name a point that is on $\overleftrightarrow{CD}$ but not on $\overleftrightarrow{AB}$ or $\overleftrightarrow{EF}$.

   g. Name a point that is on $\overleftrightarrow{EF}$ but not on $\overleftrightarrow{AB}$ or $\overleftrightarrow{CD}$.

3. Use the figure to find the following.

   a. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at point ____.

   b. $\overleftrightarrow{AB}$ and $\overleftrightarrow{EF}$ intersect at point ____.

   c. $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ intersect at point ____.

   d. Name a point that is not on $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$, or $\overleftrightarrow{EF}$.

   e. Name a point that is on $\overleftrightarrow{AB}$ but not on $\overleftrightarrow{CD}$ or $\overleftrightarrow{EF}$.

   f. Name a point that is on $\overleftrightarrow{CD}$ but not on $\overleftrightarrow{AB}$ or $\overleftrightarrow{EF}$.

   g. Name a point that is on $\overleftrightarrow{EF}$ but not on $\overleftrightarrow{AB}$ or $\overleftrightarrow{CD}$.

4. Use the figure to find the following.

   a. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at point ____.

   b. $\overleftrightarrow{AB}$ and $\overleftrightarrow{EF}$ intersect at point ____.

   c. $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ intersect at point ____.

   d. Name a point that is not on $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$, or $\overleftrightarrow{EF}$.

   e. Name a point that is on $\overleftrightarrow{AB}$ but not on $\overleftrightarrow{CD}$ or $\overleftrightarrow{EF}$.

   f. Name a point that is on $\overleftrightarrow{CD}$ but not on $\overleftrightarrow{AB}$ or $\overleftrightarrow{EF}$.

   g. Name a point that is on $\overleftrightarrow{EF}$ but not on $\overleftrightarrow{AB}$ or $\overleftrightarrow{CD}$.

5. Use the figure to find the following.

   a. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at point ____.

   b. $\overleftrightarrow{AB}$ and $\overleftrightarrow{EF}$ intersect at point ____.

   c. $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ intersect at point ____.

   d. Name a point that is not on $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$, or $\overleftrightarrow{EF}$.

   e. Name a point that is on $\overleftrightarrow{AB}$ but not on $\overleftrightarrow{CD}$ or $\overleftrightarrow{EF}$.

   f. Name a point that is on $\overleftrightarrow{CD}$ but not on $\overleftrightarrow{AB}$ or $\overleftrightarrow{EF}$.

   g. Name a point that is on $\overleftrightarrow{EF}$ but not on $\overleftrightarrow{AB}$ or $\overleftrightarrow{CD}$.

6. Use the figure to find the following.

   a. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at point ____.

   b. $\overleftrightarrow{AB}$ and $\overleftrightarrow{EF}$ intersect at point ____.

   c. $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ intersect at point ____.

   d. Name a point that is not on $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$, or $\overleftrightarrow{EF}$.

   e. Name a point that is on $\overleftrightarrow{AB}$ but not on $\overleftrightarrow{CD}$ or $\overleftrightarrow{EF}$.

   f. Name a point that is on $\overleftrightarrow{CD}$ but not on $\overleftrightarrow{AB}$ or $\overleftrightarrow{EF}$.

   g. Name a point that is on $\overleftrightarrow{EF}$ but not on $\overleftrightarrow{AB}$ or $\overleftrightarrow{CD}$.

7. Use the figure to find the following.

   a. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at point ____.

   b. $\overleftrightarrow{AB}$ and $\overleftrightarrow{EF}$ intersect at point ____.

   c. $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ intersect at point ____.

   d. Name a point that is not on $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$, or $\overleftrightarrow{EF}$.

   e. Name a point that is on $\overleftrightarrow{AB}$ but not on $\overleftrightarrow{CD}$ or $\overleftrightarrow{EF}$.

   f. Name a point that is on $\overleftrightarrow{CD}$ but not on $\overleftrightarrow{AB}$ or $\overleftrightarrow{EF}$.

   g. Name a point that is on $\overleftrightarrow{EF}$ but not on $\overleftrightarrow{AB}$ or $\overleftrightarrow{CD}$.

8. Use the figure to find the following.

   a. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at point ____.

   b. $\overleftrightarrow{AB}$ and $\overleftrightarrow{EF}$ intersect at point ____.

   c. $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ intersect at point ____.

   d. Name a point that is not on $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$, or $\overleftrightarrow{EF}$.

   e. Name a point that is on $\overleftrightarrow{AB}$ but not on $\overleftrightarrow{CD}$ or $\overleftrightarrow{EF}$.

   f. Name a point that is on $\overleftrightarrow{CD}$ but not on $\overleftrightarrow{AB}$ or $\overleftrightarrow{EF}$.

   g. Name a point that is on $\overleftrightarrow{EF}$ but not on $\overleftrightarrow{AB}$ or $\overleftrightarrow{CD}$.

9. Use the figure to find the following.

   a. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at point ____.

   b. $\overleftrightarrow{AB}$ and $\overleftrightarrow{EF}$ intersect at point ____.

   c. $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ intersect at point ____.

   d. Name a point that is not on $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$, or $\overleftrightarrow{EF}$.

   e. Name a point that is on $\overleftrightarrow{AB}$ but not on $\overleftrightarrow{CD}$ or $\overleftrightarrow{EF}$.

   f. Name a point that is on $\overleftrightarrow{CD}$ but not on $\overleftrightarrow{AB}$ or $\overleftrightarrow{EF}$.

   g. Name a point that is on $\overleftrightarrow{EF}$ but not on $\overleftrightarrow{AB}$ or $\overleftrightarrow{CD}$.

10. Use the figure to find the following.

    a. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at point ____.

    b. $\overleftrightarrow{AB}$ and $\overleftrightarrow{EF}$ intersect at point ____.

    c. $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ intersect at point ____.

    d. Name a point that is not on $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$, or $\overleftrightarrow{EF}$.

    e. Name a point that is on $\overleftrightarrow{AB}$ but not on $\overleftrightarrow{CD}$ or $\overleftrightarrow{EF}$.

    f. Name a point that is on $\overleftrightarrow{CD}$ but not on $\overleftrightarrow{AB}$ or $\overleftrightarrow{EF}$.

    g. Name a point that is on $\overleftrightarrow{EF}$ but not on $\overleftrightarrow{AB}$ or $\overleftrightarrow{CD}$.


HOLY CROP 1951


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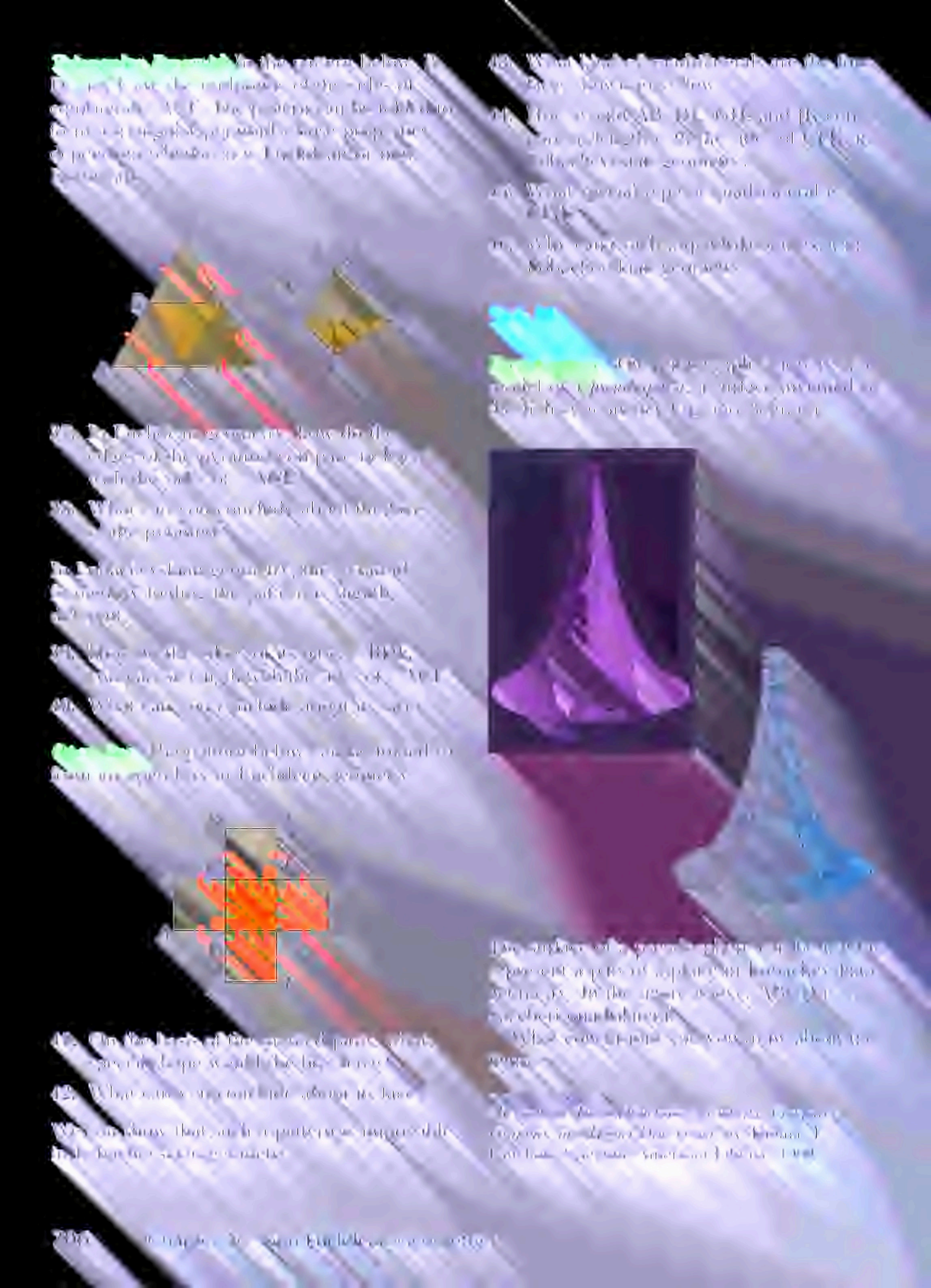


[illegible]

[illegible]

Corollary to Euclid's Fifth Postulate

[illegible]

Euclidean Theorem 4

[illegible]

[illegible]

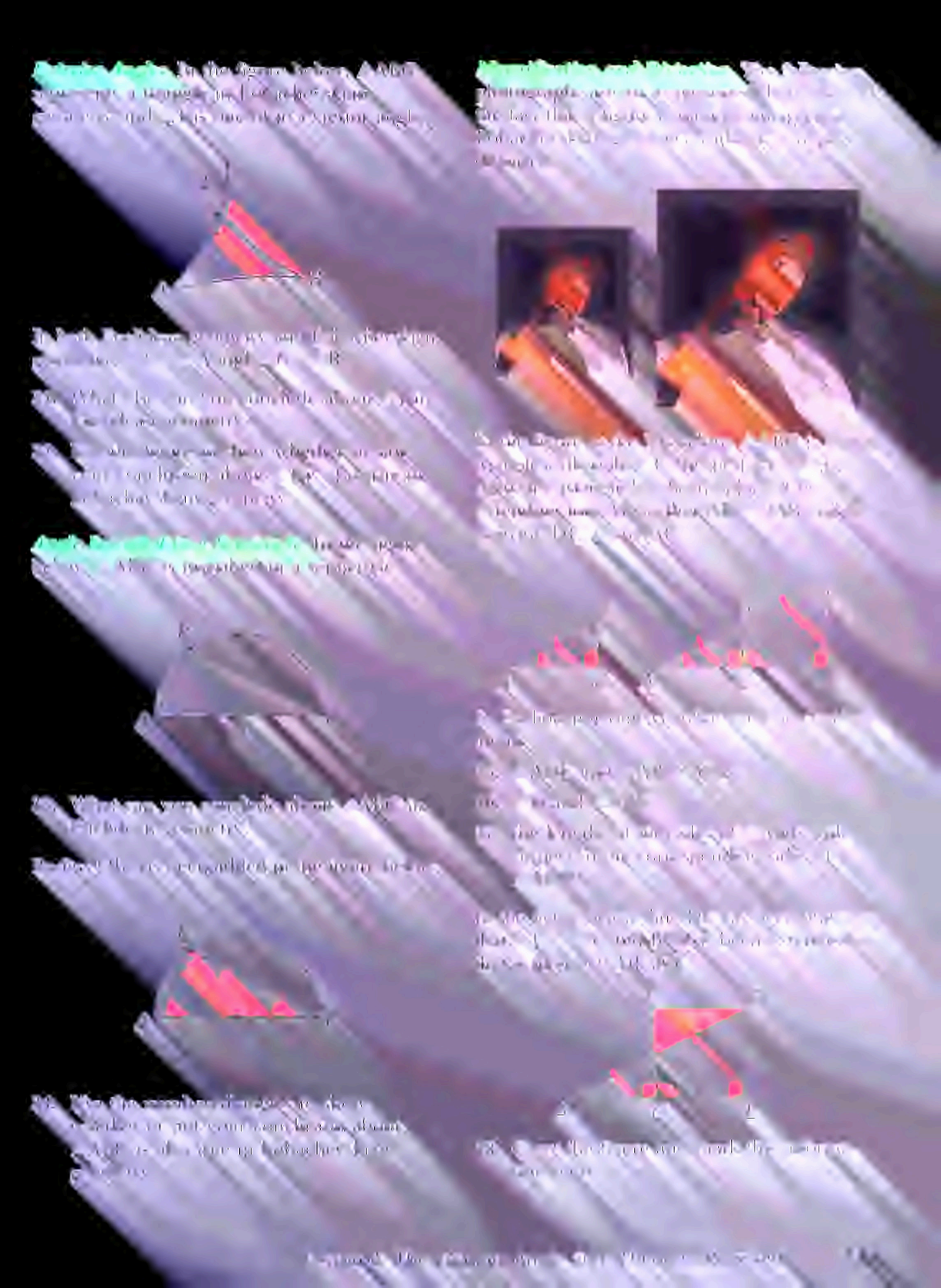
> [illegible]
> [illegible]
> [illegible]

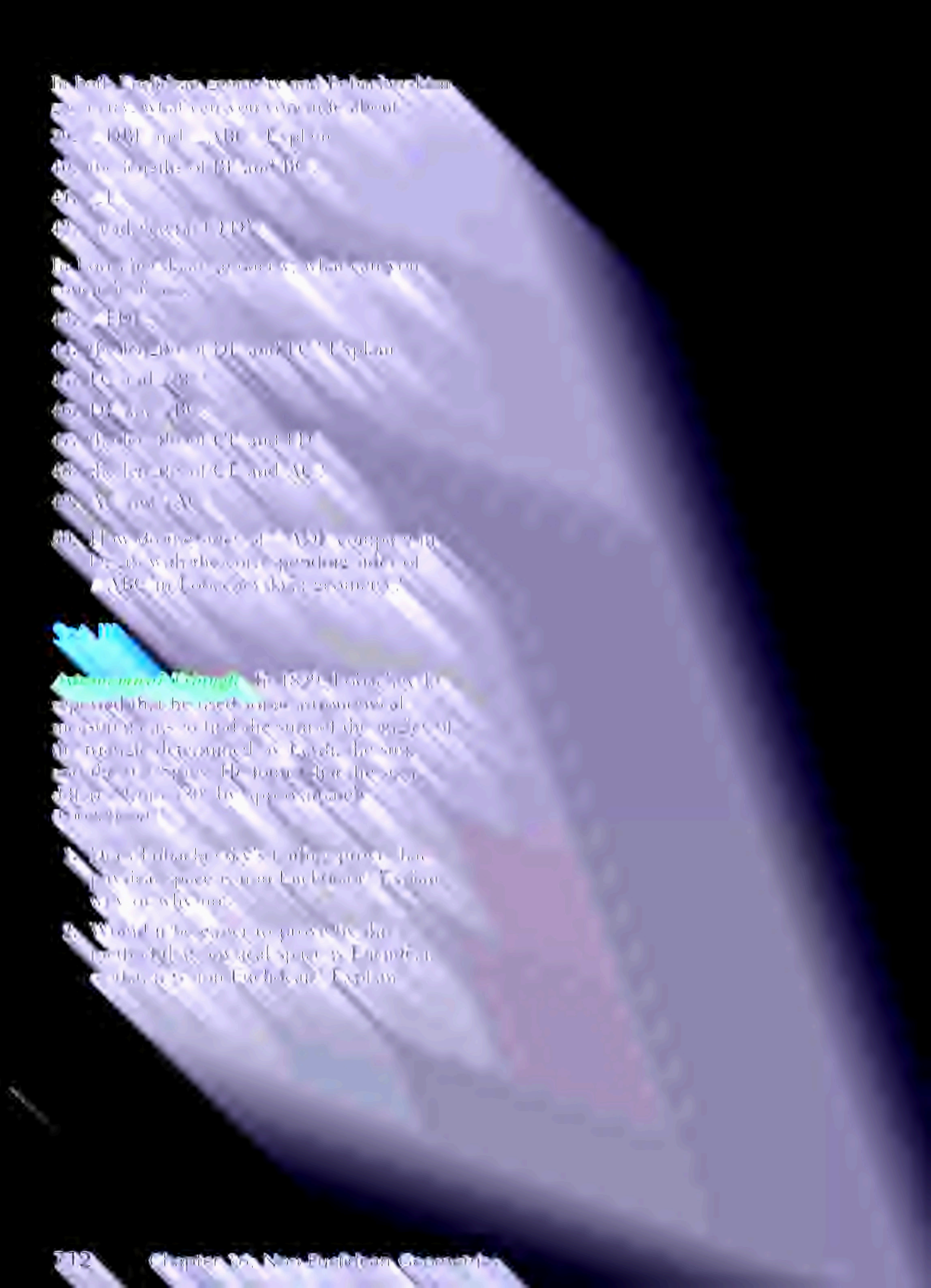
[illegible]







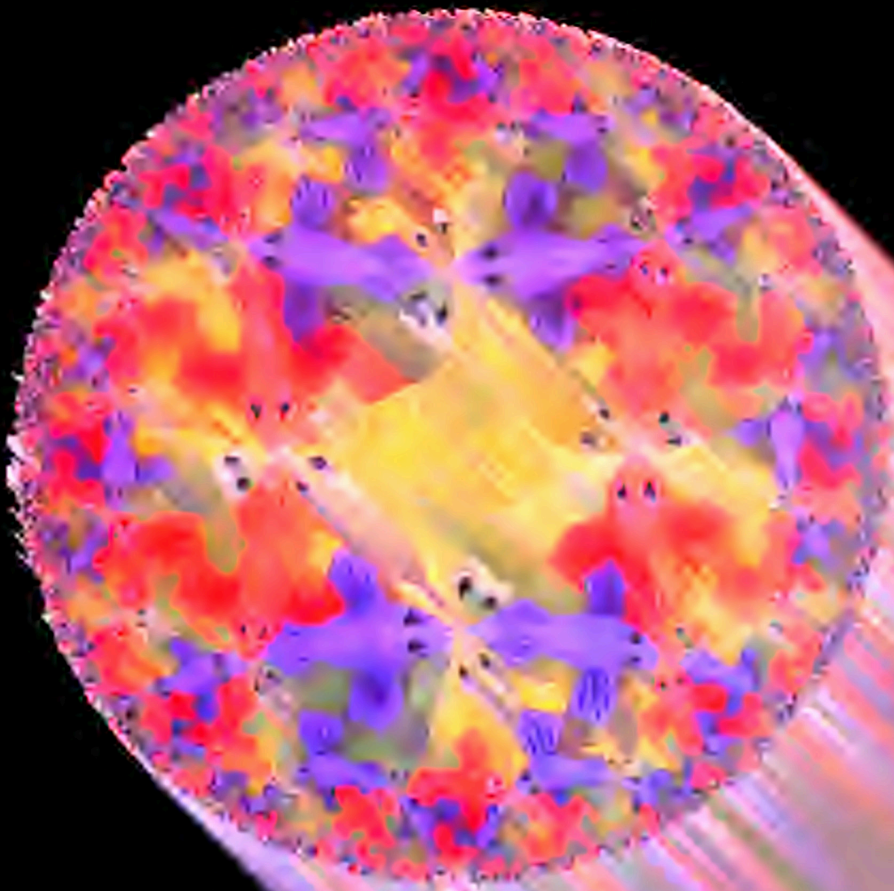
















Final Review

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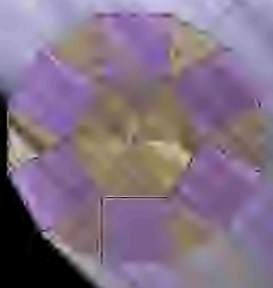
[illegible]



1. [illegible]
2. [illegible]
3. [illegible]

[illegible]



4. [illegible]
5. [illegible]
6. [illegible]
7. [illegible]
8. [illegible]
9. [illegible]

[illegible]

10. [illegible]
11. [illegible]
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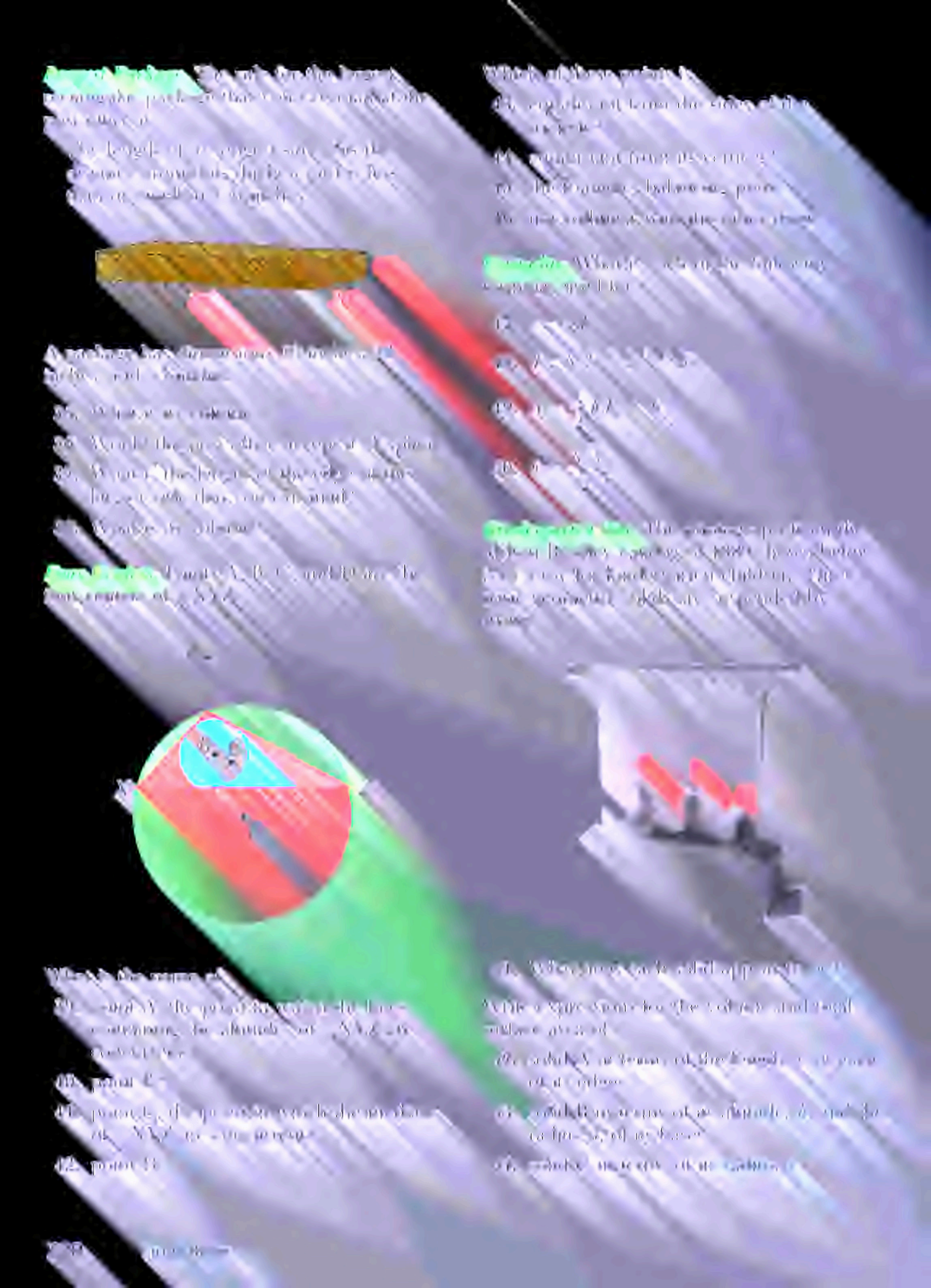
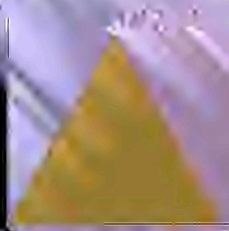
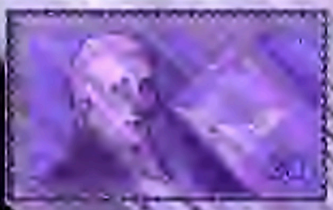






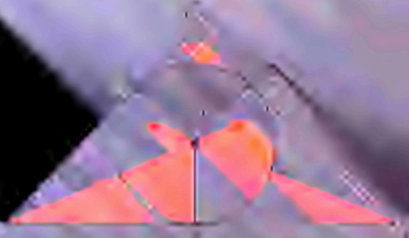


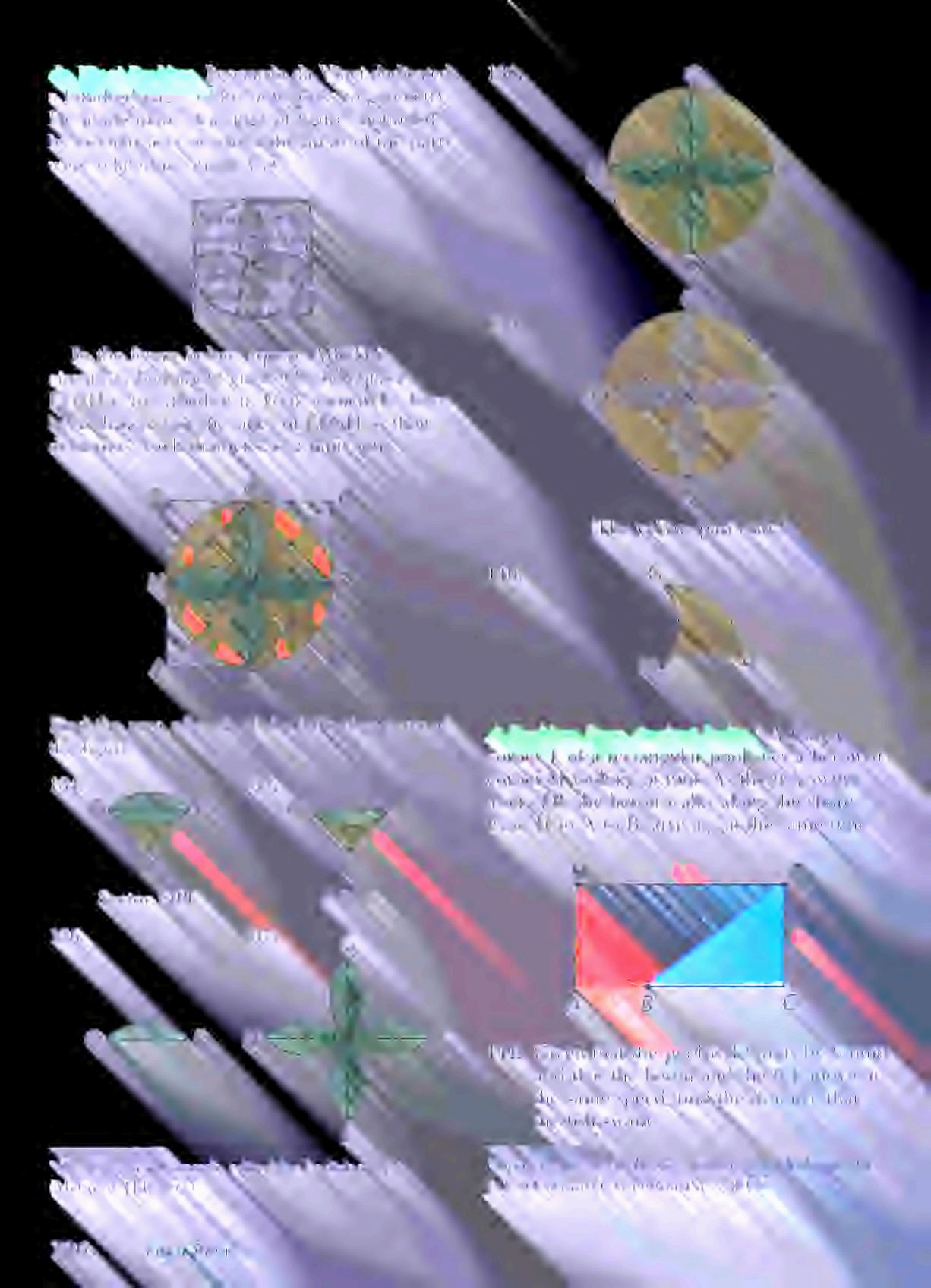




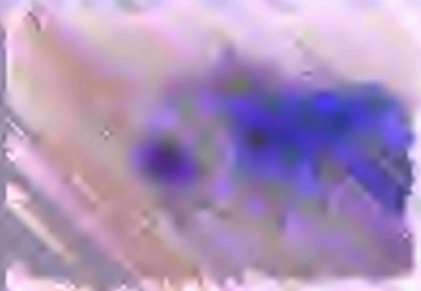












Glossary

Acute angle: An angle that is less than 90°.

Acute triangle: A triangle with three acute angles.

Altitude of a prism: The perpendicular distance between the bases of a prism. The altitude is also known as the height of the prism.

Altitude of a pyramid: The perpendicular distance between the apex of a pyramid and the plane that contains the base. The altitude is also known as the height of the pyramid.



Apothem of a regular polygon: A segment that is perpendicular to a side of a regular polygon and has one endpoint at the center of the polygon. The other endpoint is on the side of the polygon. The length of this segment is also called the apothem.

Arc: A curved segment of a circle. The endpoints of an arc are the endpoints of the chord that cuts off the arc. The measure of an arc is the measure of the central angle that intercepts the arc.

Area: The number of square units that are needed to cover a surface.

Area of a circle: The area of a circle is the product of π and the square of the radius of the circle. In symbols, $A = \pi r^2$.

Area of a sector: The area of a sector of a circle is the product of the measure of the central angle that intercepts the arc of the sector and the area of the circle, divided by 360°.

Area of a triangle: The area of a triangle is one-half the product of the base and the height of the triangle. In symbols, $A = \frac{1}{2}bh$.

Arithmetic mean: The sum of a set of numbers divided by the number of numbers in the set. The arithmetic mean is also called the average.

Arithmetic sequence: A sequence of numbers in which each term after the first is found by adding a constant to the previous term. The constant is called the common difference.

Arithmetic series: The sum of the terms of an arithmetic sequence.

Axiom: A statement that is accepted without proof.

Axis of symmetry: A line that divides a figure into two congruent halves that are mirror images of each other.

Base of a prism: Either of the two congruent polygons that are parallel to each other in a prism.

Base of a pyramid: The polygon that is opposite the apex of a pyramid.

Base of a triangle: Any side of a triangle.

Base of an isosceles triangle: The side of an isosceles triangle that is not congruent to the other two sides.

Base of an isosceles trapezoid: Either of the two parallel sides of an isosceles trapezoid.

Bearing: A measure of direction that is given as an acute angle measured from the north or south ray of the vertical line through the point of observation. The bearing is also given as a direction, either east or west, from the north or south ray.

Betweenness of points: A property of points on a line that states that if A, B, and C are collinear points, then B is between A and C if and only if $AB + BC = AC$.



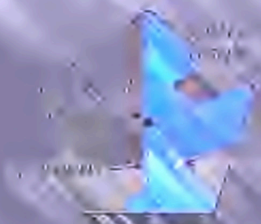
Biconditional statement: A statement that is formed by combining a conditional statement and its converse with the word *and*. A biconditional statement is true if and only if both the conditional statement and its converse are true.

Bisector of an angle: A ray that divides an angle into two congruent angles.

Bisector of a segment: A line, segment, or ray that intersects a segment at its midpoint.

Center of a circle: The point in the interior of a circle that is equidistant from all points on the circle.

Center of a regular polygon: The center of the circle that circumscribes a regular polygon.



Center of a sphere: The point in the interior of a sphere that is equidistant from all points on the sphere.

Central angle of a circle: An angle whose vertex is the center of a circle and whose sides are radii of the circle.

Central angle of a regular polygon: An angle whose vertex is the center of a regular polygon and whose sides are radii of the polygon.

Chord: A segment whose endpoints are on a circle.

Circle: The set of all points in a plane that are equidistant from a given point in the plane. The given point is the center of the circle.

Circumference: The distance around a circle. The circumference of a circle is π times the diameter of the circle. In symbols, $C = \pi d$ or $C = 2\pi r$.

Circumscribed circle: A circle that contains all of the vertices of a polygon.

Collinear points: Points that lie on the same line.

Common tangent: A line, segment, or ray that is tangent to two coplanar circles.

Compass: A tool used to draw circles and arcs.

Complement of an angle: The difference between 90° and the measure of an acute angle.

Complementary angles: Two angles whose measures have a sum of 90°.

Composite figure: A figure that is made up of two or more figures.

Concave polygon: A polygon that is not convex.

Concentric circles: Coplanar circles that have the same center.

Conclusion: The part of a conditional statement that follows the word *then*.

Concurrent lines: Three or more lines that intersect at the same point.

Conditional statement: A statement that can be written in the form "If p, then q," where p and q are statements. The statement p is called the hypothesis, and the statement q is called the conclusion.

Cone: A solid with a circular base, a vertex that is not in the plane of the base, and all segments that connect the vertex to points on the edge of the base.

Congruent angles: Angles that have the same measure.

Congruent arcs: Arcs that have the same measure and are in the same circle or congruent circles.

Congruent circles: Circles that have congruent radii.

Congruent figures: Figures that have the same size and shape.

Congruent segments: Segments that have the same length.

Conjecture: A statement that is believed to be true based on observations.

Consecutive angles: Two angles of a polygon that share a side.

Consecutive sides: Two sides of a polygon that share a vertex.

Contrapositive: The statement formed by negating both the hypothesis and the conclusion of the converse of a conditional statement.

Converse: The statement formed by switching the hypothesis and the conclusion of a conditional statement.

Convex polygon: A polygon such that no line that contains a side of the polygon contains a point in the interior of the polygon.

Coordinate: The number that corresponds to a point on a line.

Coordinate plane: A plane that is divided into four regions by a horizontal number line and a vertical number line that intersect at their zero points. The horizontal number line is usually labeled x, and the vertical number line is usually labeled y.



Coordinate proof: A proof that uses coordinate geometry.

Corollary: A theorem that can be proved easily using another theorem.

Corresponding angles: Angles that are in the same position relative to two lines and a transversal.

Cosine: In a right triangle, the ratio of the length of the leg adjacent to an acute angle to the length of the hypotenuse.

Counterexample: An example that shows that a statement is false.

Cube: A rectangular prism with six congruent square faces.

Cylinder: A solid with congruent circular bases in parallel planes and all segments that connect points on the edges of the bases.

Congruent figures: Informally, two figures [illegible]

Congruent triangles: [illegible]

Converse of a conditional statement: [illegible]

Convex polygon: [illegible]

Coplanar points: [illegible]

Corollary: [illegible]

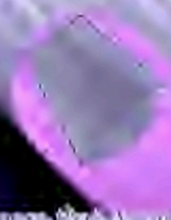
Cosine of an acute angle of a right triangle: [illegible]



Cross section of a geometric solid: [illegible]

Cube: [illegible]

Cyclic polygon: A polygon for which [illegible]



Cylinder: [illegible]



Decagon: [illegible]

Degree measure of an arc: [illegible]

Diagonal of a polygon: [illegible]

Diameter of a circle: [illegible]

Dilation: [illegible]





Icosahedron: A polyhedron that has 20 faces. [illegible]



Incenter of a polygon: [illegible]

Inscribed angle of a circle: [illegible]



Inscribed circle in a polygon: [illegible]



Inscribed polygon in a circle: [illegible]

Isometry: [illegible]

Isosceles trapezoid: [illegible]



Isosceles triangle: A triangle that has at least two equal sides.

Long diagonal of a prism: [illegible]

Length of a line segment: [illegible]

Line segment: [illegible]

Linear pair: [illegible]



Median of a triangle: [illegible]



Midpoint of a line segment: [illegible]

Midpoint of a triangle: [illegible]



Rate: A [illegible]

Rectangle: A [illegible]

Rectangular points: [illegible]

Perpendicular and plane for two planes: [illegible]

Right angle: [illegible]

Right triangle: [illegible]

Rectangle: [illegible]

Perpendicular: [illegible]



Opposite rays: [illegible]



Perpendicular triangle: [illegible]

Perpendicular line and plane: [illegible]

Parallel lines: [illegible]

Parallel planes: [illegible]

Parallelogram: [illegible]

Perpendicular: [illegible]



Perpendicular line and plane: [illegible]



Perpendicular: [illegible]

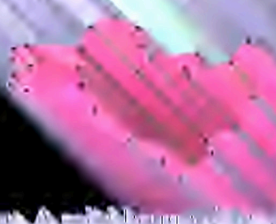
Perpendicular plane: [illegible]

Plane: [illegible]

Polygon: [illegible]



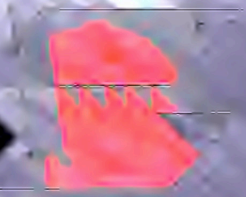
Polygon Angles: The order of a polygon [illegible]



Polyhedron: A solid bounded [illegible]

Polygon: [illegible]

Polygon: [illegible]



Perpendicular: A [illegible]

Perpendicular: [illegible]

Perpendicular: [illegible]

Perpendicular triple: A set of [illegible]

Quadrilateral: A polygon [illegible]

Radius of a circle: [illegible]

Radius of a regular polygon: [illegible]

Ratio: [illegible]

Ray: [illegible]



Rectangle: [illegible]

Rectangular solid: [illegible]



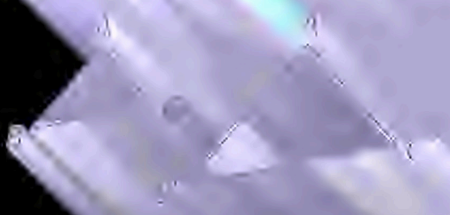
Reflection of a point: [illegible]







Corresponding angles: Two angles in which [illegible]



Sine of an angle: [illegible]



Same-side interior angles: Two lines intersected by a third [illegible]

Sphere: The set of all points in space that are [illegible] a given point.

Square: A quadrilateral all of whose side [illegible] angles are equal.

Straight angle: An angle whose measure is [illegible]

Supplementary angles: [illegible]

Symmetry with respect to a line: [illegible]

Symmetry with respect to a point: [illegible]

Tangent: [illegible]

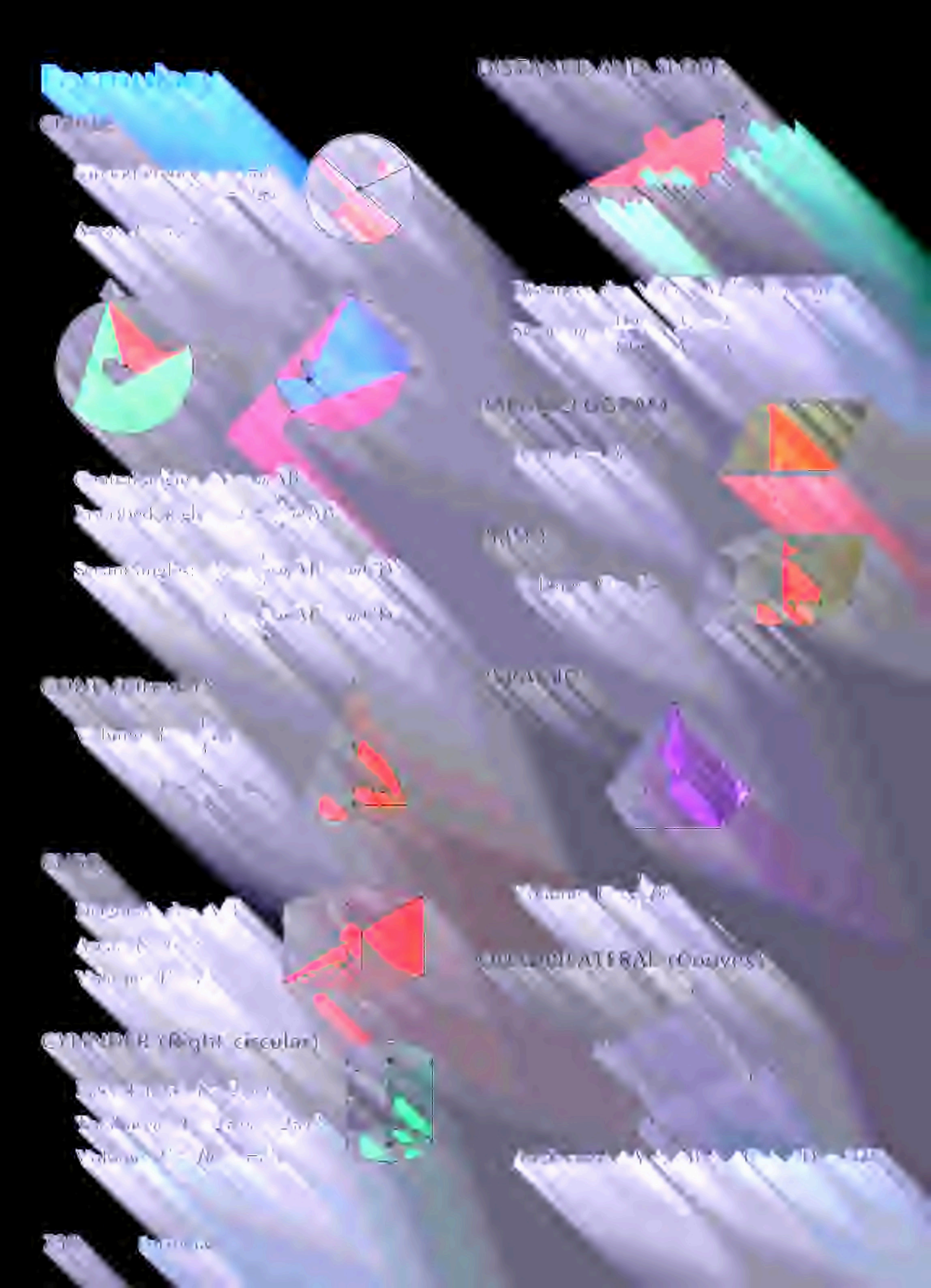
Tangent of an acute angle of a right triangle: [illegible]



Triangle inequality: [illegible]



Triangle: A polygon [illegible] of three line segments [illegible] equilateral triangle.





TRAPEZOID

TRIANGLE

TRIANGLE, EQUILATERAL

TRIANGLE, RIGHT

TRIANGLE, RIGHT (Isosceles)

TRIANGLE, RIGHT (30° - 60°)



Corollary to the Pythagoras Parabola. An [illegible]

Theorem 3. [illegible]

Theorem 4. [illegible]

Theorem 5. [illegible]

Theorem 6. [illegible]

Theorem 7. [illegible]

Corollary to the definition of a right angle. [illegible]

Theorem 8. [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Corollary to the definition of a [illegible] triangle. [illegible]

Theorem 9. [illegible]

Corollary. [illegible]

Theorem 10. [illegible]

Corollary. [illegible]

Theorem 11. [illegible]

[illegible]

Theorem 12. [illegible]

Theorem 13. [illegible]

Theorem 14. [illegible]

Theorem 15. [illegible]

[illegible]

[illegible]

Theorem 16. [illegible]

Theorem 17. [illegible]

Corollary 1. [illegible]

Corollary 2. [illegible]

Corollary 3. [illegible]

Theorem 18. [illegible]

Theorem 19. [illegible]

Corollary 1. [illegible]

Theorem 41. [illegible]

Theorem 42. [illegible]

Theorem 43. [illegible]

Chapter 12

Theorem 44. [illegible]

Corollary 1. [illegible]

Theorem 45. [illegible]

Corollary 2. [illegible]

Theorem 46. [illegible]

Theorem 47. [illegible]

Theorem 48. [illegible]

Theorem 49. [illegible]

Corollary 1. [illegible]

Corollary 2. [illegible]

Theorem 50. [illegible]

Corollary 1. [illegible]

Theorem 51. [illegible]

Corollary 2. [illegible]

Theorem 52. [illegible]

Theorem 53. [illegible]

Theorem 54. [illegible]

Theorem 55. [illegible]

Chapter 12

Proposition 10. *Do the [illegible]*

Questions on the definition of a circle. [illegible]

Theorem 56. [illegible]

Theorem 57. [illegible]

Theorem 58. [illegible]

Theorem 59. [illegible]

Theorem 60. [illegible]

Theorem 61. [illegible]

Theorem 62. [illegible]

Theorem 63. [illegible]

Theorem 64. [illegible]

Theorem 65. [illegible]

Theorem 66. [illegible]

Theorem 67. [illegible]

[illegible]

Theorem 68. [illegible]

Theorem 69. [illegible]

Theorem 70. [illegible]

Theorem 71. [illegible]

Theorem 72. [illegible]

Theorem 73. [illegible]

Theorem 74. [illegible]

Theorem 75. [illegible]

Theorem 76. [illegible]

Theorem 77. [illegible]

Theorem 78. [illegible]

Theorem 79. [illegible]

Theorem 80. [illegible]

Theorem 81. [illegible]

Theorem 82. [illegible]

Theorem 83. [illegible]

Theorem 84. [illegible]

Theorem 85. [illegible]

Theorem 86. [illegible]

Theorem 87. [illegible]

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Postulate 80. [illegible]

Postulate 81. [illegible]

Postulate 82. [illegible]

Postulate 83. [illegible]

Postulate 84. [illegible]

Theorem 80. [illegible]

Corollary. [illegible]

Corollary 1 to Postulate 11. [illegible]

Corollary 2 to Postulate 11. [illegible]

Theorem 81. [illegible]

Theorem 82. [illegible]

Theorem 83. [illegible]

Theorem 84. [illegible]

Theorem 85. [illegible]

Theorem 86. [illegible]

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Theorem 87. [illegible]

Theorem 88. [illegible]

Theorem 89. [illegible]

Theorem 90. [illegible]

Theorem 91. [illegible]

Lobachevskian Theorem 1. [illegible]

Lobachevskian Theorem 2. [illegible]

Lobachevskian Theorem 3. [illegible]

Corollary. [illegible]

Lobachevskian Theorem 4. [illegible]

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[illegible]

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Chapter 1, Lesson 4 (pages [illegible])

[illegible]

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[illegible]

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[illegible]

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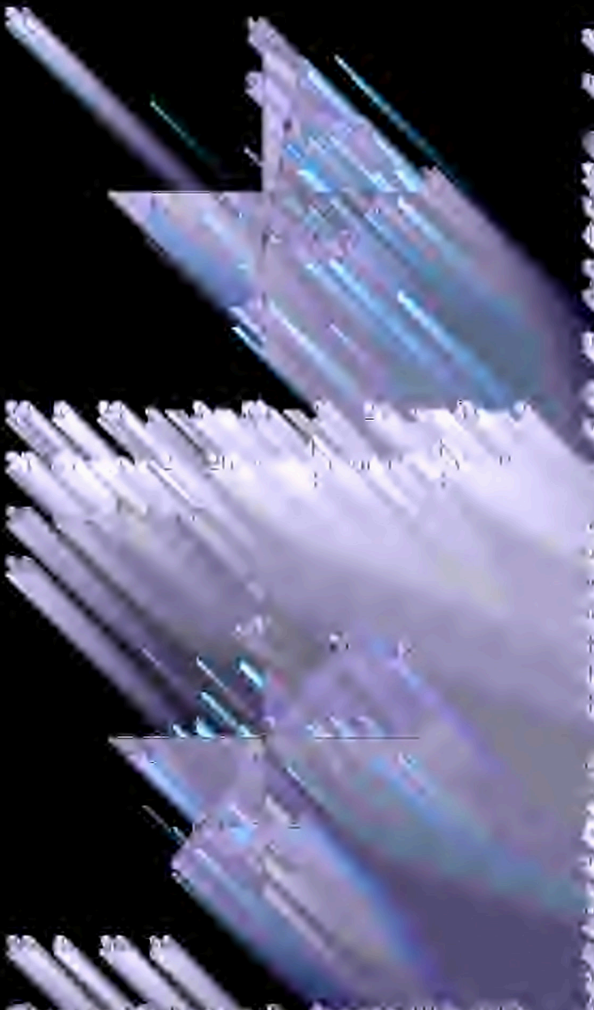

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[illegible]

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[illegible]

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[illegible]

[illegible]

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[illegible]

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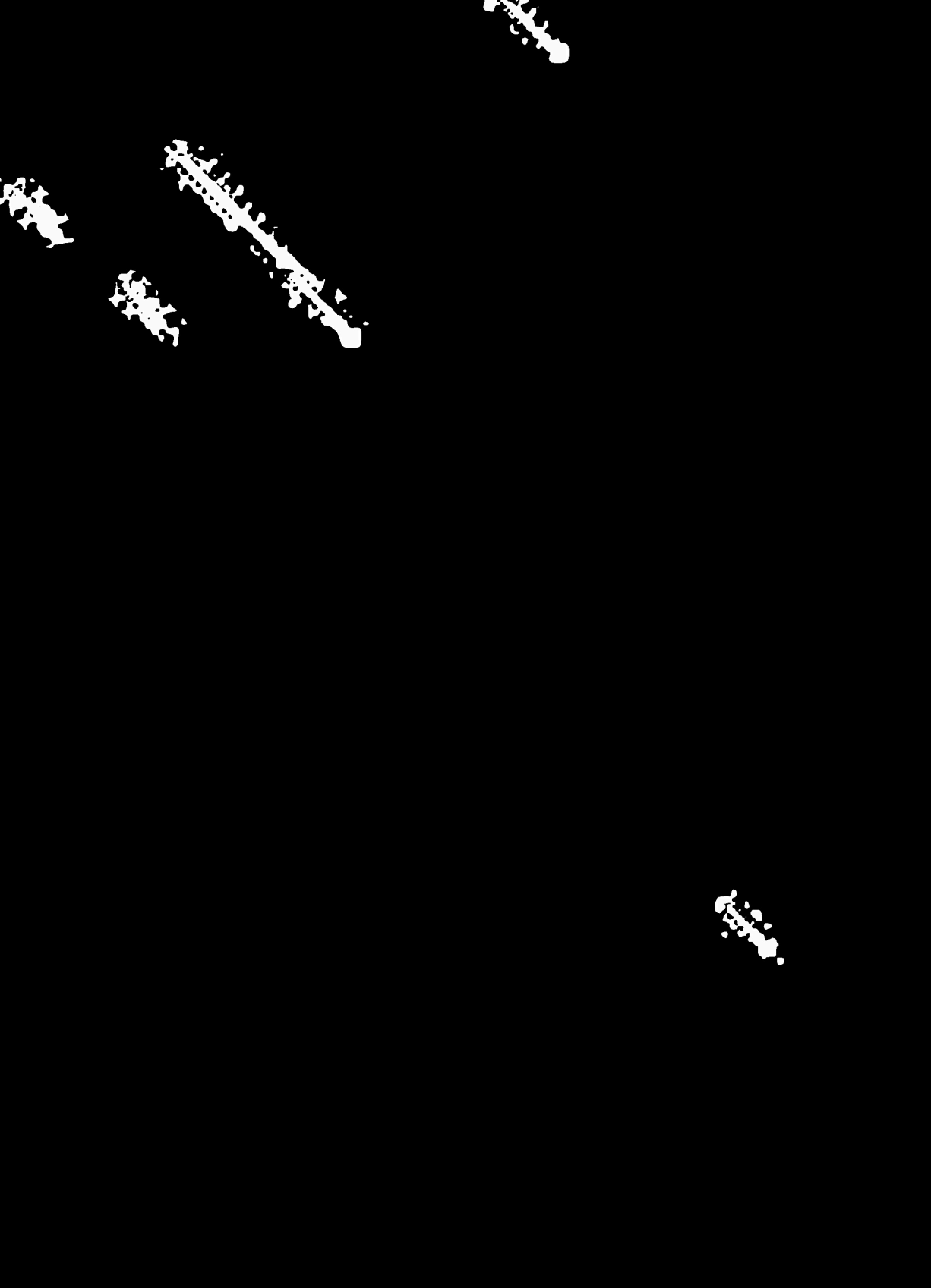



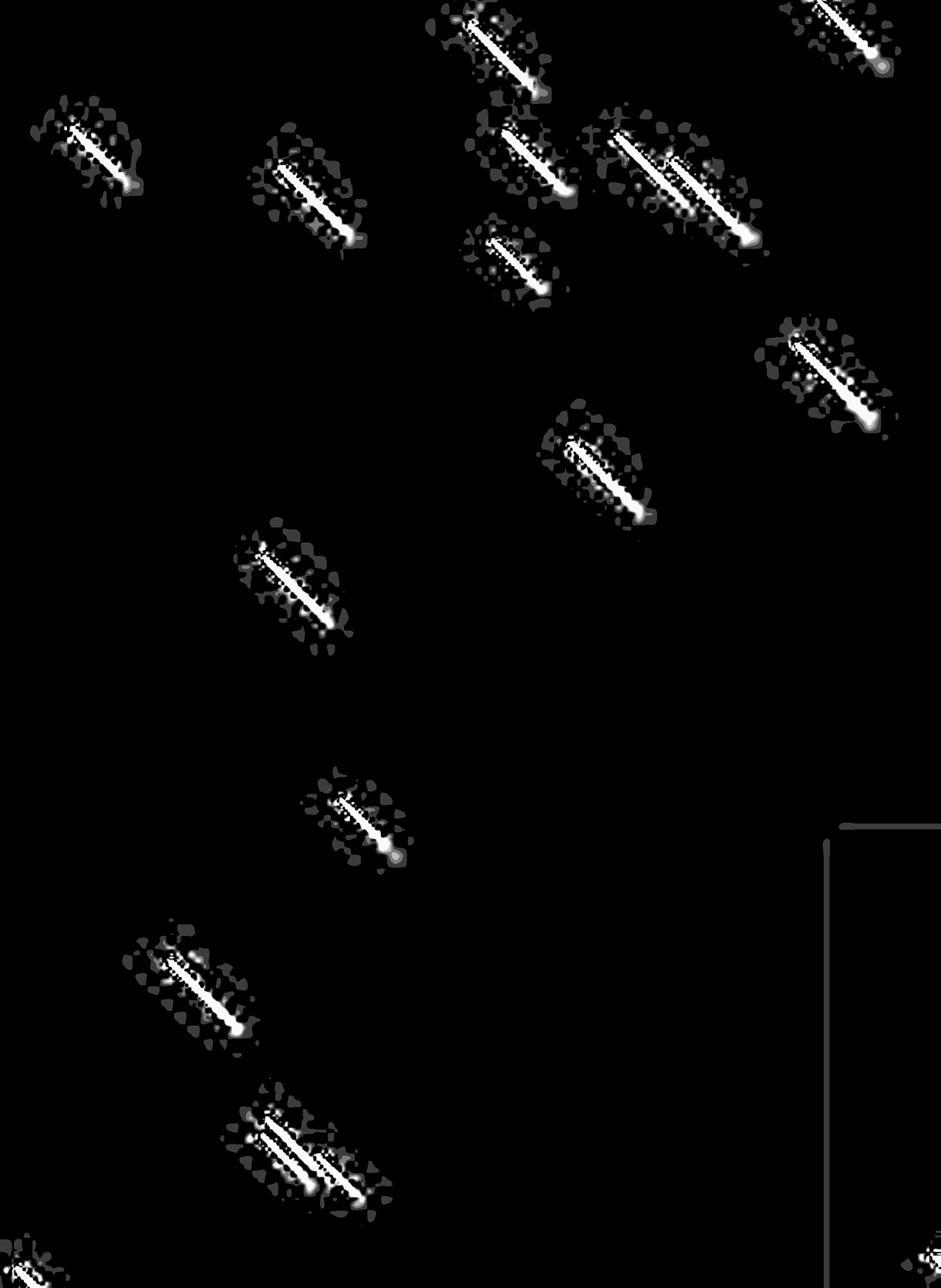

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