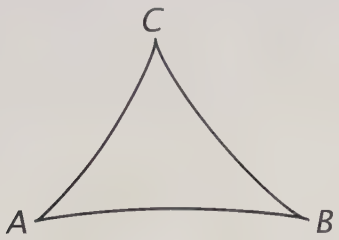
Exercises

Set I

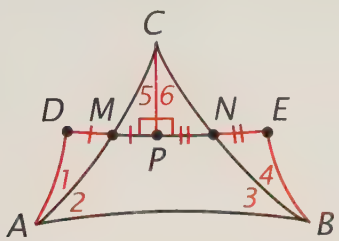
Complete the proofs of the theorems of this lesson by answering the questions.

Lobachevskian Theorem 3. The sum of the angles of a triangle is less than 180°.



Given: △ABC.
Prove: ∠A + ∠B + ∠C < 180°.

Proof



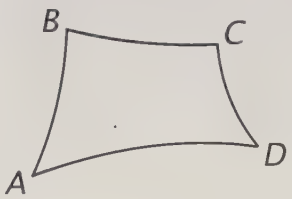
Let M and N be the midpoints of sides AC and BC, and draw MN. Through C, draw CP ⊥ MN. Choose points D and E on line MN so that MD = MP and NE = NP. Draw AD and BE.

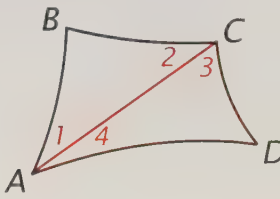
1. △ADM ≅ △CPM and △BEN ≅ △CPN. Why?
2. Which angles of ADEB are right angles?
3. Which sides of ADEB are equal?
4. What kind of quadrilateral is ADEB?
5. ∠DAB and ∠EBA are acute. Why?
6. What can be concluded about ∠DAB + ∠EBA from the fact that ∠DAB < 90° and ∠EBA < 90°?

Because ∠DAB = ∠1 + ∠2 and ∠EBA = ∠3 + ∠4, it follows that ∠1 + ∠2 + ∠3 + ∠4 < 180°.

7. To which angles in the figure are ∠1 and ∠4 equal?
8. Why is ∠5 + ∠2 + ∠3 + ∠6 < 180°?
9. Why does it follow from this that ∠CAB + ∠CBA + ∠ACB < 180°?

Corollary to Lobachevskian Theorem 3. The sum of the angles of a quadrilateral is less than 360°.



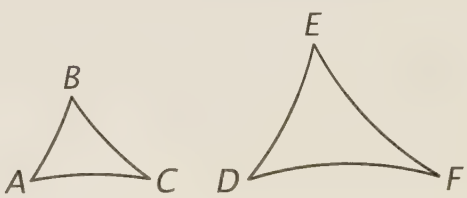


Given: Quadrilateral ABCD.
Prove: ∠A + ∠B + ∠C + ∠D < 360°.

Proof
Draw AC.

10. ∠1 + ∠2 + ∠B < 180° and ∠3 + ∠4 + ∠D < 180°. Why?
11. ∠1 + ∠2 + ∠B + ∠3 + ∠4 + ∠D < 360°. Why?
12. ∠1 + ∠4 = ∠BAD and ∠2 + ∠3 = ∠BCD. Why?
13. ∠BAD + ∠B + ∠BCD + ∠D < 360°. Why?

Lobachevskian Theorem 4. If two triangles are similar, they must also be congruent.

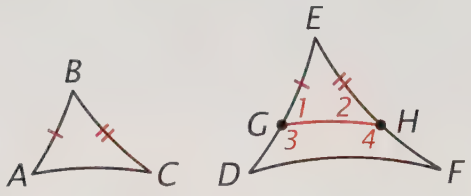


Given: △ABC ~ △DEF.
Prove: △ABC ≅ △DEF.

Proof
By hypothesis, △ABC ~ △DEF.

14. Why is ∠D = ∠A, ∠E = ∠B, and ∠F = ∠C?

Our proof will be indirect. Suppose that $\triangle ABC$ and $\triangle DEF$ are not congruent. If this is the case, then $BA \neq ED$ and $BC \neq EF$, because, if either pair of these sides were equal, then the triangles would be congruent by ASA.



We will now copy the smaller triangle on the larger one as shown above. Choose point G on ED so that $EG = BA$ and point H on EF so that $EH = BC$. Draw GH.

15. Why is $\triangle GEH \cong \triangle ABC$?

It follows that $\angle 1 = \angle A$ and $\angle 2 = \angle C$ because they are corresponding parts of these triangles.

16. Why is $\angle D = \angle 1$ and $\angle F = \angle 2$?

17. Why is $\angle 1 + \angle 3 = 180°$ and $\angle 2 + \angle 4 = 180°$?

18. Why is $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360°$?

19. Why is $\angle D + \angle F + \angle 3 + \angle 4 = 360°$?

20. If we assume from the figure that DGHF is a quadrilateral, the equation in Exercise 19 is impossible. Why?

Because we have arrived at a contradiction, our assumption that $\triangle ABC$ and $\triangle DEF$ are not congruent is false! So $\triangle ABC \cong \triangle DEF$.

Set II

Lobachevskian Triangles. In Lobachevskian geometry, the sum of the angles of every triangle is less than 180°. The larger the triangle's area, the smaller the angle sum is.

Use these facts to decide on answers to the following questions about triangles in Lobachevskian geometry.

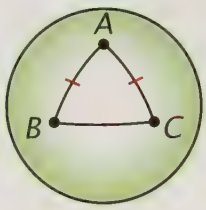
21. If two angles of one triangle are equal to two angles of another triangle, can you conclude that the angles of the third pair are equal? Explain.

22. What can you conclude about the acute angles of a right triangle?

23. What can you conclude about the measure of each angle of an equilateral triangle?

24. How does the measure of each angle of an equilateral triangle depend on the lengths of its sides?

Triangle on a Sphere. The figure below illustrates an isosceles $\triangle ABC$ with $AB = AC$ on a sphere.



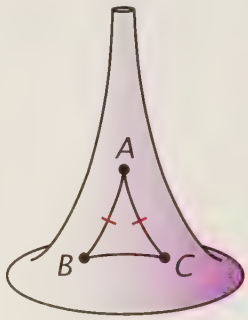
On the basis of the figure, what

25. seems to be true about $\angle B$ and $\angle C$?

26. seems to be true about $\angle A + \angle B + \angle C$?

27. kind of geometry seems to be illustrated?

Triangle on a Pseudosphere. The figure below illustrates an isosceles $\triangle ABC$ with $AB = AC$ on a pseudosphere.



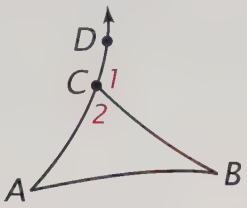
On the basis of the figure, what

28. seems to be true about $\angle B$ and $\angle C$?

29. seems to be true about $\angle A + \angle B + \angle C$?

30. kind of geometry seems to be illustrated?

Exterior Angles. In the figure below, $\triangle ABC$ represents a triangle in Lobachevskian geometry and $\angle 1$ is one of its exterior angles.

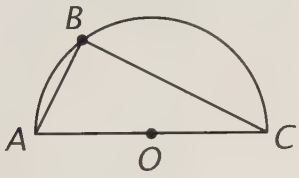


In both Euclidean geometry and Lobachevskian geometry, $\angle 1 > \angle A$ and $\angle 1 > \angle B$.

31. What else can you conclude about $\angle 1$ in Euclidean geometry?

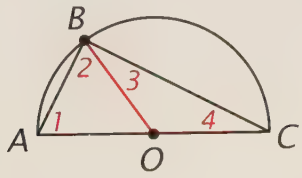
32. Use the figure to show whether or not your conclusion about $\angle 1$ is also true in Lobachevskian geometry.

Angle Inscribed in a Semicircle. In the figure below, $\angle ABC$ is inscribed in a semicircle.



33. What can you conclude about $\angle ABC$ in Euclidean geometry?

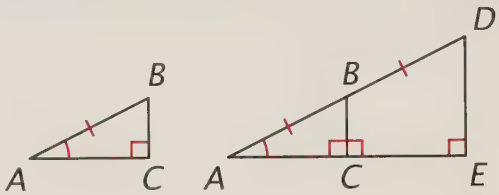
Radius OB has been added in the figure below.



34. Use the numbered angles to show whether or not your conclusion about $\angle ABC$ is also true in Lobachevskian geometry.

Magnification and Distortion. The following photographs are an exaggerated illustration of the fact that a figure cannot be enlarged in Lobachevskian geometry without its being distorted.

In the figure at the left below, $\triangle ABC$ is a right triangle with right $\angle C$. In the figure at the right, the triangle has been enlarged by extending line AB so that AD = 2AB and by drawing DE $\perp$ line AC.



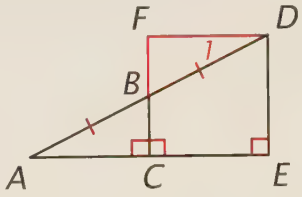
In Euclidean geometry, what can you conclude about

35. $\triangle ADE$ and $\triangle ABC$? Why?

36. $\angle D$ and $\angle ABC$?

37. the lengths of the sides of $\triangle ADE$ with respect to the corresponding sides of $\triangle ABC$?

In the figure below, line DF has been drawn so that $\angle 1 = \angle A$, and BC has been extended so that it intersects DF at F.



38. Copy the figure and mark this information on it.

In both Euclidean geometry and Lobachevskian geometry, what can you conclude about

39. $\triangle DBF$ and $\triangle ABC$? Explain.

40. the lengths of BF and BC?

41. $\angle F$?

42. quadrilateral CFDE?

In Lobachevskian geometry, what can you conclude about

43. $\angle FDE$?

44. the lengths of DE and FC? Explain.

45. FC and 2BC?

46. DE and 2BC?

47. the lengths of CE and FD?

48. the lengths of CE and AC?

49. AE and 2AC?

50. How do the sides of $\triangle ADE$ compare in length with the corresponding sides of $\triangle ABC$ in Lobachevskian geometry?

Set III

Astronomical Triangle. In 1829, Lobachevsky reported that he used some astronomical measurements to find the sum of the angles of the triangle determined by Earth, the sun, and the star Sirius. He found that the sum differed from 180° by approximately 0.000000001°.

1. Does Lobachevsky's finding prove that physical space is non-Euclidean? Explain why or why not.
2. Would it be easier to prove by this method that physical space is Euclidean or that it is non-Euclidean? Explain.

CHAPTER 16 Summary and Review

Basic Ideas

Antipodal points 691
Birectangular quadrilateral 697
Great circle 690
Lobachevskian geometry 703
Riemannian geometry 703
Saccheri quadrilateral 697

Postulate

The Lobachevskian Postulate. The summit angles of a Saccheri quadrilateral are acute. 703

Theorems

87. The summit angles of a Saccheri quadrilateral are equal. 698
88. The line segment connecting the midpoints of the base and summit of a Saccheri quadrilateral is perpendicular to both of them. 698
89. If the legs of a birectangular quadrilateral are unequal, the summit angles opposite them are unequal in the same order. 698
90. If the summit angles of a birectangular quadrilateral are unequal, the legs opposite them are unequal in the same order. 698

L1. The summit of a Saccheri quadrilateral is longer than its base. 703

L2. A midsegment of a triangle is less than half as long as the third side. 703

L3. The sum of the angles of a triangle is less than 180°. 708

Corollary. The sum of the angles of a quadrilateral is less than 360°. 708

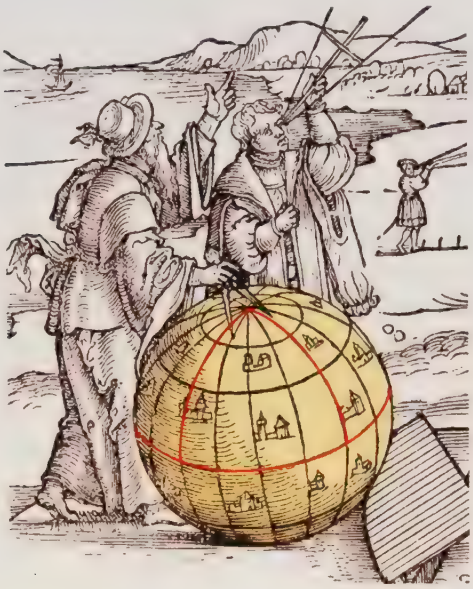
L4. If two triangles are similar, they must also be congruent. 708

Statement	*Euclid*	*Lobachevsky*	*Riemann*
Through a point not on a line, there is	exactly one line parallel to the line.	more than one line parallel to the line.	no line parallel to the line.
The summit angles of a Saccheri quadrilateral are	right.	acute.	obtuse.
The sum of the angles of a triangle is	180°.	less than 180°.	more than 180°.

Exercises

Set I

Sphere Geometry. The figure below appeared in a book published in 1533.*



Most of the curves on the globe are "lines" in sphere geometry.

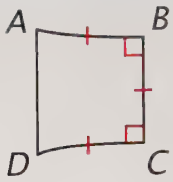
1. What is a "line" in sphere geometry?

Tell whether each of the following statements is true or false in sphere geometry.

2. Through a point not on a line, there is exactly one line perpendicular to the line.
3. In a plane, two lines perpendicular to a third line are parallel to each other.
4. If two lines intersect, they intersect in no more than one point.
5. The sum of the angles of a triangle is 180°.
6. An exterior angle of a triangle is greater than either remote interior angle.

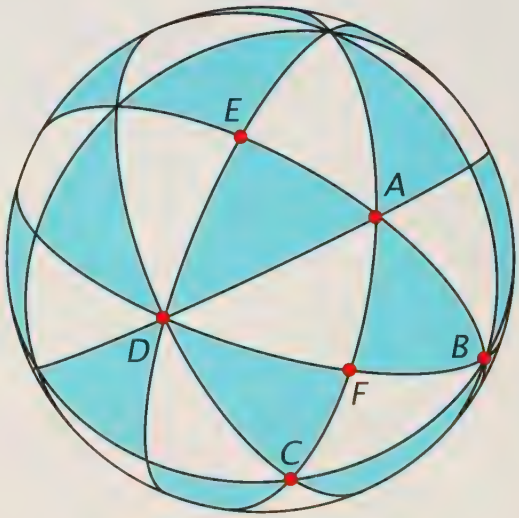
*Introduction to Geography, by Peter Apian.

Saccheri Quadrilateral. In the figure below, ABCD is a Saccheri quadrilateral in Lobachevskian geometry; ∠B and ∠C are right angles, and AB = BC = CD.



7. Which side of ABCD is its summit?
8. What can you conclude about the length of AD?
9. Can ABCD be a rhombus? Explain.
10. What kind of angles are ∠A and ∠D?
11. Can ABCD be a rectangle? Explain.

Beach Ball. This beach ball has been divided by great circles into congruent triangles.



The angles at E and F are right angles, the angles at A and C are 60°, and the angles at B and D are 45°.

What is the sum of the measures of the angles surrounding

12. point A?
13. point B?
14. point C?

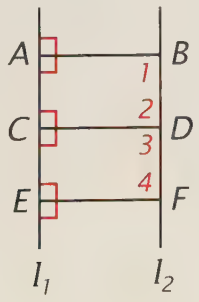
15. Make a large copy of the part of the figure bounded by quadrilateral EBCD. Label the measures of all of the angles in your figure.
16. Find the measures of the angles in quadrilateral ABCD.
17. If the opposite angles of a quadrilateral in Euclidean geometry are equal, what can you conclude?
18. Does the same conclusion appear to hold in sphere geometry? Explain.
19. What can you conclude from the fact that quadrilateral ABCD is equilateral?
20. How are the diagonals of ABCD related to each other?
21. Is this relation also true for such quadrilaterals in Euclidean geometry?
22. Find the measures of the angles in quadrilateral EAFD.

Can a quadrilateral have exactly three right angles in

23. Euclidean geometry?
24. sphere geometry?

Set II

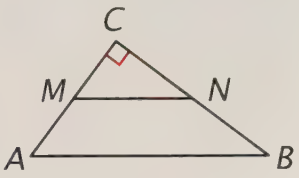
Lobachevsky's Ladder. In the figure below, l_1 and l_2 represent the two parallel side rails of a ladder. Segments AB, CD, and EF represent three rungs that are perpendicular to l_1.



In Lobachevskian geometry, no more than two of the three rungs AB, CD, and EF can have equal lengths.

25. Copy the figure and mark it to suggest that AB = CD = EF.
26. If AB = CD = EF, what kind of quadrilaterals are ABDC and CDFE?
27. In Lobachevskian geometry, what kind of angles are $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$?
28. What can you conclude about $\angle 2 + \angle 3$?
29. How does this conclusion contradict the fact that $\angle 2$ and $\angle 3$ are a linear pair?
30. What does this contradiction prove about the rungs of the ladder?

Pythagoras Meets Lobachevsky. In the figure below, $\triangle ABC$ is a right triangle and MN is one of its midsegments.



Suppose that $CA^2 + CB^2 = AB^2$ and $CM^2 + CN^2 = MN^2$.

31. On what theorem of Euclidean geometry are these equations based?

Because M and N are the midpoints of CA and CB, $CM = \frac{1}{2}CA$ and $CN = \frac{1}{2}CB$.

32. Why does it follow that $\left(\frac{1}{2}CA\right)^2 + \left(\frac{1}{2}CB\right)^2 = MN^2$?
33. Use this equation and the initial assumption that $CA^2 + CB^2 = AB^2$ to show that $MN = \frac{1}{2}AB$.
34. What theorem in Lobachevskian geometry does this result contradict?
35. What does this contradiction indicate about the Pythagorean Theorem in Lobachevskian geometry?

M.C. Escher's "Circle Limit III"
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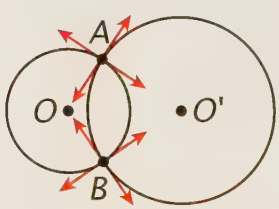
Lobachevsky Meets Escher. The print by Maurits Escher at the beginning of this chapter illustrates a model devised by a French mathematician, Henri Poincaré, for visualizing the theorems of Lobachevskian geometry. Silvio Levy used a computer to create the above version of it.*

To understand this model, it is necessary to know what *orthogonal circles* are.

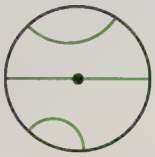
The figure at the right shows two orthogonal circles O and O′ intersecting at points A and B. The tangents to the circles at these points have been drawn.



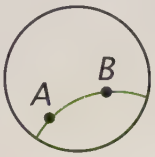
36. On the basis of this figure, define *orthogonal circles.*

**The Visual Mind: Art and Mathematics*, edited by Michele Emmer (MIT Press, 1993).

In Poincaré's model of Lobachevskian geometry, points of the plane are represented by points inside a circle, and lines are represented by both the diameters of the circle and the arcs of circles orthogonal to it. Examples of some lines in this model are shown in the figure below.

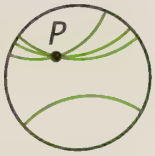


The figure below shows two points in a circle and the only arc through them that is orthogonal to the circle.



37. What postulate does this figure illustrate?

The figure below shows an "orthogonal arc" and several such arcs through point P that do not intersect it.



38. What idea in Lobachevskian geometry does this figure illustrate?

State a theorem or postulate in Lobachevskian geometry suggested by each of the following figures.

39.

40.

41.

The Geometry of the Universe. According to one model of the universe, called Einstein's spherical universe, the geometrical properties of space are comparable to those of the surface of a sphere.

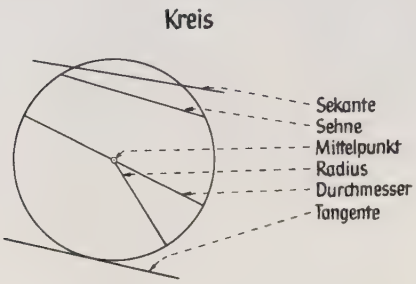
42. If this model is correct, what conclusion follows about the volume of the space of the universe? Explain.

43. If an astronaut were to travel far enough along a straight line, what would this model predict? Explain.

Final Review

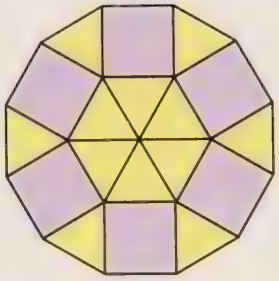
Set I

German Terms. The figure below from a German geometry book names lines and line segments related to the circle.



1. Which word in the figure is identical in German and English?
2. It appears from the figure that a *durchmesser* is a *sehne* of a *kreis* that contains its *mittelpunkt.* Explain.
3. What is the difference between a *sekante* and a *tangente*?

Regular Polygons. The figure below consists of equilateral triangles and squares.

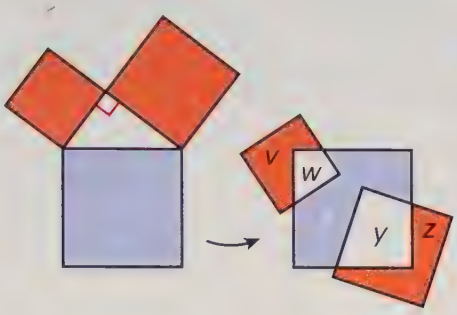


4. What other regular polygons do you see in it?
5. What properties do all regular polygons have in common?
6. What are the measures of the angles of the rhombuses in the figure that are not squares?
7. What other type of quadrilateral in the figure has angles of these measures?
8. What is the sum of the measures of the angles of a regular hexagon?
9. What is the sum of the measures of the angles of a regular dodecagon?

What Follows? Complete the statements of the following postulates and theorems.

10. The area of a parallelogram is the product of . . .
11. If a line through the center of a circle bisects a chord that is not a diameter, it . . .
12. If two points lie in a plane, the line that contains them . . .
13. If the square of one side of a triangle is equal to the sum of the squares of the other two sides, . . .
14. Triangles with equal bases and equal altitudes . . .
15. Two nonvertical lines are parallel iff their slopes . . .
16. If a line is perpendicular to a radius at its outer endpoint, it is . . .
17. If two planes intersect, they intersect in . . .
18. The length of a diagonal of a cube with edges of length e is . . .
19. If a line parallel to one side of a triangle intersects the other two sides in different points, it divides the sides . . .
20. Each leg of a right triangle is the geometric mean between . . .
21. The volume of a pyramid is one-third of the product of . . .
22. The ratio of the perimeters of two similar polygons is equal to . . .
23. Two nonvertical lines are perpendicular iff the product of their slopes . . .
24. A secant angle whose vertex is outside a circle is equal in measure to . . .

Pythagorean Squares. In the figure below, the squares on the legs of a right triangle have been moved so that they partly overlap the square on its hypotenuse. The letters represent the areas of the five regions that result.



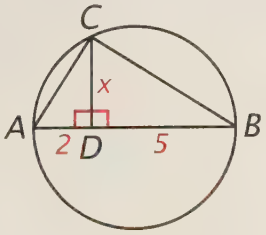
According to the Area Postulate, the area of a polygonal region is equal to the sum of the areas of its nonoverlapping parts.

25. In terms of the letters, what is the area of the largest square?

26. Which is greater: the area shaded red, $v + z$, or the area shaded blue, x? Explain.

The Value of Pi. Brahmagupta, a seventh-century Indian mathematician, proposed the following estimate for π.

AB is a diameter of the circle and CD $\perp$ AB.



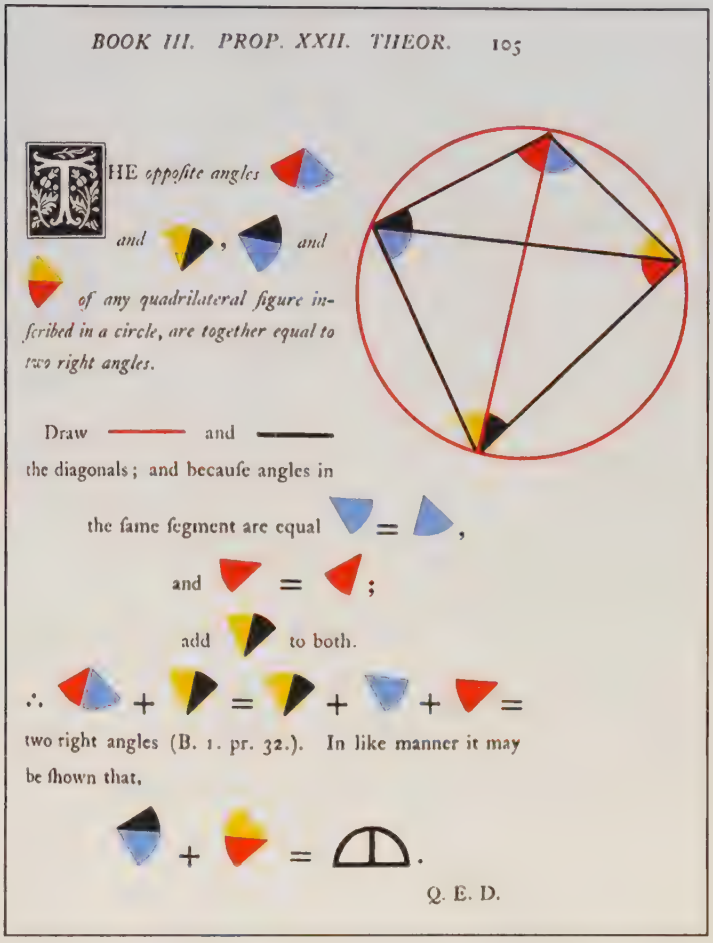
27. What can you conclude about $\triangle ABC$? Explain.

28. Write a proportion relating the lengths AD, CD, and DB.

29. Using the fact that AD = 2 and DB = 5, find CD.

Brahmagupta's estimate was CD = π.

30. Is this estimate correct? Explain.



BOOK III. PROP. XXII. THEOR. 105

THE *opposite angles* and , and , *of any quadrilateral figure inscribed in a circle, are together equal to two right angles.*

Draw —— and —— the diagonals; and because angles in the same segment are equal = ,

and = ;

add to both.

∴ + = + + =

two right angles (B. 1. pr. 32.). In like manner it may be shown that,

+ = .

Q. E. D.

Euclid in Color. The most colorful edition of Euclid's *Elements* was published in London in 1847. The figure above shows a page from this book.

31. What theorem is being proved?

32. Why are the angles that are labeled with the same color equal?

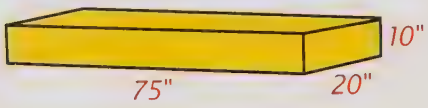
33. Why is the part of the proof shown below true?

+ + = two right angles

34. What do you think the symbol means?

Largest Package. The rule for the largest rectangular package that you can mail at the post office is:

> The length of its longest side plus the distance around its thickest part is less than or equal to 130 inches.



A package has dimensions 10 inches, 20 inches, and 75 inches.

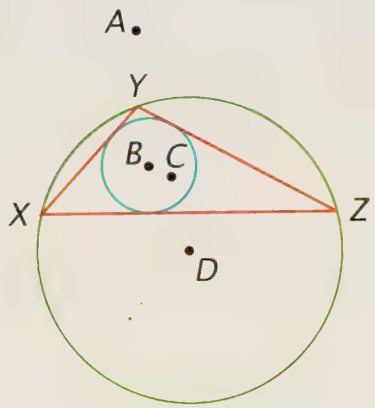
35. What is its volume?

36. Would the post office accept it? Explain.

37. What is the length of the edge of the largest *cube* that you can mail?

38. What is its volume?

Four Centers. Points A, B, C, and D are the four centers of $\triangle XYZ$.



What is the name of

39. point A, the point in which the lines containing the altitudes of $\triangle XYZ$ are concurrent?

40. point B?

41. point C, the point in which the medians of $\triangle XYZ$ are concurrent?

42. point D?

Which of these points is

43. equidistant from the sides of the triangle?

44. equidistant from its vertices?

45. the triangle's balancing point?

46. not collinear with the other three?

Formulas. What is each of the following formulas used for?

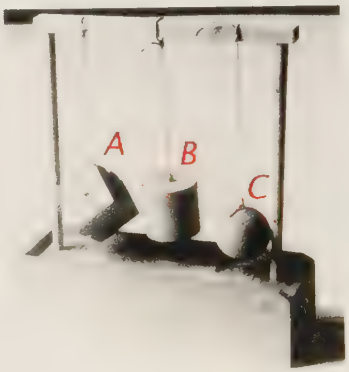
47. $c = \pi d$.

48. $d = \sqrt{l^2 + w^2 + h^2}$.

49. $A = \frac{1}{2}h(b_1 + b_2)$.

50. $h = \frac{\sqrt{3}}{2}s$.

Kindergarten Toy. The photograph from the Milton Bradley catalog of 1889 shown below is of a toy for kindergarten children. Three basic geometric solids are suspended by strings.



51. What does each solid appear to be?

Write expressions for the volume and total surface area of

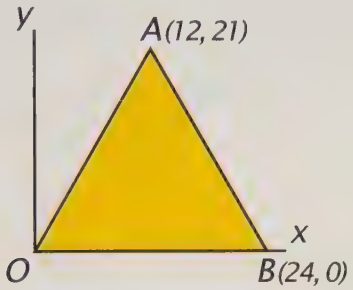
52. solid A in terms of the length, e, of each of its edges.

53. solid B in terms of its altitude, h, and the radius, r, of its bases.

54. solid C in terms of its radius, r.

Steep Roof. The steep roof of an A-frame house helps it to shed snow easily.

The figure below is a side view of an A-frame roof in a coordinate system; the units are in feet.



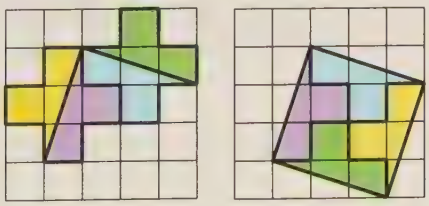
55. Find the slopes of OA and AB.

56. Find ∠AOB, the angle of inclination of the roof, to the nearest degree.

57. Find OA and AB to the nearest foot.

58. △OAB looks equilateral. Is it?

Greek Crosses. The figures below show how two congruent Greek crosses can be cut and put together to form a square.



Given that the length of each side of one of the crosses is x units, find each of the following measures in terms of x.

59. The perimeter of one of the crosses.

60. The area of one of the crosses.

61. The area of the square.

62. The length of a side of the square.

63. The perimeter of the square.

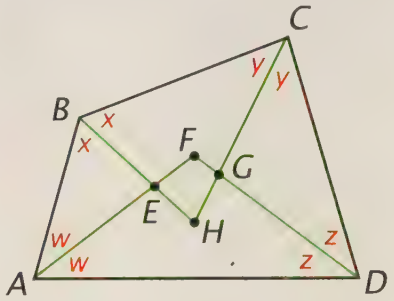
Baseball in Orbit. The smaller a planet, the weaker its gravity. If the radius of the earth were 22 miles, it would be possible to throw a baseball into orbit!*

64. Given that the orbit is circular, find the distance that the ball would travel in going once around the earth.

To be put into this orbit, the ball would have to be thrown at a speed of 100 miles per hour, something that professional pitchers can easily do.

65. Approximately how many minutes would it take the ball to return to the spot from which it was thrown?

Two Quadrilaterals. ABCD represents any convex quadrilateral. The four lines that bisect its angles intersect at E, F, G, and H to form quadrilateral EFGH.



Letting the measures of the angles produced by the bisectors be w, x, y, and z as indicated, show why each of the following statements is true.

66. $w + x + y + z = 180°$.

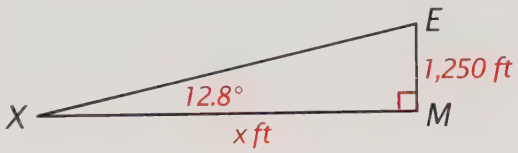
67. $\angle F = 180° - w - z$ and $\angle H = 180° - x - y$.

68. $\angle F + \angle H = 180°$.

69. Quadrilateral EFGH is cyclic.

**The Cosmological Milkshake*, by Robert Ehrlich (Rutgers University Press, 1994).

Long Shadows. Tall buildings cast long shadows. At 3 P.M. on the winter solstice, December 21, the Empire State Building casts a shadow more than a mile long.*

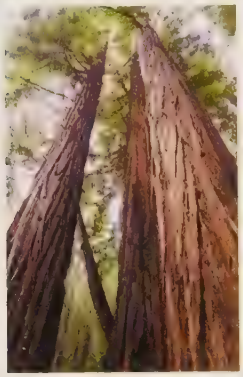


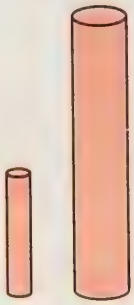
70. Given that the angle of elevation of the sun at this time is 12.8° and that the Empire State Building is 1,250 feet tall, find the approximate length of its shadow.

71. How long would the shadow cast by a person 6 feet tall be at the same time?

Tree Geometry. Galileo calculated that trees cannot grow beyond a certain size without collapsing from their own weight.

The figures at the right below represent the trunks of two trees as similar cylinders.





If the ratio of their diameters is k, what is the ratio of

72. their heights?

73. the areas of their bases?

74. their volumes?

75. their weights?

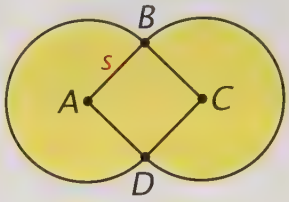
Suppose two trees are similar in shape and that one tree is 10 times as tall as the other.

76. How would corresponding cross sections of their trunks compare in area?

77. How would their trunks and branches compare in weight?

78. On the basis of these comparisons, which tree trunk would be more likely to fail because of the load that it bears?

SAT Problem. The figure at the right appeared in a problem on an SAT test.



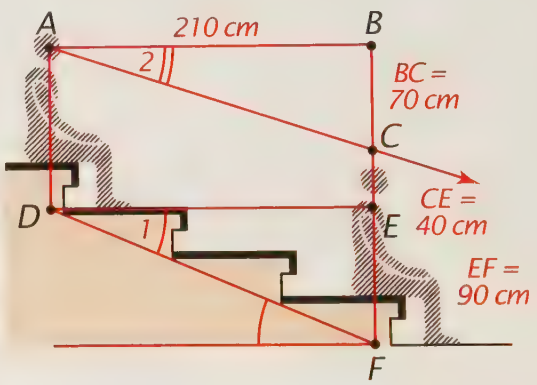
It consists of a square with side of length s and the arcs of two circles centered at A and C and having radius s.

Write an expression in terms of s for

79. the perimeter of the figure.

80. its area.

Sight Line. In the design of seating that enables each spectator to have a clear view, the sight line is projected 15 cm above the head of the spectator seated three rows below.†



Given the measurements in the figure above, find

81. $\angle 1$, which equals the angle at which the seats rise with respect to the horizontal.

82. $\angle 2$, the angle of the sight lines with the horizontal.

**The Empire State Building*, by John Tauranac (Scribner's, 1995).

†*Indoor Sports Spaces*, by Robin Crane and Malcolm Dixon (Van Nostrand Reinhold, 1991).

Set II

Dog Crates. Dog crates usually have the shape of rectangular solids.

Crates for beagles are 30 inches long and 22 inches high and have a volume of 12,540 cubic inches.

83. How wide are they?

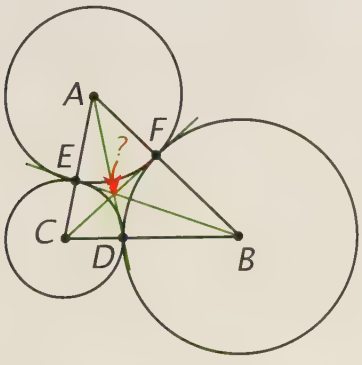
Crates for boxers are 36 inches long, 26 inches high, and 23 inches wide.

84. Are the crates for beagles and boxers similar? Explain why or why not.

Crates for Great Danes are 48 inches long.

85. Given that these crates are similar to those for beagles, find the volume of such a crate to the nearest cubic inch.

Tangent Circles. The three circles in the figure below are centered on the vertices of $\triangle ABC$ and meet on its sides at points D, E, and F.



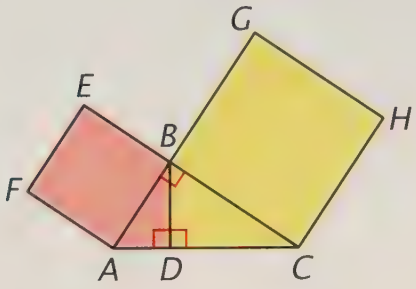
Lines AD, BE, and CF look as if they might be concurrent.

86. Write an equation relating the lengths of the segments of the sides of $\triangle ABC$ that, if true, would show that lines AD, BE, and CF are concurrent.

87. Would this equation be true if the three circles had equal radii? Explain.

88. Is the equation true, given that the circles have radii of different lengths? Explain.

Squares on the Legs. In the figure below, squares ABEF and BCHG have been drawn on the legs of right $\triangle ABC$, and BD is the altitude to its hypotenuse.



89. $\triangle ABD \sim \triangle BCD$. Why?

90. $\dfrac{\alpha\triangle ABD}{\alpha\triangle BCD} = \left(\dfrac{AB}{BC}\right)^2$. Why?

91. How does $\dfrac{\alpha\triangle ABD}{\alpha\triangle BCD}$ compare with $\dfrac{\alpha ABEF}{\alpha BCHG}$? Explain.

92. Is the yellow region similar to the light red one? If so, write the similarity.

Formula Confusion. On the final exam, Obtuse Ollie started confusing different formulas with the one for the area of a triangle.

For the area of a circle, he wrote $A = \frac{1}{2}rc$.

93. If by r and c he meant the radius and circumference of the circle, how many points out of 5 would you give Ollie? Explain.

For the volume of a sphere, Ollie wrote

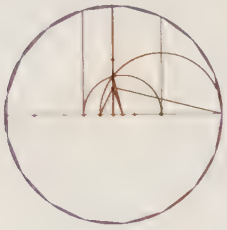
$V = \frac{1}{2}rA$.

94. If by r and A he meant the radius and surface area of the sphere, should Ollie get any credit for this answer? Explain.

Ollie also wrote $V = \frac{1}{2}rA$ for the volume of a right cylinder.

95. If by r and A he meant the radius and area of the lateral surface of the cylinder, should Ollie get any credit for this answer? Explain.

Regular 17-gon. The great German mathematician Carl Friedrich Gauss made a surprising discovery when he was 18 years old. He proved that a circle can be divided into 17 equal arcs with just a straightedge and compass.



96. Find the value of $N = n \sin\frac{180}{n}$ for a regular 17-gon to the nearest hundredth.

97. Find the value of $M = n \sin\frac{180}{n} \cos\frac{180}{n}$ for the same polygon to the nearest hundredth.

Given that the radius of a regular 17-gon is 10 centimeters, use the formula

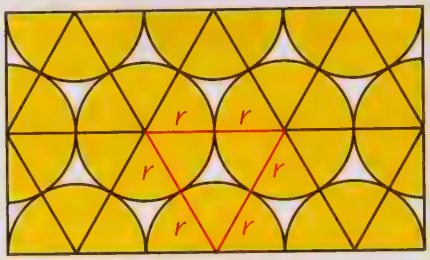
98. $p = 2Nr$ to find its perimeter to the nearest centimeter.

99. $A = Mr^2$ to find its area to the nearest square centimeter.

100. Without calculating the numbers, how would you expect the corresponding measurements of a circle with the same radius to compare with those of the 17-gon?

101. Check your answer to exercise 100 by calculating the circumference and area of the circle, each to the nearest square centimeter.

Packing Circles. When equal circles are packed together as closely as possible, their centers are at the vertices of equilateral triangles.

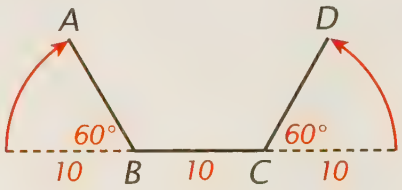


102. Find the area of one of these triangles in terms of r, the radius of the circles.

103. Find the area inside the triangle filled by the three sectors of the circles.

104. Use your answers to exercises 102 and 103 to find the percentage of the plane filled by the circles.

Rain Gutter. A rain gutter is to be constructed from a metal sheet of width 30 cm by bending upward one-third of the sheet on each side through an angle of 60°.*



105. Copy the figure and draw perpendicular line segments from A and D to line BC to form right △AEB and right △DFC. Also draw AD.

*From a problem in *Calculus*, by James Stewart (Brooks/Cole, 1995).

106. What can you conclude about △AEB and △DFC? Explain.

107. Why is AE ∥ DF?

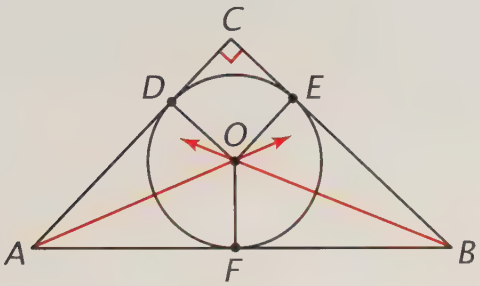
108. Why is AE = DF?

109. Why is AD ∥ EF?

110. Find the length of AE.

111. Find the area of ABCD (the cross section of the water when the rain gutter is full) to the nearest square centimeter.

Incircle. In right △ABC, the bisectors of ∠A and ∠B intersect in point O.



112. Why does it follow that the bisector of ∠C also passes through point O?

113. What is point O called with respect to the triangle?

The sides of the triangle are tangent to circle O at points D, E, and F.

114. Why is CD = CE?

115. Why is OD ⊥ AC and OE ⊥ CB?

116. Why is OD ∥ EC and OE ∥ DC?

117. Why does it follow that OD = EC and OE = DC?

118. Given that CA = 20 and CB = 21, find AB.

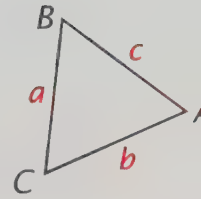
Express each of the following lengths in terms of the given side lengths and r, the radius of the incircle.

119. AD and AF.

120. BE and BF.

121. Use AB to write and solve an equation for r.

Angles and Sides. The sides opposite ∠A, ∠B, and ∠C of △ABC have lengths a, b, and c.



122. Why does it follow that $c^2 = a^2 + b^2 - 2ab \cos C$?

123. Solve this equation for cos C, given that $a = b = c$.

124. Use your calculator to find ∠C.

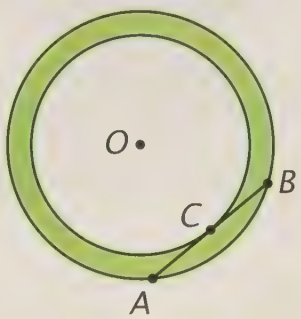
125. Is the result the number that you would expect? Explain.

126. Solve the equation of exercise 122 for cos C, given that $c^2 = a^2 + b^2$.

127. Use your calculator to find ∠C.

128. Is the result the number that you would expect? Explain.

Circular Track. The figure below shows a circular race track bordered by two concentric circles.



Chord AB represents one of the longest possible straight paths on the track. It is tangent to the inside of the track at C and is 200 meters long.*

129. Copy the figure and draw OA and OC. Let R be the length of OA and r be the length of OC.

130. AB ⊥ OC. Why?

131. OC bisects AB. Why?

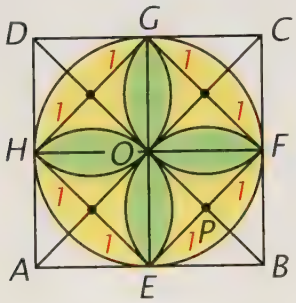
132. $R^2 - r^2 = 10{,}000$. Why?

133. Find the area of the track to the nearest square meter.

**Mathematical Nuts*, by Samuel I. Jones (S. I. Jones Co., 1932).

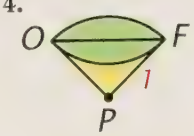
da Vinci Problem. Leonardo da Vinci dedicated a hundred pages of his notebooks to geometry. He made many drawings of figures bounded by circular arcs in which the areas of the parts were related in simple ways.*

In the figure below, square ABCD is circumscribed about circle O, and square EFGH is inscribed in it. Four semicircles have been drawn with the sides of EFGH as their diameters; each diameter is 2 units long.



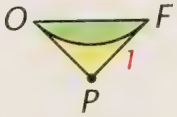
Find the area of each of the following parts of the figure.

134.



Sector OPF.

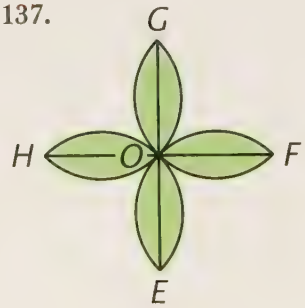
135.



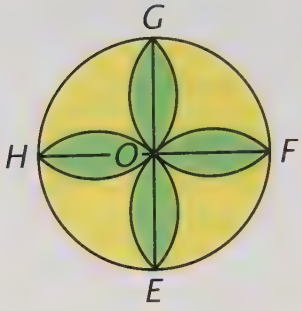
△OPF.

136.

137.



138.



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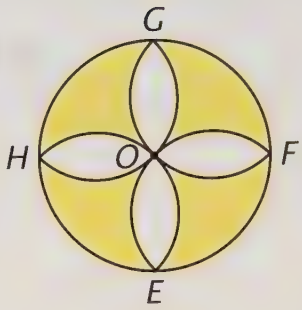
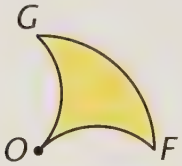
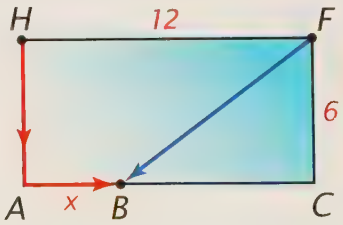


(The yellow part only.)

140.



A Problem from Ancient India.† A fish at corner F of a rectangular pool sees a heron at corner H looking at him. As the fish swims along FB, the heron walks along the shore from H to A to B, arriving at the same time.



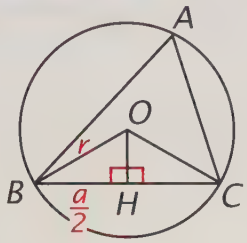
141. Given that the pool is 12 units by 6 units and that the heron and the fish move at the same speed, find the distance that the fish swam.

**The Unknown Leonardo*, edited by Ladislao Reti (McGraw-Hill, 1974).

†*Geometry Civilized: History, Culture, and Technique*, by J. L. Heilbron (Clarendon Press, 1998).

Circumradius. The figure on this stamp from Guinea-Bissau can be used to derive an equation relating a triangle and its circumcircle.

The figure at the right shows $\triangle ABC$ with circumcircle O; $OH \perp BC$, $BC = a$, and $OB = r$.



142. State the theorem that tells us that, regardless of its shape, $\triangle ABC$ has a circumcircle.

143. Why is $BH = \frac{a}{2}$?

144. Why is $\angle BOH = \frac{1}{2}\angle BOC$?

145. Why is $\angle BOC = m\overset{\frown}{BC}$?

146. Why is $\angle A = \frac{1}{2} m\overset{\frown}{BC}$?

147. Why is $\angle BOH = \angle A$?

148. Why is $\sin \angle BOH = \frac{\frac{a}{2}}{r}$?

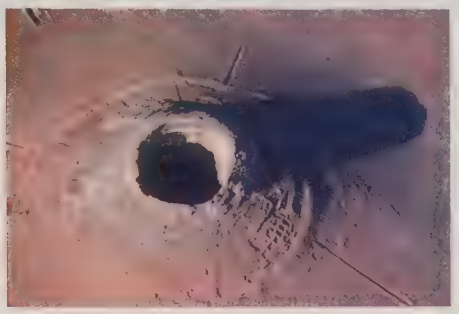
149. Why does it follow that $\frac{a}{\sin A} = 2r$?

150. Given that the sides of $\triangle ABC$ opposite $\angle B$ and $\angle C$ have lengths b and c, why does it follow that $\frac{b}{\sin B} = \frac{c}{\sin C} = 2r$?

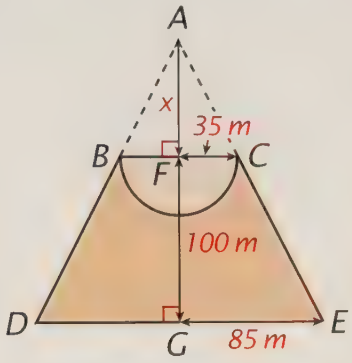
151. Use the equation of exercise 149 to find the radius of the circumcircle of a triangle for which $a = 3$ cm and $\angle A = 30°$.

152. Draw a figure to show that your answer is reasonable.

Conical Mountain. The unusual mountain in Iran in the photograph below was shaped by a gaseous spring. Its crater once contained a lake.*



In the side view of the mountain below, the crater is represented as the lower part of a circular cone, and the bowl of the crater is represented as half of a sphere.



153. Use the numbers in the figure to find x, the length of AF.

Find each of the following volumes to the nearest cubic meter.

154. The volume of the solid cone whose altitude is AG.

155. The volume of the solid cone whose altitude is AF.

156. The volume of the bowl of the crater.

157. Use your answers to exercises 154 through 156 to estimate the volume of material in the actual mountain to the nearest 100,000 cubic meters.

**Below from Above*, by George Gerster (Abbeville Press, 1986).

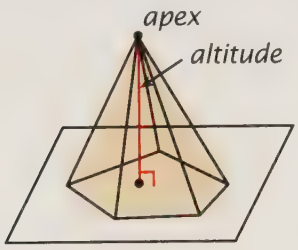
Glossary

Acute angle: An angle whose measure is less than 90°. 92

Acute triangle: A triangle all of whose angles are acute. 158

Altitude of a prism: (1) A line segment that connects the planes of the bases of a prism and that is perpendicular to both of them. (2) The length of that line segment. 640

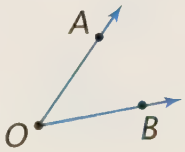
Altitude of a pyramid: (1) The perpendicular line segment from the apex of a pyramid to the plane of its base. (2) The length of that line segment. 648



Altitude of a quadrilateral that has parallel sides: (1) A perpendicular line segment that connects points on the lines of the parallel sides. (2) The length of that line segment. 359

Altitude of a triangle: (1) A perpendicular line segment from a vertex of a triangle to the line of the opposite side. (2) The length of that line segment. 352

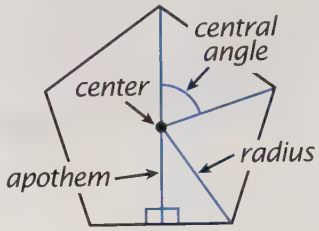
Angle: A pair of rays that have the same endpoint. The angle in the figure at the right can be named ∠O or ∠AOB or ∠BOA; this notation also stands for the measure of this angle. The measure might either be found by using a protractor or worked out by the geometry of a given problem. 13



Angle of inclination: The angle that a line forms with the horizontal. 461

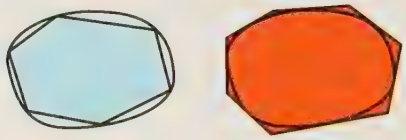
Antipodal points: The two points of intersection of a sphere with a line through its center. 691

Apothem of a regular polygon: A perpendicular line segment from the center of the polygon to one of its sides. 574



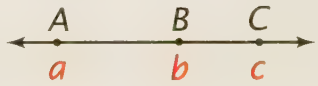
Area: Measure of extent for plane figures. Determined by using the Area Postulates for polygonal regions or as limits of the areas of polygons (as described for the area of the circle) for curved regions. The methods can be extended to finding the areas of surfaces in three dimensions, including the area of a sphere. 20, 664–665

Area of a circle: The limit of the areas of inscribed regular polygons. Archimedes extended this method to find the areas of figures bounded by curves by using areas of inscribed and circumscribed polygons. 598

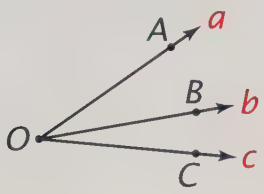


blue area < *curved region area* < *red area*

Betweenness of Points: An intuitive notion, formally defined as follows by using the Ruler Postulate. A point is between two other points on the same line iff its coordinate is between their coordinates. (More briefly, A-B-C iff $a < b < c$ or $a > b > c$.) 85



Betweenness of Rays: An intuitive notion, formally defined as follows by using the Protractor Postulate. A ray is between two others in the same half-rotation iff its coordinate is between their coordinates. (More briefly, OA-OB-OC iff $a < b < c$ or $a > b > c$.) 93

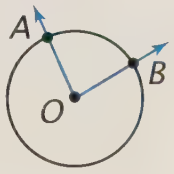


Birectangular quadrilateral: A quadrilateral that has two sides perpendicular to a third side. 697

Bisector of an angle: A line or ray that divides an angle into two equal angles. 99

Center of a regular polygon: The center of the circumscribed circle of the polygon. See *Apothem of a regular polygon.* 574

Central angle of a circle: An angle whose vertex is the center of a circle. 498



Central angle of a regular polygon: An angle formed by radii drawn to two consecutive vertices of the polygon. See *Apothem of a regular polygon.* 574

Centroid of a triangle: The point in which the medians of a triangle are concurrent. Physically, a uniform triangular region balances perfectly in any position when supported at its centroid. 549

Cevian of a triangle: A line segment that connects a vertex of a triangle to a point on the opposite side. 554

Chord of a circle: A line segment that connects two points of a circle. 485

Circle: The set of all points in a plane that are at a given distance from a given point in the plane. 484

Circumcenter of a polygon: The center of the circle circumscribed about the polygon. 531

Circumference of a circle: The limit of the perimeters of the inscribed regular polygons. 592

Circumscribed circle about a polygon: The circle that contains all of the vertices of the polygon. 531

Circumscribed polygon about a circle: A polygon each of whose sides is tangent to the circle. 542

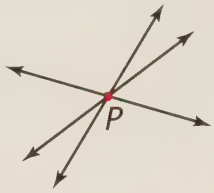
Collinear points: Points that are contained by the same line. 9

Complementary angles: Two angles whose sum is 90°. 105

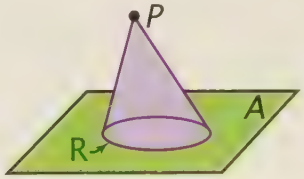
Concave polygon: A polygon that is not convex. 258

Concentric circles: Circles that lie in the same plane and have the same center. 484

Concurrent lines: Lines that contain the same point. 9



Cone: Suppose that A is a plane, R is a circular region in plane A, and P is a point not in plane A. The solid made up of all segments that connect P to a point of region R is a cone. 654



Congruent figures: Informally, two figures are congruent if they have the same size and shape so that, if one is superimposed on the other, they match exactly. Formally, two figures are congruent if there is an isometry such that one figure is the image of the other. This definition of congruence for figures in general agrees with the definition of *congruent triangles* and extends the idea of same size and same shape to all plane and solid shapes. 140, 312

Congruent triangles: Two triangles for which there is a correspondence between their vertices such that all of their corresponding sides and angles are equal. 140

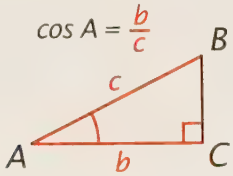
Converse of a conditional statement: The statement formed by interchanging the hypothesis and conclusion of the conditional statement. 47

Convex polygon: A polygon such that every line segment that connects two points inside it lies entirely inside the polygon. More generally, a plane figure or a solid shape is convex iff every line segment connecting two points inside the figure or shape also lies inside it. 258

Coplanar points: Points that are contained in the same plane. 9

Corollary: A theorem that can be easily proved as a consequence of a definition, a postulate or another theorem. 99

Cosine of an acute angle of a right triangle: The ratio of the length of the adjacent leg to the length of the hypotenuse. More generally, see *Trigonometric functions.* 455

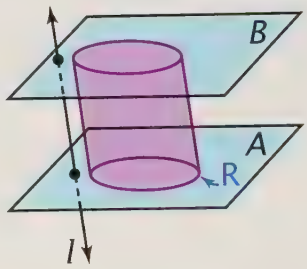


Cross section of a geometric solid: The intersection of a plane and the solid. 641

Cube: A rectangular solid whose length, width, and height are equal. 629

Cyclic polygon: A polygon for which there exists a circle that contains all of the vertices of the polygon. 530

Cylinder: Suppose that A and B are two parallel planes, R is a circular region in one plane, and l is a line that intersects both planes but not R. The solid made up of all segments parallel to line l that connect a point of region R to a point of the other plane is a cylinder. 655



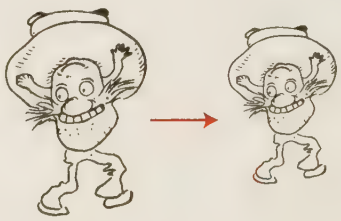
Decagon: A polygon that has 10 sides. 573

Degree measure of an arc: The measure of the central angle of an arc. 498

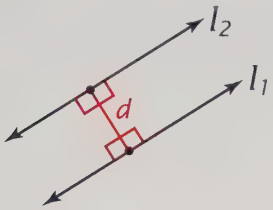
Diagonal of a polygon: A line segment that connects any two nonconsecutive vertices of a polygon. 259

Diameter of a circle: (1) A chord that contains the center of a circle. (2) The length of a chord that contains the center. 485

Dilation: Informally, a transformation in which a figure is reduced or enlarged. 300

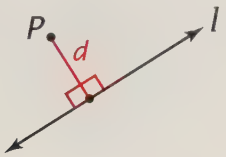


Distance between two parallel lines: In general, the length of any perpendicular segment connecting one line to the other.



Distance between two points: An intuitive idea of what we measure by using a ruler. Formalized by the Ruler Postulate. The distance between the endpoints of a line segment gives the length of the segment. 85

Distance from a point to a line: In general, the length of the perpendicular segment from the point to the line.



Dodecagon: A polygon that has 12 sides. 585

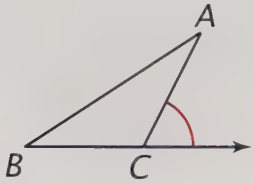
Dodecahedron: A polyhedron that has 12 faces. The faces of a regular dodecahedron are regular pentagons. 678

Equiangular polygon: A convex polygon all of whose angles are equal. 572

Equiangular triangle: A triangle all of whose angles are equal. 158

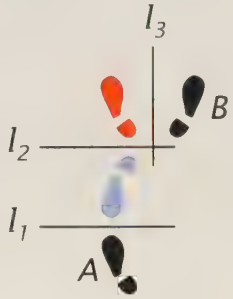
Equilateral polygon: A polygon all of whose sides are equal. 572

Exterior angle of a triangle: An angle that forms a linear pair with an angle of the triangle. 191



Geometric mean: The number b is the geometric mean between the numbers a and c iff a, b, and c are positive and $\frac{a}{b} = \frac{b}{c}$. 380

Glide reflection: A transformation that is the composite of a translation and a reflection in a line parallel to the direction of the translation. 313



Great circle of a sphere: A set of points that is the intersection of the sphere and a plane containing its center. 690

Heptagon: A polygon that has seven sides. 573

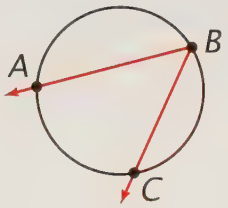
Hexagon: A polygon that has six sides. 19

Hypotenuse of a right triangle: The side opposite the right angle of the triangle. 66

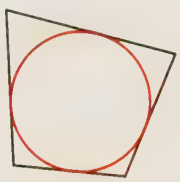
Icosahedron: A polyhedron that has 20 faces. The faces of a regular icosahedron are equilateral triangles. 678

Incenter of a polygon: The center of the inscribed circle of the polygon. 542

Inscribed angle of a circle: An angle whose vertex is on a circle and whose sides each intersect the circle in another point. 505



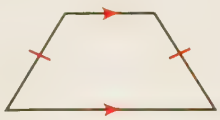
Inscribed circle in a polygon: A circle for which each side of the polygon is tangent to the circle. 542



Inscribed polygon in a circle: A polygon each of whose vertices lies on the circle. 531

Isometry: A transformation that preserves distance and angle measure. 300

Isosceles trapezoid: A trapezoid whose legs are equal. 281



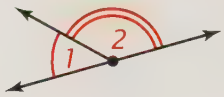
Isosceles triangle: A triangle that has at least two equal sides. 157

Lateral area of a prism: The sum of the areas of the lateral faces of a prism. Lateral area excludes the areas of the bases of a prism. 635

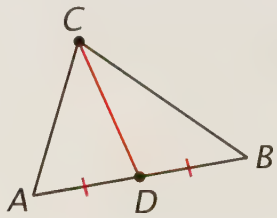
Length of a line segment: The distance between its endpoints. 85

Line segment: Part of a line bounded by two endpoints. If the endpoints are A and B, then the segment is written as AB (or BA). 9

Linear pair: Two angles that have a common side and whose other sides are opposite rays. 111

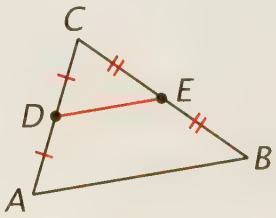


Median of a triangle: A line segment that connects a vertex of a triangle to the midpoint of the opposite side. 549



Midpoint of a line segment: The point that divides a line segment into two equal segments. 98

Midsegment of a triangle: A line segment that connects the midpoints of two of a triangle's sides. 286



Net: A two-dimensional pattern of polygons that, when folded together, forms a polyhedron. 635

Nonagon: A polygon that has nine sides. 573

Noncollinear points: Points that do not lie on the same line. 9

Oblique line and plane (or two planes): A line and a plane (or two planes) that are neither parallel nor perpendicular. 620

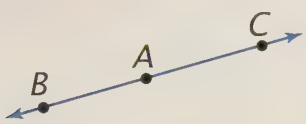
Obtuse angle: An angle whose measure is more than 90° but less than 180°. 92

Obtuse triangle: A triangle that has an obtuse angle. 158

Octagon: A polygon that has eight sides. 19

Octahedron: A polyhedron that has eight faces. The faces of a regular octahedron are equilateral triangles. 678

Opposite rays: Informally, two rays are opposite rays if they have a common endpoint and point in opposite directions. Formally, rays AB and AC are opposite rays iff B-A-C. 19



Orthocenter of a triangle: The point in which the lines containing the altitudes of a triangle are concurrent. 549

Parallel line and plane: A line and a plane that do not intersect. 619

Parallel lines: Lines that lie in the same plane and do not intersect. 118

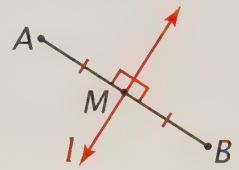
Parallel planes: Planes that do not intersect. 619

Parallelogram: A quadrilateral whose opposite sides are parallel. 265

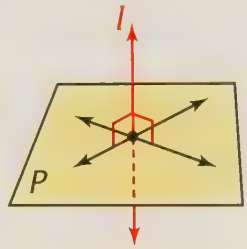
Pentagon: A polygon that has five sides. 19

Perimeter of a polygon: The sum of the lengths of the sides of a polygon. 20

Perpendicular bisector of a line segment: The line that is perpendicular to the line segment and that divides the segment into two equal parts. 212



Perpendicular line and plane: A line and a plane that intersect such that the line is perpendicular to every line in the plane that passes through the point of intersection. 620



Perpendicular lines: Lines that form a right angle. 117

Perpendicular planes: Two planes such that one plane contains a line that is perpendicular to the other plane. 620

Plane: A flat unbounded surface. 9

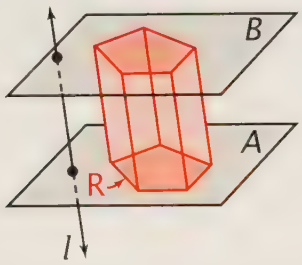
Polygon: A connected set of at least three line segments in the same plane such that each segment intersects exactly two others, one at each endpoint. 139

Polygonal region: The union of a polygon and its interior. 339

Polyhedron: A solid bounded by parts of intersecting planes. 628

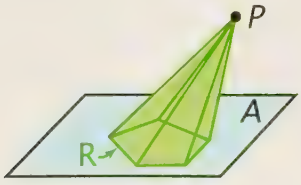
Postulate: A statement that is assumed to be true without proof. We choose our postulates for their natural intuitive appeal and for their ability to capture the basic properties of space from which the rest of geometry can be developed. 61

Prism: Suppose that A and B are two parallel planes, R is a polygonal region in one plane, and l is a line that intersects both planes but not R. The solid made up of all segments parallel to line l that connect a point of region R to a point of the other plane is a prism. 634



Proportion: An equality between two ratios. 379

Pyramid: Suppose that A is a plane, R is a polygonal region in plane A, and P is a point not in plane A. The solid made up of all segments that connect P to a point of region R is a pyramid. Point P is the apex of the pyramid. 647



Pythagorean triple: A set of three integers that can be the lengths of the sides of a right triangle. 437

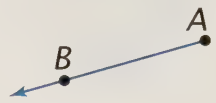
Quadrilateral: A polygon that has four sides. 19

Radius of a circle: (1) The distance between the center of a circle and any point on it. (2) A line segment that connects the center of the circle to any point on it. 485

Radius of a regular polygon: A line segment that connects the center of the polygon to a vertex of the polygon. See *Apothem of a regular polygon.* 574

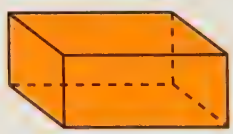
Ratio: The ratio of the number a to the number b is the number $\frac{a}{b}$. 379

Ray: Part of a line that extends endlessly in one direction. If a ruler is placed on a line, points with coordinates that are zero or positive form a ray. The ray in the figure below is named ray AB. 13

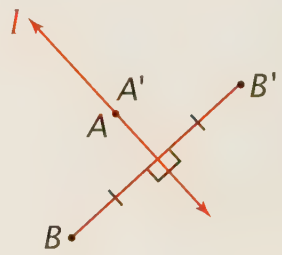


Rectangle: A quadrilateral each of whose angles is a right angle. 259

Rectangular solid: A polyhedron that has six rectangular faces. 628

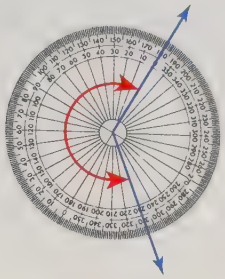


Reflection of a point through a line: The reflection of point P through line l is point P itself if P is on l. Otherwise, it is point P′ such that l is the perpendicular bisector of PP′. 305



Reflection symmetry: A figure has reflection (line) symmetry with respect to a line iff it coincides with its reflection image through the line. 320

Reflex angle: An angle whose measure is more than 180°. 498



Regular polygon: A convex polygon that is both equilateral and equiangular. 572

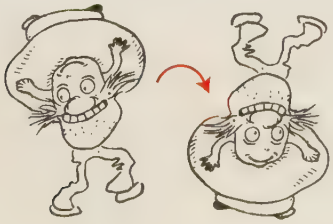
Regular polyhedron: A convex solid having faces that are congruent regular polygons and having an equal number of polygons that meet at each vertex. Euclid showed that there are five regular polyhedra: the tetrahedron, the cube, the octahedron, the dodecahedron, and the icosahedron. 678

Rhombus: A quadrilateral all of whose sides are equal. 276

Right angle: An angle whose measure is 90°. 92

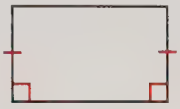
Right triangle: A triangle that has a right angle. 158

Rotation: Informally, a transformation in which a figure is rotated a certain number of degrees about a fixed point. Formally, a transformation that is the composite of two successive reflections through intersecting lines. 307



Rotation symmetry: A figure has rotation symmetry with respect to a point iff it coincides with its rotation image through less than 360° about the point. 319

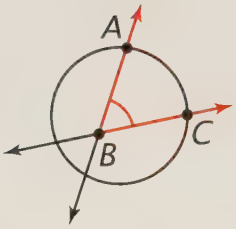
Saccheri quadrilateral: A birectangular quadrilateral whose legs are equal. 697



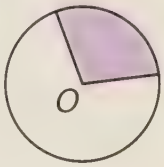
Scalene triangle: A triangle that has no equal sides. 157

Secant: A line that intersects a circle in two points. 511

Secant angle: An angle whose sides are contained in two secants of a circle so that each side intersects the circle in at least one point other than the angle's vertex. 511



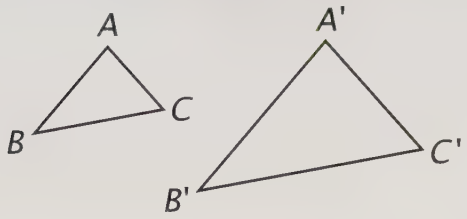
Sector of a circle: A region bounded by an arc of the circle and the two radii to the endpoints of the arc. 605



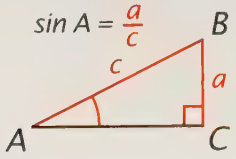
Similar figures: Informally, two figures are similar if they have the same shape, though they may differ in size. In similar figures, corresponding segments are proportional and corresponding angles are equal. 386–387

Similar solids: See *Similar figures.* 670

Similar triangles: Two triangles for which there is a correspondence between their vertices such that their corresponding sides are proportional and their corresponding angles are equal. 386

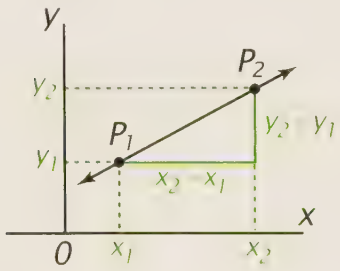


Sine of an acute angle of a right triangle: The ratio of the length of the opposite leg to the length of the hypotenuse. More generally, see *Trigonometric functions.* 455



Skew lines: Two lines that are not parallel and do not intersect. 619

Slope: The slope m of a nonvertical line that contains the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is $m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$. 462



Sphere: The set of all points in space that are at a given distance from a given point. 662

Square: A quadrilateral all of whose sides and angles are equal. 276

Straight angle: An angle whose measure is 180°. 92

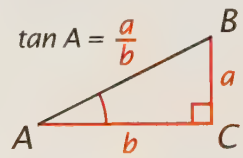
Supplementary angles: Two angles whose sum is 180°. 106

Symmetry with respect to a line: Two points are symmetric with respect to a line iff the line is the perpendicular bisector of the line segment connecting the two points. See *Reflection symmetry.* 212

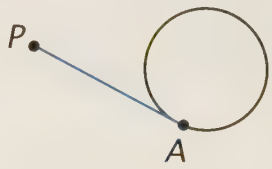
Symmetry with respect to a point: Two points are symmetric with respect to a point iff the point is the midpoint of the line segment connecting the two points. A figure that has symmetry with respect to a point coincides with its rotation image through 180° about the point. 266

Tangent: A line in the plane of a circle that intersects the circle in exactly one point. 491

Tangent of an acute angle of a right triangle: The ratio of the length of the opposite leg to the length of the adjacent leg. More generally, see *Trigonometric functions.* 449



Tangent segment: Any segment of a line that is tangent to a circle that has the point of tangency as one of the segment's endpoints. 517

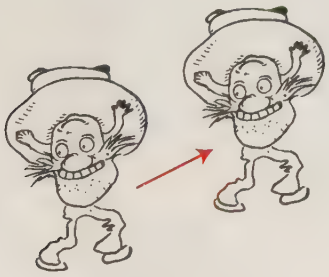


Tetrahedron: A polyhedron that has four faces. The faces of a regular tetrahedron are equilateral triangles. 678

Theorem: A statement that is proved by reasoning deductively from already accepted statements. 51

Transformation: A one-to-one correspondence between two sets of points. Examples of transformations include isometries, which preserve the size and shape of figures. Isometries give us the general definition of congruent figures. Isometries include reflections, translations, rotations, and glide reflections. Also important are dilations, which preserve shape but may alter the size of figures. 299–300

Translation: Informally, a transformation in which a figure is slid a certain distance in a given direction without being turned. Formally, a transformation that is the composite of two successive reflections through parallel lines. 307



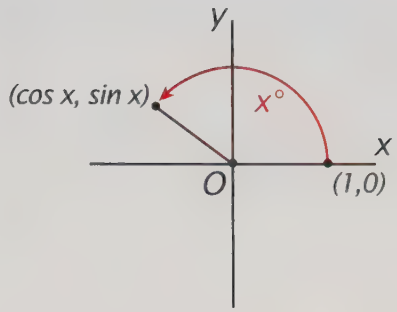
Translation symmetry: A pattern has translation symmetry if it coincides with a translation image. 320

Transversal: A line that intersects two or more lines that lie in the same plane in different points. 219

Trapezoid: A quadrilateral that has exactly one pair of parallel sides. 281

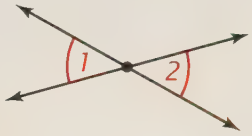
Triangle: A polygon that has three sides. 19

Trigonometric functions: For acute angles, see *Cosine, Sine,* and *Tangent.* More generally, you can discover by working with a calculator that you can find cos x, sin x, and tan x where the number of degrees of x ranges widely. The interpretation of



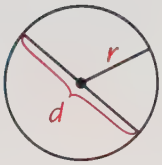
these trigonometric functions is as follows. If a unit segment with one endpoint at the origin is rotated counterclockwise through an angle of x degrees starting with its other endpoint at (1, 0) on the x-axis, the other endpoint of this segment will end up at the point having coordinates (cos x, sin x). Other trigonometric functions such as the tangent can be defined for general angles by the equations that define them in terms of sines and cosines of acute angles; for example, $\tan x = \dfrac{\sin x}{\cos x}$.

Vertical angles: Two angles such that the sides of one angle are opposite rays to the sides of the other. 111



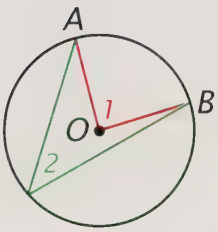
Formulary

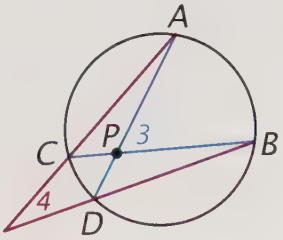
CIRCLE



Circumference: $c = \pi d$
$c = 2\pi r$

Area: $A = \pi r^2$





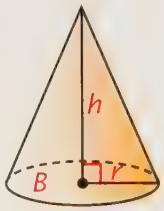
Central angle: $\angle 1 = m\overset{\frown}{AB}$

Inscribed angle: $\angle 2 = \frac{1}{2} m\overset{\frown}{AB}$

Secant angles: $\angle 3 = \frac{1}{2}(m\overset{\frown}{AB} + m\overset{\frown}{CD})$

$\angle 4 = \frac{1}{2}(m\overset{\frown}{AB} - m\overset{\frown}{CD})$

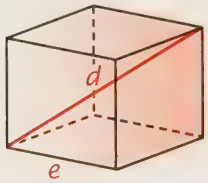
CONE (Circular)



Volume: $V = \frac{1}{3}Bh$

$V = \frac{1}{3}\pi r^2 h$

CUBE

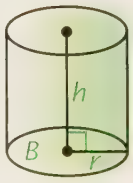


Diagonal: $d = e\sqrt{3}$

Area: $A = 6e^2$

Volume: $V = e^3$

CYLINDER (Right circular)

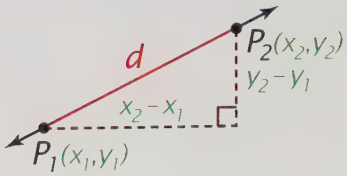


Lateral area: $A = 2\pi rh$

Total area: $A = 2\pi rh + 2\pi r^2$

Volume: $V = Bh = \pi r^2 h$

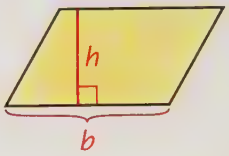
DISTANCE AND SLOPE



Distance: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

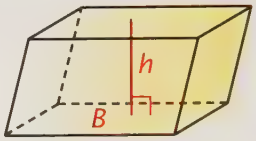
Slope: $m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$

PARALLELOGRAM



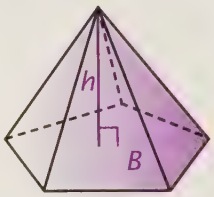
Area: $A = bh$

PRISM



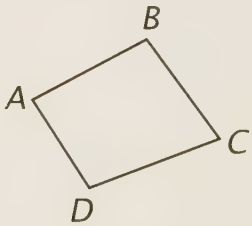
Volume: $V = Bh$

PYRAMID



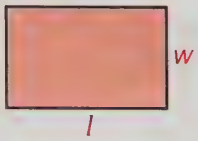
Volume: $V = \frac{1}{3}Bh$

QUADRILATERAL (Convex)



Angle sum: $\angle A + \angle B + \angle C + \angle D = 360°$

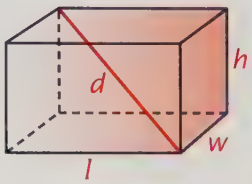
RECTANGLE



Perimeter: $p = 2l + 2w$

Area: $A = lw$

RECTANGULAR SOLID

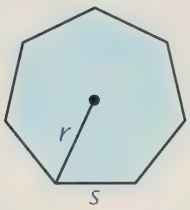


Diagonal: $d = \sqrt{l^2 + w^2 + h^2}$

Area: $A = 2lw + 2wh + 2lh$

Volume: $V = lwh$

REGULAR POLYGON (n sides)



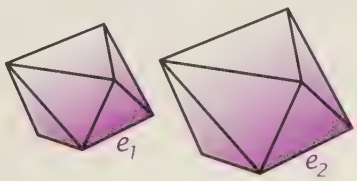
$$N = n \sin \frac{180}{n}$$

$$M = n \sin \frac{180}{n} \cos \frac{180}{n}$$

Perimeter: $p = ns = 2Nr$

Area: $A = Mr^2$

SIMILAR SOLIDS

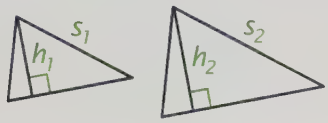


Given the edge ratio, $\frac{e_2}{e_1} = r$

Area ratio: $\frac{A_2}{A_1} = r^2$

Volume ratio: $\frac{V_2}{V_1} = r^3$

SIMILAR TRIANGLES (AND POLYGONS)



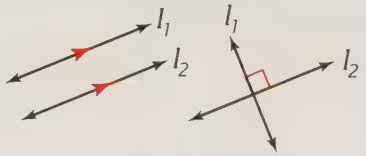
Given the side ratio, $\frac{s_2}{s_1} = r$

Altitude ratio: $\frac{h_2}{h_1} = r$

Perimeter ratio: $\frac{p_2}{p_1} = r$

Area ratio: $\frac{A_2}{A_1} = r^2$

SLOPE RELATIONS

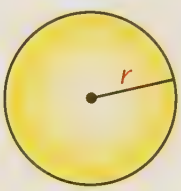


Parallel lines: $m_1 = m_2$

Perpendicular lines: $m_1 m_2 = -1$,

so $m_1 = -\frac{1}{m_2}$

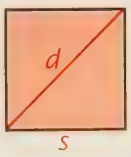
SPHERE



Area: $A = 4\pi r^2$

Volume: $V = \frac{4}{3}\pi r^3$

SQUARE

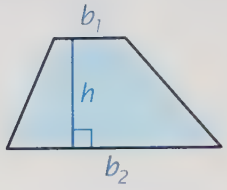


Diagonal: $d = s\sqrt{2}$

Perimeter: $p = 4s$

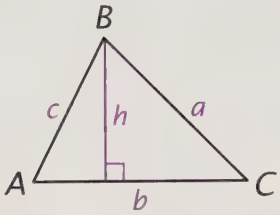
Area: $A = s^2$

TRAPEZOID



Area: $A = \frac{1}{2}h(b_1 + b_2)$

TRIANGLE



Perimeter: $p = a + b + c$

Area: $A = \frac{1}{2}bh$

Angle sum: $\angle A + \angle B + \angle C = 180°$

Law of sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc \cos A$

$b^2 = a^2 + c^2 - 2ac \cos B$

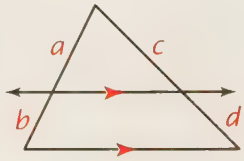
$c^2 = a^2 + b^2 - 2ab \cos C$

Side-Splitter Theorem:

$\frac{a}{b} = \frac{c}{d}$

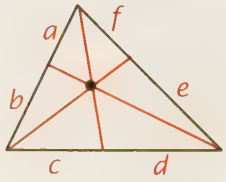
$\frac{a}{a+b} = \frac{c}{c+d}$

$\frac{b}{a+b} = \frac{d}{c+d}$



Ceva's Theorem:

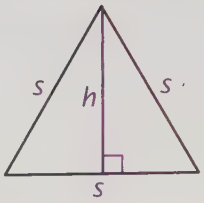
$\frac{a}{b} \cdot \frac{c}{d} \cdot \frac{e}{f} = 1$



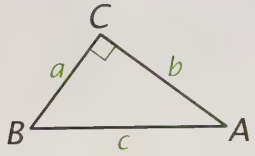
TRIANGLE, EQUILATERAL

Altitude: $h = \frac{\sqrt{3}}{2}s$

Area: $A = \frac{\sqrt{3}}{4}s^2$



TRIANGLE, RIGHT



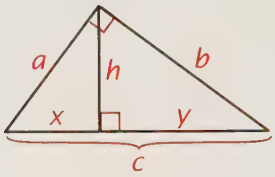
Area: $A = \frac{1}{2}ab$

Pythagorean Theorem: $c^2 = a^2 + b^2$

Sine ratio: $\sin A = \frac{a}{c}$, $\sin B = \frac{b}{c}$

Cosine ratio: $\cos A = \frac{b}{c}$, $\cos B = \frac{a}{c}$

Tangent ratio: $\tan A = \frac{a}{b}$, $\tan B = \frac{b}{a}$



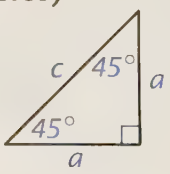
Altitude mean: $\frac{x}{h} = \frac{h}{y}$

Leg means: $\frac{c}{a} = \frac{a}{x}$, $\frac{c}{b} = \frac{b}{y}$

TRIANGLE, RIGHT (Isosceles)

Leg: a

Hypotenuse: $c = a\sqrt{2}$

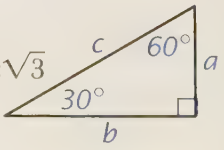


TRIANGLE, RIGHT (30°–60°)

Shorter leg: a

Longer leg: $b = \sqrt{3}a = a\sqrt{3}$

Hypotenuse: $c = 2a$



Teacher: ". . . and now I want to prove this theorem."
Pupil: "Why bother to prove it, teacher? I take your word for it."

Postulates and Theorems

Chapter 2

Postulate 1 Two points determine a line. 61

Postulate 2 Three noncollinear points determine a plane. 61

Theorem *The Pythagorean Theorem.* The square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides. 65

Theorem *The Triangle Angle Sum Theorem.* The sum of the angles of a triangle is 180°. 66

Theorem If the diameter of a circle is d, its circumference is πd. 66

Theorem If the radius of a circle is r, its area is πr^2. 66

Chapter 3

Postulate 3 *The Ruler Postulate.* The points on a line can be numbered so that positive number differences measure distances. 85

Postulate 4 *The Protractor Postulate.* The rays in a half-rotation can be numbered from 0 to 180 so that positive number differences measure angles. 92

Theorem 1 *The Betweenness of Points Theorem.* If A-B-C, then AB + BC = AC. 86

Theorem 2 *The Betweenness of Rays Theorem.* If OA-OB-OC, then $\angle AOB + \angle BOC = \angle AOC$. 93

Corollary to the Ruler Postulate A line segment has exactly one midpoint. 99

Corollary to the Protractor Postulate An angle has exactly one ray that bisects it. 100

Theorem 3 Complements of the same angle are equal. 106

Theorem 4 Supplements of the same angle are equal. 106

Theorem 5 The angles in a linear pair are supplementary. 111

Theorem 6 Vertical angles are equal. 112

Theorem 7 Perpendicular lines form right angles. 118

Corollary to the definition of a right angle All right angles are equal. 118

Theorem 8 If the angles in a linear pair are equal, then their sides are perpendicular. 118

Chapter 4

The Distance Formula The distance between the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. 134

Postulate 5 *The ASA Postulate.* If two angles and the included side of one triangle are equal to two angles and the included side of another triangle, the triangles are congruent. 147

Postulate 6 *The SAS Postulate.* If two sides and the included angle of one triangle are equal to two sides and the included angle of another triangle, the triangles are congruent. 147

Corollary to the definition of congruent triangles Two triangles congruent to a third triangle are congruent to each other. 141

Theorem 9 If two sides of a triangle are equal, the angles opposite them are equal. 158

Corollary An equilateral triangle is equiangular. 159

Theorem 10 If two angles of a triangle are equal, the sides opposite them are equal. 159

Corollary An equiangular triangle is equilateral. 159

Theorem 11 *The SSS Theorem.* If the three sides of one triangle are equal to the three sides of another triangle, the triangles are congruent. 164

Chapter 5

Theorem 12 *The Exterior Angle Theorem.* An exterior angle of a triangle is greater than either remote interior angle. 191

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Theorem 14 If two angles of a triangle are unequal, the sides opposite them are unequal in the same order. 196

Theorem 15 *The Triangle Inequality Theorem.* The sum of any two sides of a triangle is greater than the third side. 201

Chapter 6

Postulate 7 *The Parallel Postulate.* Through a point not on a line, there is exactly one line parallel to the given line. 226

Theorem 16 In a plane, two points each equidistant from the endpoints of a line segment determine the perpendicular bisector of the line segment. 212

Theorem 17 Equal corresponding angles mean that lines are parallel. 220

Corollary 1 Equal alternate interior angles mean that lines are parallel. 220

Corollary 2 Supplementary interior angles on the same side of a transversal mean that lines are parallel. 220

Corollary 3 In a plane, two lines perpendicular to a third line are parallel. 220

Theorem 18 In a plane, two lines parallel to a third line are parallel to each other. 226

Theorem 19 Parallel lines form equal corresponding angles. 231

Corollary 1 Parallel lines form equal alternate interior angles. 231

Corollary 2 Parallel lines form supplementary interior angles on the same side of a transversal. 231

Corollary 3 In a plane, a line perpendicular to one of two parallel lines is also perpendicular to the other. 231

Theorem 20 *The Angle Sum Theorem.* The sum of the angles of a triangle is 180°. 237

Corollary 1 If two angles of one triangle are equal to two angles of another triangle, the third angles are equal. 237

Corollary 2 The acute angles of a right triangle are complementary. 237

Corollary 3 Each angle of an equilateral triangle is 60°. 237

Theorem 21 An exterior angle of a triangle is equal to the sum of the remote interior angles. 237

Theorem 22 *The AAS Theorem.* If two angles and the side opposite one of them in one triangle are equal to the corresponding parts of another triangle, the triangles are congruent. 243

Theorem 23 *The HL Theorem.* If the hypotenuse and a leg of one right triangle are equal to the corresponding parts of another right triangle, the triangles are congruent. 244

Chapter 7

Theorem 24 The sum of the angles of a quadrilateral is 360°. 259

Corollary A quadrilateral is equiangular iff it is a rectangle. 259

Theorem 25 The opposite sides and angles of a parallelogram are equal. 266

Theorem 26 The diagonals of a parallelogram bisect each other. 266

Theorem 27 A quadrilateral is a parallelogram if its opposite sides are equal. 271

Theorem 28 A quadrilateral is a parallelogram if its opposite angles are equal. 271

Theorem 29 A quadrilateral is a parallelogram if two opposite sides are both parallel and equal. 271

Theorem 30 A quadrilateral is a parallelogram if its diagonals bisect each other. 271

Theorem 31 All rectangles are parallelograms. 276

Theorem 32 All rhombuses are parallelograms. 276

Theorem 33 The diagonals of a rectangle are equal. 277

Theorem 34 The diagonals of a rhombus are perpendicular. 277

Theorem 35 The base angles of an isosceles trapezoid are equal. 282

Theorem 36 The diagonals of an isosceles trapezoid are equal. 282

Theorem 37 *The Midsegment Theorem.* A midsegment of a triangle is parallel to the third side and half as long. 287

Chapter 9

Postulate 8 *The Area Postulate.* Every polygonal region has a positive number called its area such that (1) congruent triangles have equal areas and (2) the area of a polygonal region is equal to the sum of the areas of its nonoverlapping parts. 339

Postulate 9 The area of a rectangle is the product of its base and altitude. 345

Corollary to Postulate 9 The area of a square is the square of its side. 345

Theorem 38 The area of a right triangle is half the product of its legs. 352

Theorem 39 The area of a triangle is half the product of any base and corresponding altitude. 353

Corollary Triangles with equal bases and equal altitudes have equal areas. 353

Theorem 40 The area of a parallelogram is the product of any base and corresponding altitude. 359

Theorem 41 The area of a trapezoid is half the product of its altitude and the sum of its bases. 359

Theorem 42 *The Pythagorean Theorem.* The square of the hypotenuse of a right triangle is equal to the sum of the squares of its legs. 366

Theorem 43 *Converse of the Pythagorean Theorem.* If the square of one side of a triangle is equal to the sum of the squares of the other two sides, the triangle is a right triangle. 366

Chapter 10

Theorem 44 *The Side-Splitter Theorem.* If a line parallel to one side of a triangle intersects the other two sides in different points, it divides the sides in the same ratio. 393

Corollary If a line parallel to one side of a triangle intersects the other two sides in different points, it cuts off segments proportional to the sides. 394

Theorem 45 *The AA Theorem.* If two angles of one triangle are equal to two angles of another triangle, the triangles are similar. 400

Corollary Two triangles similar to a third triangle are similar to each other. 401

Theorem 46 Corresponding altitudes of similar triangles have the same ratio as that of the corresponding sides. 408

Theorem *The SAS Similarity Theorem.* If an angle of one triangle is equal to an angle of another triangle and the sides including these angles are proportional, then the triangles are similar. 412

Theorem *The SSS Similarity Theorem.* If the sides of one triangle are proportional to the sides of another triangle, then the triangles are similar. 412

Theorem 47 The ratio of the perimeters of two similar polygons is equal to the ratio of the corresponding sides. 415

Theorem 48 The ratio of the areas of two similar polygons is equal to the square of the ratio of the corresponding sides. 416

Chapter 11

Theorem 49 The altitude to the hypotenuse of a right triangle forms two triangles similar to it and to each other. 429

Corollary 1 The altitude to the hypotenuse of a right triangle is the geometric mean between the segments into which it divides the hypotenuse. 429

Corollary 2 Each leg of a right triangle is the geometric mean between the hypotenuse and its projection on the hypotenuse. 429

Theorem 50 *The Isosceles Right Triangle Theorem.* In an isosceles right triangle, the hypotenuse is $\sqrt{2}$ times the length of a leg. 442

Corollary Each diagonal of a square is $\sqrt{2}$ times the length of one side. 442

Theorem 51 *The 30°-60° Right Triangle Theorem.* In a 30°-60° right triangle, the hypotenuse is twice the shorter leg and the longer leg is $\sqrt{3}$ times the shorter leg. 442

Corollary An altitude of an equilateral triangle having side s is $\frac{\sqrt{3}}{2}s$ and its area is $\frac{\sqrt{3}}{4}s^2$. 443

Theorem 52 Two nonvertical lines are parallel iff their slopes are equal. 463

Theorem 53 Two nonvertical lines are perpendicular iff the product of their slopes is -1. 463

Theorem 54 *The Law of Sines.* If the sides opposite $\angle A$, $\angle B$, and $\angle C$ of $\triangle ABC$ have lengths a, b, and c, then $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$. 469

Theorem 55 *The Law of Cosines.* If the sides opposite $\angle A$, $\angle B$, and $\angle C$ of $\triangle ABC$ have lengths a, b, and c, then $c^2 = a^2 + b^2 - 2ab \cos C$. 469

Chapter 12

Postulate 10 *The Arc Addition Postulate.* If C is on $\overset{\frown}{AB}$, then $m\overset{\frown}{AC} + m\overset{\frown}{CB} = m\overset{\frown}{ACB}$. 499

Corollary to the definition of a circle All radii of a circle are equal. 485

Theorem 56 If a line through the center of a circle is perpendicular to a chord, it also bisects the chord. 485

Theorem 57 If a line through the center of a circle bisects a chord that is not a diameter, it is also perpendicular to the chord. 485

Theorem 58 The perpendicular bisector of a chord of a circle contains the center of the circle. 485

Theorem 59 If a line is tangent to a circle, it is perpendicular to the radius drawn to the point of contact. 492

Theorem 60 If a line is perpendicular to a radius at its outer endpoint, it is tangent to the circle. 492

Theorem 61 In a circle, equal chords have equal arcs. 499

Theorem 62 In a circle, equal arcs have equal chords. 499

Theorem 63 An inscribed angle is equal in measure to half its intercepted arc. 505

Corollary 1 Inscribed angles that intercept the same arc are equal. 505

Corollary 2 An angle inscribed in a semicircle is a right angle. 505

Theorem 64 A secant angle whose vertex is inside a circle is equal in measure to half the sum of the arcs intercepted by it and its vertical angle. 511

Theorem 65 A secant angle whose vertex is outside a circle is equal in measure to half the difference of its larger and smaller intercepted arcs. 511

Theorem 66 *The Tangent Segments Theorem.* The tangent segments to a circle from an external point are equal. 517

Theorem 67 *The Intersecting Chords Theorem.* If two chords intersect in a circle, the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord. 517

Chapter 13

Theorem 68 Every triangle is cyclic. 531

Corollary The perpendicular bisectors of the sides of a triangle are concurrent. 531

Theorem 69 A quadrilateral is cyclic iff a pair of its opposite angles are supplementary. 536

Theorem 70 Every triangle has an incircle. 543

Corollary The angle bisectors of a triangle are concurrent. 543

Theorem 71 The medians of a triangle are concurrent. 549

Theorem 72 The lines containing the altitudes of a triangle are concurrent. 549

Theorem 73 *Ceva's Theorem.* Three cevians, AY, BZ, and CX, of $\triangle ABC$ are concurrent iff $\frac{AX}{XB} \cdot \frac{BY}{YC} \cdot \frac{CZ}{ZA}$. 555

Chapter 14

Theorem 74 Every regular polygon is cyclic. 573

Theorem 75 The perimeter of a regular polygon having n sides is $2Nr$, in which $N = n \sin \frac{180}{n}$ and r is its radius. 581

Theorem 76 The area of a regular polygon having n sides is Mr^2, in which $M = n \sin \frac{180}{n} \cos \frac{180}{n}$ and r is its radius. 587

Theorem 77 If the radius of a circle is r, its circumference is $2\pi r$. 592

Corollary If the diameter of a circle is d, its circumference is πd. 592

Theorem 78 If the radius of a circle is r, its area is πr^2. 599

Chapter 15

Postulate 11 If two points lie in a plane, the line that contains them lies in the plane. 619

Postulate 12 If two planes intersect, they intersect in a line. 619

Postulate 13 *Cavalieri's Principle.* Consider two geometric solids and a plane. If every plane parallel to this plane that intersects one of the solids also intersects the other so that the resulting cross sections have equal areas, then the two solids have equal volumes. 641

Postulate 14 The volume of any prism is the product of the area of its base and its altitude: $V = Bh$. 642

Theorem 79 The length of a diagonal of a rectangular solid with dimensions l, w, and h is $\sqrt{l^2 + w^2 + h^2}$. 629

Corollary The length of a diagonal of a cube with edges of length e is $e\sqrt{3}$. 629

Corollary 1 to Postulate 14 The volume of a rectangular solid is the product of its length, width, and height: $V = lwh$. 642

Corollary 2 to Postulate 14 The volume of a cube is the cube of its edge: $V = e^3$. 642

Theorem 80 The volume of any pyramid is one-third of the product of the area of its base and its altitude: $V = \frac{1}{3}Bh$. 648

Theorem 81 The volume of a cylinder is the product of the area of its base and its altitude: $V = Bh = \pi r^2 h$. 655

Theorem 82 The volume of a cone is one-third of the product of the area of its base and its altitude: $V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h$. 656

Theorem 83 The volume of a sphere is $\frac{4}{3}\pi$ times the cube of its radius: $V = \frac{4}{3}\pi r^3$. 664

Theorem 84 The surface area of a sphere is 4π times the square of its radius: $V = 4\pi r^2$. 665

Theorem 85 If the ratio of a pair of corresponding dimensions of two similar solids is r, then the ratio of their surface areas is r^2. 672

Theorem 86 If the ratio of a pair of corresponding dimensions of two similar solids is r, then the ratio of their volumes is r^3. 672

Chapter 16

Theorem 87 The summit angles of a Saccheri quadrilateral are equal. 698

Theorem 88 The line segment connecting the midpoints of the base and summit of a Saccheri quadrilateral is perpendicular to both of them. 698

Theorem 89 If the legs of a birectangular quadrilateral are unequal, the summit angles opposite them are unequal in the same order. 698

Theorem 90 If the summit angles of a birectangular quadrilateral are unequal, the legs opposite them are unequal in the same order. 698

The Lobachevskian Postulate The summit angles of a Saccheri quadrilateral are acute. 703

Lobachevskian Theorem 1 The summit of a Saccheri quadrilateral is longer than its base. 703

Lobachevskian Theorem 2 A midsegment of a triangle is less than half as long as the third side. 703

Lobachevskian Theorem 3 The sum of the angles of a triangle is less than 180°. 708

Corollary The sum of the angles of a quadrilateral is less than 360°. 708

Lobachevskian Theorem 4 If two triangles are similar, they must also be congruent. 708

Answers to Selected Exercises

Chapter 1, Lesson 1 (pages 10–11)

2. 12 cm. 5. About 1.3 cm. 7. 3 mi. 14. Line segments. 21. Three. 25. Six; AB, AG, AC, AF, AD, AI.

Chapter 1, Lesson 2 (pages 15–17)

2. $\angle C$. 6. Rays AC and AB. 10. $\angle B = 30°$. 13. Collinear. 16. $\angle 1 = 67°$. 22. $\angle I = 117°$. 25. Approximately $1\frac{3}{4}$ ft. 28. Approximately $3\frac{1}{4}$ ft. 30. 34°.

Chapter 1, Lesson 3 (pages 21–22)

2. 9 in^2. 3. 12 in. 4. 36 in^2. 5. 24 in. 13. Piece I because it has four sides. 16. Ranch A. 23. Polygons. 25. Eight. 28. Three. 30. A line. 32. 75.6 ft.

Chapter 1, Lesson 4 (pages 26–29)

5. Noncollinear. 11. The bisecting lines seem to be concurrent. 16. No. 17. West. 21. About 6 A.M. and 6 P.M. 23. OI bisects $\angle HOJ$. 25. $\angle DOB$ and $\angle EOA$. 31. 60°. 33. 180°.

Chapter 1, Lesson 5 (pages 31–34)

1. 100 square units. 6. It contains 80 square units. 8. 54 square units. 11. It contains 84 square units.

Chapter 1, Review (pages 36–38)

2. Polygons (or squares). 5. 12. 8. Coplanar. 10. PQ. 13. T. 18. Constructions. 20. $\angle DAB = 110°$. 25. $\angle DCF = \angle FCE = 40°$.

Chapter 1, Algebra Review (page 40)

1. Associative for addition. 2. Commutative for multiplication. 3. Definition of subtraction. 4. Identity for addition. 5. Definition of division. 6. Inverse for multiplication. 7. 1,728. 8. 12,000. 9. 20. 10. −75. 11. 225. 12. 81. 13. 49π. 14. 14π. 15. $5x$. 16. x^5. 17. $5x$. 18. x^5. 19. $2x + 3y$. 20. x^2y^3. 21. $x^2 + y^3$. 22. $x^3 + y^2$. 23. $x + 2y$. 24. xy^2. 25. $7x^2$. 26. $7x^2$. 27. $5x^2$. 28. $5x^2$. 29. $4x^2 + 3x$. 30. $3x^3 + 2x^2 + x$. 31. x^{14}. 32. x^{14}. 33. $6x^5$. 34. $6x^5$. 35. $36x^2$. 36. $72x^5$. 37. $4x + 4y$. 38. $4x + 4y$. 39. $2x$. 40. $2x$. 41. 0. 42. 1. 43. $5x + 10y$. 44. $3x + 4y$. 45. $5x - 10y$. 46. $3x - 4y$. 47. $31xy$. 48. $25xy$. 49. $7 + x^3 + x^5$. 50. $10x^8$.

Chapter 2, Lesson 1 (pages 43–45)

1. A conditional statement. 4. You live in the Ozarks. 6. Yes. 14. If an animal is a koala bear, then it eats only eucalyptus leaves. 18. If I don't understand, then I ask questions. 21. If at first you don't succeed, then try again. 24. Regions 1 and 2. 28. The second. 35. Statements c and d.

Chapter 2, Lesson 2 (pages 47–49)

3. If a creature has two legs, then it is a goose. 5. You are not more than six feet tall. 9. If you do not know how to reason deductively. 13. They have the same meaning. 21. Yes. The converse must be true because this is a definition. 26. No. 28. It is the converse. 32. Sentence 2. 35. It is the converse.

Chapter 2, Lesson 3 (pages 52–54)

2. If you keep quiet, others will think you are wise. 4. All Greeks are humans. All humans are mortals. Therefore, all Greeks are mortals. 7. No. 8. The hypothesis. 12. It also may be false. 16. A theorem. 18. If you go to Dallas, you will see the yellow rows of taxis. 22. 21. 26. Red. Because the label is wrong.

Chapter 2, Lesson 4 (pages 57–59)

1. Contradiction. 3. Suppose that it would not speak foul language. 6. No. 10. It is the opposite of the theorem's conclusion. 11. The earth is not flat. 14. That it is false.

15. *Beginning assumption:* Suppose that the pupils can obey the teacher. *Contradiction:* There are 13 pupils at the black desks but only 12 brown desks. *Conclusion:* The pupils can't obey the teacher.

Chapter 2, Lesson 5 (pages 61–64)

3. You go around in circles. 6. Two points determine a line. 8. Theorems. 12. The points. 18. Converse. 20. True. 22. False. 26. A picul is 1,600 taels. 32. Statements 1 and 3. 37. Two points determine a line.

Chapter 2, Lesson 6 (pages 67–70)

1. The length of the hypotenuse of a right triangle. 5. The area of a circle. 9. No. 12. 2π, 4π, 6π, 8π, and 10π units. 13. 9π square units. 18. 30°. 21. 70°. 24. No. 26. Approximately 50 mm^2. 32. Yes, because the sum of the other two angles could be 1°. 34. a^2. 38. To the left.

Chapter 2, Review (pages 71–74)

1. If something is a limerick, then it has five lines. 9. Collinear. 11. A postulate. 13. If it rains, then it pours. 18. 50°. 20. Three noncollinear points determine a plane. 23. Square. 32. Theorems. 34. 144 square units. 37. 20 ft. 41. Direct. 44. $2\pi r$. 48. $4r^2$.

Chapter 2, Algebra Review (page 76)

1. $10x$. 2. $16x^2$. 3. $6x + 5$. 4. $4x - 5$. 5. $6x^2$. 6. $-6x^2$. 7. $9x + 4y$. 8. $3x + 2y$. 9. $5x + 15$. 10. $44 - 11x$. 11. $18x + 6$. 12. $40 - 56x$. 13. $9x^2 - 18x$. 14. $20x + 6x^2$. 15. $x^2y + xy^2$. 16. $2x^2 - 4x + 8$. 17. 10. 18. 8. 19. 3. 20. 6. 21. 13. 22. 9. 23. −7. 24. −5. 25. 60. 26. 0. 27. −12. 28. 37. 29. −2. 30. 4.5. 31. 12. 32. Any number.

Chapter 3, Lesson 1 (pages 80–82)

1. Subtraction. 2. Substitution. 10. Addition. 14. No number (because there is no number when multiplied by 0 that gives 6). 19. If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$. If $c = d$, then $\frac{a}{c} = \frac{b}{d}$ by substitution. 21. Substitution. 25. a^2 and b^2. 26. $2ab$. 28. $(a + b)^2$. 32. Substitution. 38. Division. 41. A direct proof.

Chapter 3, Lesson 2 (pages 86–90)

4. The definition of betweenness of points. 7. 52. 9. BC + CD = BD, or 5 + 6 = 11. 15. 92,760,000 mi. 17. 18. 20. 9 − 2 = 7. 26. A, −25; B, 15. 30. B. 35. AB + BC is greater than AC. 38. AC = AB + BC.

Chapter 3, Lesson 3 (pages 93–97)

6. The definition of betweenness of rays. 8. Rays. 11. The coordinates. 13. Acute. 6. 15. 22. Obtuse. 25. 14.4°. 26. Yes. 29. 28.8°. 31. $\angle SCP + \angle PCK = \angle SCK$. 35. 27. 38. They are the same. 41. Betweenness of Rays Theorem. 44. 20°. 50. 360. 51. 15.

Chapter 3, Lesson 4 (pages 100–104)

3. Figures that can be made to coincide (or fit exactly together). 7. Then it divides it into two equal segments. 9. Each is the converse of the other. 15. No. 19. 45°. 22. 67.5°. 26. The midpoint of a line segment divides it into two equal segments. 28. 1.5. 31. 2 cm. 32. 2π (or about 6.28). 42. 180. 44. They are each 36°.

Chapter 3, Lesson 5 (pages 107–109)

1. 90°. 4. 87°. 8. 30°. 9. 150°. 16. $\angle DOW + \angle WOF = \angle DOF$. 19. 27°. 26. Betweenness of Rays Theorem. 28. Substitution. 35. 75°. 37. $\angle S$ seems to be 90° larger than $\angle C$. 42. They are supplementary.

Chapter 3, Lesson 6 (pages 112–115)

1. Because they are vertical angles. 7. They are a linear pair and they are supplementary. 12. 80°. 14. They are vertical angles. 16. No. 23. Two. 26. 180°. 30. These angles are linear pairs; so they are supplementary. 36. Vertical angles are equal. 37. $\angle 3 = \angle 1 + 90°$. 45. 179°59′58.25″.

Chapter 3, Lesson 7 (pages 119–121)

1. Two angles are complementary if their sum is 90°. 4. Supplements of the same angle are equal. 6. If the angles in a linear pair are equal, then their sides are perpendicular. 10. Vertical angles are equal. 14. DH, AE, and, if perspective is taken into account, HG, EF. 17. They are a linear pair. 21. They are perpendicular. 24. $\angle 1$ and $\angle 3$ look like vertical angles, yet they are not equal. 30. $\angle BOX$, 25°; $\angle BOY$, 65°. 33. 180°. 37. They are perpendicular.

Chapter 3, Review (pages 124–128)

1. Yes. E-F-H because $6.1 < 7.7 < 10.8$. 5. $(180 - x)°$. 8. The handle is perpendicular to the blade. 11. CE-CS-CD. 13. 41.5. 18. 110. 23. They would be perpendicular. 28. Any number is equal to itself. 34. $(6 + x)$ ft. 38. $\angle AOB = (90 - x + y)°$. 43. Hexagons. 46. Six. 50. 150. 53. 120 steps per minute. 56. 6,480 inches per minute.

Chapter 3, Algebra Review (page 130)

1. $6x + 1$. 2. $2x - 4y$. 3. $4x^2 + 8$. 4. $3x^2 + 6x - 5$. 5. $12x - 3$. 6. $x^3 + 4x^2 - x - 8$. 7. $14x - 5y$. 8. $7x + 4$. 9. $6x$. 10. $-x^2 - 11x$. 11. $2x^2 + 10x - 1$. 12. $2x^3 + y^3$. 13. $x^2 + 14x + 24$. 14. $6x^2 - 7x - 20$. 15. $16x^2 - 1$. 16. $x^3 - 10x^2 - 23x + 6$. 17. $4y^3 - 8y^2 + 13y - 5$. 18. $x^3 + y^3$. 19. $5x^3 + 3x^2 - 20x - 12$. 20. $x^3 + 5x^2 - 12x - 36$. 21. -49. 22. $100x^2 - 20x + 1$. 23. $x^3 + 12x^2 + 48x + 64$. 24. $-4x + 5$.

Chapter 4, Lesson 1 (pages 134–138)

3. A and E. 5. B. 7. A(4, 3), C(−3, −4). 9. On the y-axis. 12. Two. 16. The fourth (IV). 19. The hummingbirds. 23. Point A. 27. A(4, 1), B(6, 6). AB = $\sqrt{29} \approx 5.39$. 31. $A_1(5, 2)$, $A_2(15, 2)$. 35. (900, 200). 37. 250 mm. 41. $\sqrt{18}$ or $3\sqrt{2}$.

Chapter 4, Lesson 2 (pages 141–144)

2. The vertices. 4. A 32-gon. 6. That they are congruent. 8. They are right angles (and equal). 11. Line segments AB and EF intersect two others but not at their endpoints. 12. △AEG. 17. CBEG. 25. An octagon (an 8-gon). 27. D. 31. H. 34. HBCFIE ↔ MNIEHK. 40. Convex. 45. No. 48. 21 square units. 53. No.

Chapter 4, Lesson 3 (pages 148–150)

1. BF and BP. 3. ∠B. 5. SAS. 10. TR = AN. 11. ∠I = ∠G. 15. Two triangles congruent to a third triangle are congruent to each other. 17. All the angles are acute. 20. None of the sides are equal. 22. All three. 29. 35°. 31. They get larger. 34. About 5.9 cm. 38. Perpendicular lines form right angles. 41. ASA. 44. Both are pairs of opposite rays.

Chapter 4, Lesson 4 (pages 153–155)

2. The midpoint of a line segment divides it into two equal segments. 7. Corresponding parts of congruent triangles are equal. 10. ASA. 17. If the angles in a linear pair are equal, then their sides are perpendicular. 19. Hexagons. 20. △ABX ≅ △CBX. 26. No. △BDC is larger than △ADB. 35. AB = 7, AC = 4, BC = $\sqrt{65} \approx 8.1$. 37. △ABC and △DEF would coincide. 38. The x-coordinates are equal; the y-coordinates are opposites. 40. SAS. 44. Yes.

Chapter 4, Lesson 5 (pages 159–162)

3. ∠ABD and ∠ADB. 4. BD. 6. No. 8. If two sides of a triangle are equal, the angles opposite them are equal. 12. That it has at least two equal sides. 21. Addition. 22. Betweenness of Rays Theorem. 23. Substitution. 28. They appear to be parallel. 30. ∠S = ∠B = 72°. 38. 92.2°. 39. Obtuse. 43. A pentagon. 44. Because its sides intersect in points other than their endpoints. 50. They seem to be congruent. 52. It seems to be equilateral (and equiangular).

Chapter 4, Lesson 6 (pages 165–167)

1. SSS. 2. Corresponding parts of congruent triangles are equal. 5. They are a linear pair. 10. Five. 11. No. 16. CA = CB and AD = BD. 18. SSS. 24. SAS. 29. $\angle ADC = \angle CBA$ because they are corresponding parts of congruent triangles. 38. SSS. 40. OC bisects $\angle AOB$. 42. Because an angle has exactly one ray that bisects it (the corollary to the Protractor Postulate).

Chapter 4, Lesson 7 (pages 172–175)

1. A compass. 2. Distances between pairs of points. 5. A line segment is bisected. 11. $\angle 2$ is complementary to $\angle 1$. 12. Copying an angle. 18. It means equally distant. 25. Addition. 26. Substitution. 36. SSS. 37. Height = 7.5 in; width = 10 in.

Chapter 4, Review (pages 176–180)

2. Because they have the same size and shape (they can be made to coincide). 4. They are all equal. 8. Statement 4. 13. An equilateral triangle is equiangular. 15. Substitution. 17. 60°. 19. 120°. 21. 30°. 25. SSS. 29. 90°. 33. 45°. 38. $\sqrt{4 + 16} = \sqrt{20}$ (or $2\sqrt{5} \approx 4.5$). 40. At $(-5, 3)$. 46. Seven.

Chapter 4, Algebra Review (page 182)

1. $2(3x - 1)$. 2. $3(2x - 1)$. 3. $2(3x - 2)$. 4. $4(x + 3y)$. 5. $x(x + 10)$. 6. $x^2(3x - 2)$. 7. $5(3\pi - 8)$. 8. $2x(4 + y)$. 9. $x(a + b - c)$. 10. $2\pi r(h + r)$. 11. $(x + 4)(x + 4)$ or $(x + 4)^2$. 12. $(x + 4)(x - 4)$. 13. $(x + 3)(x + 10)$. 14. $(x - 2)(x + 15)$. 15. $(x + 7)(x - 6)$. 16. $(2x - 1)(x - 7)$. 17. $(3x - 5)(x + 2)$. 18. $x(x - 4y)$. 19. $(x - 2y)(x - 2y)$ or $(x - 2y)^2$. 20. $(x + 2y)(x - 2y)$. 21. $\pi(a + r)(a - r)$. 22. $5x(x + 2)(x - 2)$. 23. $x(x + 4)(x - 3)$. 24. $z(2x - 3)$. 25. $(x + 2)(2x - 3)$.

Chapter 5, Lesson 1 (pages 186–189)

1. Either $h > w$, $h = w$, or $h < w$. 6. The transitive property. 8. Multiplication. 11. Addition. 14. AB > BC. 15. Substitution. 20. $\angle APB > \angle HPI$. 22. $\angle PIH < \angle PBA$. 26. Transitive. 30. The "whole greater than part" theorem. 34. $\angle A \approx 83°$, $\angle B \approx 56°$, $\angle C \approx 41°$. 37. 9 units. 41. That the line is "upright" (contains the center of the earth).

Chapter 5, Lesson 2 (pages 192–194)

1. $\angle BCY$, $\angle BAX$, $\angle CAD$($\angle YAD$). 3. It gets smaller. 6. An exterior angle of a triangle is greater than either remote interior angle. 9. $\angle 2$, $\angle 5$, $\angle 8$. 10. Two. 16. Two. 19. $\triangle ROS$. 20. $\angle ROA > \angle SRO$ and $\angle ROA > \angle S$. 25. No. It does not form a linear pair with an angle of the triangle. 26. 360°. 28. 180°. 32. A line segment has exactly one midpoint. 37. The "whole greater than part" theorem. 39. Two points determine a line. 41. The angles in a linear pair are supplementary. 43. An exterior angle of a triangle is greater than either remote interior angle.

Chapter 5, Lesson 3 (pages 197–199)

3. Each is the converse of the other. 4. BC < AC. 8. $\angle A > \angle B$. 10. If two sides of a triangle are unequal, the angles opposite them are unequal in the same order. 14. $\angle B$. 18. $\angle A \approx 29°$, $\angle B \approx 104°$, $\angle C \approx 47°$. 19. DE. 22. DF ≈ 4.5 cm, EF ≈ 5.4 cm. 23. $\angle X > \angle Y$. 30. $\angle BDE$. 34. BE bisects $\angle ABC$. 36. Yes. They are congruent by ASA. 42. FG < GH because both are sides of $\triangle FGH$ in which GH is the longest side. 45. An equilateral triangle is equiangular. 47. The "whole greater than part" theorem.

Chapter 5, Lesson 4 (pages 202–204)

2. DP + PH > DH. 6. They are collinear. 7. Yes; 160 million miles. 10. PA + PB > 12, PA + PC > 12, and PB + PC > 12. 17. A, B, and C are not collinear. 19. AB + BC = AC. 24. An angle has exactly one line that bisects it. 27. Substitution. 28. If two angles of a triangle are unequal, the sides opposite them are unequal in the same order. 33. The sum of any two sides of a triangle is greater than the third side. 35. Substitution. 44. 5.5 m.

Chapter 5, Review (pages 206–209)

5. The "three possibilities" property. 7. Both dimensions are 2.5 in. 12. One angle has a measure of 179°, and the other two angles are each less than 1°. 16. The midpoint of a line segment divides it into two equal segments. 17. Reflexive. 20. If the angles in a linear pair are equal, their sides are perpendicular. 22. If two sides of a triangle are unequal, the angles opposite them are unequal in the same order. 24. Transitive. 26. The small angles each appear to be equal to 10°. 29. An exterior angle of a triangle is greater than either remote interior angle. 34. $n > 2$. 37. One. 39. Yes. 41. SSS. 43. Betweenness of Rays Theorem.

Chapter 5, Algebra Review (page 210)

1. $\frac{4}{5}$. 2. $\frac{3x}{8}$. 3. $\frac{1}{x}$. 4. $\frac{x+1}{2}$. 5. $\frac{4}{7}$.

6. $\frac{1}{3}$. 7. $\frac{1}{x-5}$. 8. $\frac{x}{y}$. 9. $\frac{x+3}{10}$.

10. $\frac{x-4}{x-6}$. 11. $\frac{4}{4x}$. 12. $\frac{x^3}{x^4}$. 13. $\frac{-3}{x-2}$.

14. $\frac{8}{12}$ and $\frac{9}{12}$. 15. $\frac{28}{7x}$ and $\frac{2}{7x}$.

16. $\frac{y}{xy}$ and $\frac{x}{xy}$. 17. $\frac{3x}{3x+6}$ and $\frac{2}{3x+6}$.

18. $\frac{5(x+1)}{(x-1)(x+1)}$ or $\frac{5x+5}{x^2-1}$ and $\frac{x}{(x-1)(x+1)}$ or $\frac{x}{x^2-1}$. 19. $\frac{x^2}{xyz}$ and $\frac{y^2}{xyz}$.

Chapter 6, Lesson 1 (pages 214–217)

4. Yes, because W and E are equidistant from the endpoints of NS. 12. 6 cm. 13. 90 ft. 15. In the west. 19. Seven. 21. Line l is the perpendicular bisector of ST. 22. If a line segment is bisected, it is divided into two equal segments. 31. 9.5 in. 34. C and E; D and F. 36. Because they are corresponding parts of congruent triangles ($\triangle ABC \cong \triangle ADC$ by SSS). 39. N. 43. Three.

Chapter 6, Lesson 2 (pages 221–224)

1. (1) $\angle 1 = \angle 2$. (2) $\angle 2 = \angle 3$. (3) $\angle 1 = \angle 3$. (4) $a \parallel b$. 4. Alternate interior angles. 6. In a plane, two lines perpendicular to a third line are parallel. 7. Corresponding angles. 9. Equal corresponding angles mean that lines are parallel. 14. No. 18. AE $\parallel$ BF. 27. No. These angles are not formed by a transversal. They could be equal and yet lines x and y could intersect either to the left or to the right. 32. Four. 35. SAS. 38. No.

Chapter 6, Lesson 3 (pages 226–229)

1. 12. 2. If they form a right angle. 5. $\angle 3$. 6. Equal corresponding angles mean that lines are parallel. 12. At least, no more than, exactly. 14. No more than. 20. They form equal corresponding angles with a transversal. 22. They seem to be supplementary. 25. All four of them. 27. Down, below the figure. 30. One. 33. In a plane, two lines perpendicular to a third line are parallel. 36. Through a point not on a line, there is exactly one line parallel to the line. 37. Yes. (SAS.) 42. a and b are not parallel. 43. Given.

Chapter 6, Lesson 4 (pages 232–235)

1. They are equal. 2. Parallel lines form equal corresponding angles. 5. (1) Given. (2) Parallel lines form equal corresponding angles. (3) Vertical angles are equal. (4) Substitution. 9. Betweenness of Rays Theorem. 12. Equal corresponding angles mean that lines are parallel. 13. $\angle 1 = 115°$, $\angle 2 = 65°$, $\angle 3 = 115°$. 17. They must be perpendicular because, in a plane, a line perpendicular to one of two parallel lines is also perpendicular to the other. 20. 67°, 113°, 67°, and 113°. 22. $\angle 1$ and $\angle 2$ are supplementary because parallel lines form supplementary interior angles on the same side of a transversal. 24. $\angle 3 = \angle 4$ because parallel lines form equal alternate interior angles. 30. Through a point not on a line, there is exactly one line parallel to the line (the Parallel Postulate). 31. Parallel lines form equal alternate interior angles. 37. Two points determine a line. 39. In a plane, two lines perpendicular to a third line are parallel.

44. The length of a perpendicular segment between them. 46. 70°.

Chapter 6, Lesson 5 (pages 238–241)

1. The sum of the angles of a triangle is 180°. 2. Substitution. 3. Substitution. 4. Subtraction. 9. An equilateral triangle is equiangular. 10. An equiangular triangle has three equal angles. 14. Through a point not on a line, there is exactly one line parallel to the line. 16. Parallel lines form equal corresponding angles. 19. $\angle 2 > \angle 1$ and $\angle 2 > \angle A$. 24. An equilateral triangle. 27. It is a right triangle. 28. They are complementary. 34. $\angle 2$ and $\angle 3$ are complementary. 36. Complements of the same angle are equal. 40. An exterior angle of a triangle is equal to the sum of the remote interior angles. 41. Substitution.

Chapter 6, Lesson 6 (pages 245–248)

1. ASA. 5. HL. 6. Parallel lines form equal corresponding angles. 9. Three. 12. Yes. 19. HL. 21. If two angles of a triangle are equal, the sides opposite them are equal. 24. In a plane, two lines perpendicular to a third line are parallel. 27. Substitution. 29. AAS. 32. AB = AC, AD = AE, and $\angle A = \angle A$. 40. Because $\angle DAP = \angle EAP$ (corresponding parts of congruent triangles are equal). 42. SSS. 44. 45°. 50. Substitution.

Chapter 6, Review (pages 250–254)

1. Fold the figure along the proposed line of symmetry and see if the two halves coincide. 5. It is the perpendicular bisector of AB. 7. Equal corresponding angles mean that lines are parallel. 9. To the sum of the remote interior angles. 12. They are perpendicular. 14. Are equal, their sides are perpendicular. 16. In a plane, *two points each* equidistant from the endpoints of a line segment determine the perpendicular bisector of the line segment. 26. Equilateral. 30. In a plane, two lines perpendicular to a third line are parallel. 33. In a plane, a line perpendicular to one of two parallel lines is also perpendicular to the other. 35. Concurrent. 39. $\angle B = \angle CAD =$ 45°. 42. BA = CD by substitution. 48. $\angle CDB = 2x°$. 50. $\angle ACB = 3x°$.

Chapter 6, Algebra Review (page 256)

1. $\frac{7}{6}$. 2. $\frac{x}{20}$. 3. $\frac{x+10}{2x}$. 4. $\frac{4x-3}{x^3}$.

5. $\frac{2}{3}$. 6. $\frac{x^6}{8}$. 7. $\frac{5}{7}$. 8. $\frac{x+2}{2}$.

9. $\frac{3x}{4}$. 10. $\frac{1}{5}$. 11. $\frac{1}{6}$. 12. $\frac{2}{x}$.

13. $\frac{4x+2y-z}{8}$. 14. $\frac{4\pi r^3}{3}$. 15. $\frac{x-y}{y}$.

16. $\frac{2(x-3)}{2x-3}$ or $\frac{2x-6}{2x-3}$. 17. $\frac{x+2}{x^2-1}$.

18. $\frac{1+x}{x-y}$. 19. 1. 20. $\frac{x^3}{x-1}$.

Chapter 7, Lesson 1 (pages 260–264)

1. Two points determine a line. 3. Addition. 6. The sum of the angles of a quadrilateral is 360°. 9. A quadrilateral each of whose angles is a right angle is a rectangle. 10. Each angle of a rectangle is a right angle. 14. Lines that form right angles are perpendicular. 17. They seem to be parallel and equal. 21. They are equiangular (or each of their angles is a right angle). 26. $2x + 2y = 180$. 29. 144°. 31. SSS. 37. Triangles, quadrilaterals, and an octagon. 40. Six sides, three diagonals, and four triangles. 43. $n - 3$. 46. 720°. 47. 120°. 50. $(n - 2)180°$. 52. Their sum is 360°. 54. They all appear to be smaller.

Chapter 7, Lesson 2 (pages 267–269)

2. Point. 3. See if the figure looks exactly the same when it is turned upside down. 4. Three. 5. Yes. The opposite sides of a parallelogram are equal. 8. An exterior angle of a triangle is greater than either remote interior angle. 11. The opposite sides of a parallelogram are parallel. 15. A line segment is bisected if it is divided into two equal segments. 17. The diagonals of a parallelogram bisect each other. 18. Two points are symmetric with respect to a point if it is the midpoint of the line segment

connecting them. 26. 10. 30. About 24°. 32. $\angle AEB = x°$, $\angle CED = y°$. 35. AD = 2DC. 37. 180. 45. All three.

Chapter 7, Lesson 3 (pages 272–275)

1. A quadrilateral is a parallelogram if its opposite sides are equal. 3. Parallel lines form equal corresponding angles. 4. In a plane, if a line is perpendicular to one of two parallel lines, it is also perpendicular to the other. 8. Supplementary interior angles on the same side of a transversal mean that lines are parallel. 13. SAS. 15. A quadrilateral is a parallelogram if its opposite sides are equal. 20. Equal alternate interior angles mean that lines are parallel. 21. A quadrilateral is a parallelogram if two opposite sides are both parallel and equal. 22. Line symmetry. The figure can be folded along a vertical line so that the two halves coincide. 26. No. Line symmetry (one vertical line). 29. They seem to be parallel. 32. The pairs of opposite angles. 37. A quadrilateral is a parallelogram if its diagonals bisect each other. 39. SSS. 43. A quadrilateral is a rectangle if all of its angles are equal. 47. 12. 53. They are the sides of the quadrilaterals above and below the braced square.

Chapter 7, Lesson 4 (pages 277–280)

1. 12. 2. Three. 5. 15. 7. All of the angles of a rectangle are equal. 10. A quadrilateral is a parallelogram if its opposite sides are equal. 12. All rectangles are parallelograms. 17. All of the sides of a rhombus are equal. 19. Two consecutive sides. 24. No. It must be a rectangle. 31. A quadrilateral is a parallelogram if its opposite sides are equal. 34. A quadrilateral all of whose angles are equal is a rectangle. 36. False. 39. True. 43. Its opposite sides are parallel.

Chapter 7, Lesson 5 (pages 282–285)

1. A trapezoid. 2. That it has exactly one pair of parallel sides. 4. That all of its angles are right angles (or that they are equal). 5. Yes. All rectangles are parallelograms. 8. Point. 10. They seem to be congruent. 11. Trapezoids. 13. No. The legs of a trapezoid must be equal for it to be isosceles. 16. The bases of a trapezoid are parallel. 17. Through a point not on a line, there is exactly one line parallel to the line. 19. The opposite sides of a parallelogram are equal. 22. If two sides of a triangle are equal, the angles opposite them are equal. 25. Parallel lines form supplementary interior angles with a transversal. 30. The base angles of an isosceles trapezoid are equal. 36. 75°. 40. Parallel lines form equal corresponding angles. 41. 105°. 46. AC and DB bisect each other. 47. A quadrilateral is a parallelogram if its diagonals bisect each other. 49. A trapezoid has only one pair of opposite sides parallel.

Chapter 7, Lesson 6 (pages 288–291)

2. A midsegment. 5. DE ∥ AC, and $DE = \frac{1}{2}AC$. 8. A midsegment of a triangle is parallel to the third side. 9. In a plane, two lines parallel to a third line are parallel to each other. 10. A midsegment of a triangle is half as long as the third side. 13. MNPQ is a parallelogram because two opposite sides are both parallel and equal. 17. Addition. 18. Substitution. 20. The sum of its diagonals. 24. $\angle A = \angle C$ because the base angles of an isosceles trapezoid are equal. 28. (4, 7). 29. (8, 10). 32. 5. 35. They should be parallel. 38. They are congruent by SSS. 43. A rhombus. 49. MN seems to be parallel to AB and DC, the bases of the trapezoid.

Chapter 7, Review (pages 292–295)

2. Both pairs of its opposite sides are parallel. 4. AAS. 7. Substitution. 9. Four. 13. All of the sides and angles of a square are equal. 16. If a line divides an angle into two equal parts, it bisects the angle. 21. True. 27. True. 31. The sum of the angles of a quadrilateral is 360°. 33. The opposite angles of a parallelogram are equal. 36. D, E, and F. 42. GI ⊥ HM and ML ⊥ HM. In a plane, two lines perpendicular to a third line are parallel. 45. $2c$.

49. A trapezoid. 51. No, because a trapezoid has two parallel sides. 56. IJKL is a trapezoid because IJ ∥ LK.

Chapter 7, Algebra Review (page 296)

1. 2.236. 2. 22.36. 3. 223.6. 4. 7.07. 5. 70.7. 6. $10\sqrt{5}$. 7. $100\sqrt{5}$. 8. $5\sqrt{2}$. 9. $50\sqrt{2}$. 10. $5\sqrt{x}$. 11. $x^2\sqrt{x}$. 12. $x\sqrt{\pi}$. 13. $2x^6\sqrt{3}$. 14. $\sqrt{49\sqrt{3}} + \sqrt{3} = 7\sqrt{3} + \sqrt{3} = 8\sqrt{3}$. 15. $\sqrt{150} = \sqrt{25}\sqrt{6} = 5\sqrt{6}$. 16. $5\sqrt{7}$. 17. $5\sqrt{21}$. 18. $2\sqrt{15}$. 19. $6\sqrt{5}$. 20. 14. 21. 10. 22. $7\sqrt{6}$. 23. 11. 24. $24\sqrt{10}$. 25. 63. 26. $3\sqrt{5}$. 27. 10. 28. 23. 29. $9 + \sqrt{3}$. 30. $4\sqrt{3} + 1$. 31. $2\sqrt{x} + 2\sqrt{y}$. 32. $x + 2\sqrt{xy} + y$.

Chapter 8, Lesson 1 (pages 300–304)

1. A translation. 3. A rotation. 6. A reflection. 16. Exercises 9, 11, and 14. 17. Each has a vertical line of symmetry. 20. Yes. 25. A translation. 28. They are parallelograms because they have two sides that are both parallel and equal. 31. The opposite sides of a parallelogram are equal. 36. Betweenness of Rays Theorem. 37. Substitution. 43. A dilation. 44. They seem to be twice as long. 52. A translation. 56. A rotation.

Chapter 8, Lesson 2 (pages 308–311)

4. $2x$ units. 5. It is the segment's perpendicular bisector. 7. A rotation. 9. 180°. 14. Its image looks the same. 18. B. 22. A reflection. 24. Yes, because it is the composite of two reflections in intersecting lines. 27. C and E. 28. 120°. 30. No. The figure does not look exactly the same upside down. 32. E. 35. E. 37. A translation. 41. A rotation. 44. If a point is reflected through a line, the line is the perpendicular bisector of the segment connecting the point and its image. 46. Its magnitude. 48. SAS.

Chapter 8, Lesson 3 (pages 314–316)

3. A reflection. 5. A glide reflection. 8. ∠PAB and ∠PCD. 10. The opposite sides of a parallelogram are equal. 14. A reflection. 17. The magnitude of the rotation. 18. The lines bisect these angles. 22. They are the perpendicular bisectors of these line segments. 25. Two figures are congruent if there is an isometry such that one figure is the image of the other. 27. D. 29. G. 35. No, because the two dogs are not mirror images of each other. 39. A translation and a reflection. 42. B′(14, 9), C′(9, 6). 43. $6\sqrt{2}$. 46. M(11, 4), N(12, 5), P(8, 1).

Chapter 8, Lesson 4 (pages 321–324)

1. Rotation (or point) symmetry. 6. It has reflection (line) symmetry with respect to a line through home plate and second base. 12. The point itself. 14. A line looks the same if it is rotated 180°. 16. A line has reflection symmetry because it can be reflected (folded) onto itself. 19. That rays OA and OD are opposite rays and that rays OB and OC are opposite rays. 22. In a plane, two points each equidistant from the endpoints of a line segment determine the perpendicular bisector of the line segment. 25. They bisect each other. 30. Translation. 33. 22.5°. 34. No. 36. 16. 38. The y-axis. 42. Even. 45. Because it can be translated (in various directions) and look exactly the same.

Chapter 8, Review (pages 325–329)

6. No. 8. A glide reflection. 9. A rotation. 11. That it can be rotated so that it looks the same in three positions. 17. Yes. 20. A reflection. 24. Two figures are congruent if there is an isometry such that one figure is the image of the other. 29. A translation. 31. About 47°. 36. Because △ADC is the reflection image of △ABC. 37. It has reflection (line) symmetry with respect to line AC. 41. 45°. 47. No.

Chapter 9, Lesson 1 (pages 340–342)

3. Its area is equal to the sum of the areas of its nonoverlapping parts. 7. True, because △ADC ≅ △ABC. 9. True, because $\alpha\triangle ADC - \alpha\triangle 1 - \alpha\triangle 2 = \alpha\triangle ABC - \alpha\triangle 4 - \alpha\triangle 3$. 14. $2x$ square units. 16. $(x + y)$ square units. 20. They are both concave octagons. 23. $3a + 6b$. 26. They are congruent (SSS)

and therefore equal in area. 30. No. 31. Yes. Each one contains two of the four triangles, all of whose areas are equal. 35. $\frac{1}{6}$ square unit. 37. 2 square units. 42. It is equilateral and equiangular. 43. BFED contains four of the congruent triangles, and ABCD contains two of them. 44. 4 square units. 49. They are both equal to AB. 50. Because DF = DC + CF = CF + FE = CE.

Chapter 9, Lesson 2 (pages 345–349)

1. 20 ft. 4. 3,600 in^2. 8. Four. 11. $a(b + c)$. 12. ab and ac. 14. $(a + b)^2$. 19. Two congruent triangles. 22. $\sqrt{26}$. 25. No. 28. 2,592 nails. 30. 66 ft. 34. 6,400 square chains. 37. 84 cm^2. 38. 17.5%. 40. 21.6 ft long. 43. The total surface area of the walls. 47. 1,728 in^2. 48. 189 in^2.

Chapter 9, Lesson 3 (pages 353–357)

1. 6 square units. 5. $\frac{1}{4}c^2$. 6. The area of a right triangle is half the product of its legs. 7. The area of a polygonal region is equal to the sum of the areas of its nonoverlapping parts (the Area Postulate). 9. Substitution. 15. They seem to be equal. 17. $\triangle ABC$, 40 units; $\triangle DEF$, 80 units. 21. They seem to stay the same. 27. $\alpha ABCD \approx 99.87$ square chains. 29. The area of a triangle is half the product of any base and corresponding altitude. 31. Substitution. 33. Division. 35. 84 square units. 36. 11.2 units. 41. In a plane, two points each equidistant from the endpoints of a line segment determine the perpendicular bisector of the line segment. 42. Betweenness of Points Theorem. 48. $\alpha\triangle AMB = \alpha\triangle AMD = 20.25$ in^2; $\alpha\triangle BMX = \alpha\triangle DMY = 6.75$ in^2; $\alpha\triangle CMX = \alpha\triangle CMY = 13.5$ in^2. 52. About 1,200 shingles.

Chapter 9, Lesson 4 (pages 360–364)

1. That they are equal. 2. 2,800 ft^2. 6. Area $= \frac{1}{4}(s + s)(s + s) = \frac{1}{4}(2s)(2s) = s^2$. 8. 260 square units. 14. 1,369 square units. 17. A parallelogram. (Two of its opposite sides are both parallel and equal.) 18. 1 unit. $A = bh = (1)(1) = 1$. 23. 30,794 ft^2. 26. 4.5 ft^2. 28. 8.25 ft^2. 32. 675 m^2. 35. 3,696 m^2. 41. Its area is $(a + b)h$. 43. In a plane, two lines perpendicular to a third line are parallel. 46. 165 square units. 50. $\triangle EBH$, $FHCG$ (and $FHCD$).

Chapter 9, Lesson 5 (pages 367–370)

2. 2, 2, and 4 units. 4. The Ruler Postulate. 5. The Protractor Postulate. 8. The square of the hypotenuse of a right triangle is equal to the sum of the squares of its legs. 9. Substitution. 12. Substitution. 15. 12. 16. 1,225. 18. 37. 26. No. $256 + 225 = 481 \neq 484$. 33. 6 units. 35. 25 units. 37. $2ab$. 38. $(b - a)^2$. 43. The acute angles of a right triangle are complementary (so the sum of these angles is 90°). 45. A trapezoid. 49. It appears to bisect ABDE (that is, divide it into two congruent parts having equal areas).

Chapter 9, Review (pages 371–375)

1. 125 m^2. 3. 525. 4. Camp A, 68 paces; camp B, 64 paces; camp C, 60 paces. 8. 16 units. 12. The area of a polygonal region is equal to the sum of the areas of its nonoverlapping parts. 14. $\angle DEY$. 16. AAS. 18. Corresponding parts of congruent triangles are equal. 20. Triangles with equal bases and equal altitudes have equal areas. 23. 11. 27. $4a$. 29. $4a + 4b$. 30. $a(a + 2b)$ or $a^2 + 2ab$. 34. 216 ft^2. 38. 332.5 ft^2. 40. It would be four times as great. 44. $\alpha\triangle AEF = 2x$ (because AF = 2AC and BH = EG). 46. $4x^2 - xy$. 47. $4x - y$. 51. It would be rectangular.

Chapter 9, Algebra Review (page 376)

1. 500. 2. $\frac{1}{5}$ or 0.2. 3. 2.5. 4. ± 10. 5. -3. 6. 42. 7. 15. 8. 20. 9. 41. 10. 3.5. 11. 0. 12. 8. 13. 5. 14. 5. 15. -36.

Chapter 10, Lesson 1 (pages 380–384)

1. 2.25. 4. The geometric mean. 5. $\frac{9}{6} = \frac{6}{4}$. 8. $h \approx 45$ ft. 9. $\frac{2}{7}$. 15. Division. 19. Rectangles 2 and 4. 22. 30. 28. 36 in; $\frac{36}{15} = 2.4$. 31. $\frac{19}{10}$. 36. The width of one stripe. 37. Approximately 12.3 ft. 39. Approximately 122 ft. 41. $\frac{9}{8}$. 42. $\frac{256}{243}$. 48. The means are interchanged. 50. Division.

Chapter 10, Lesson 2 (pages 387–391)

1. They are proportional. 2. They are equal. 3. $\angle DAC = \angle FBC$. 6. $\frac{AD}{BF} = \frac{DC}{FC}$. 9. OE = 4 and OG = 8. 13. 24.2. 15. The center of the dilation. 18. $\frac{1}{2}$. 20. Corresponding sides of similar polygons are proportional. 24. A midsegment of a triangle is parallel to the third side. 28. Division. 30. Betweenness of Points Theorem. 34. Two triangles are similar if their corresponding angles are equal and their corresponding sides are proportional. 35. OC = 56 mm; $r_2 = 2.0$. 37. BF = 24 mm; $\frac{BF}{AE} = \frac{24}{21} \approx 1.1$. 42. Approximately 840 mm. 44. Approximately 1 m^2. 47. $\sqrt{2}$. 49. $w = 0$.

Chapter 10, Lesson 3 (pages 394–398)

2. If a line parallel to one side of a triangle intersects the other two sides in different points. 7. The area of the rectangle. 8. As lengths: AC = 3, AD = 2, and AE = 6. 13. $\frac{AC}{CO} = \frac{AP}{PB}$. 17. Substitution. (Both are equal to $\frac{AP}{PB}$). 18. $x = 2.4$. 22. Reasonable because BE ∥ CF in △OCF (the Side-Splitter Theorem). 26. It is a parallelogram because DF ∥ EB and DF = EB. 29. If a line parallel to one side of a triangle intersects the other two sides in different points, it divides the sides in the same ratio. 36. EO. 39. Point G is the same point as point A. 41. EF = FG; so $\frac{EF}{FG} = 1$ by division. 43. In a plane, two lines perpendicular to a third line are parallel. 45. The opposite sides of a parallelogram are equal.

Chapter 10, Lesson 4 (pages 401–405)

1. Parallel lines form equal corresponding angles. 3. AA. 6. Yes. Two triangles similar to a third triangle are similar to each other. 10. 30°, 30°, and 120°. 14. Corresponding sides of similar triangles are proportional. 17. They are not collinear. 19. That they are parallel. 21. △AHK ~ △ABD and △AKL ~ △ADE. 23. $\frac{HK}{BD} = \frac{KL}{DE}$. 27. Addition. 28. Multiplication. 31. △ABD ~ △CBA. 34. $x = 8$, $y = 10$, and $z = 10$. 39. AA ($\angle A = \angle EFA$ and $\angle FHA = \angle A$). 45. $\frac{a}{s} = \frac{b}{b - s}$.

Chapter 10, Lesson 5 (pages 409–413)

1. CG and CF. 4. Corresponding altitudes. 7. CG, the distance of the fish from the camera. 10. 7.5 cm. 12. 2.5. 15. 0.4. 18. r_2 is the reciprocal of r_1. 21. Yes. Corresponding sides of similar triangles are proportional. 23. No. These areas cannot be equal if △AGB is larger. 26. B′(7, 2), C′(17, 8). 27. △A′B′C′ appears to be an enlargement of △ABC. (Also, the corresponding sides of the two triangles appear to be parallel.) 31. 555.4 ft. 34. Corresponding altitudes of similar triangles have the same ratio as that of the corresponding sides. 35. ASA. 36. AA. 40. Ruler Postulate. 41. Through a point not on a line, there is exactly one line parallel to the given line. 47. Parallel lines form equal corresponding angles. 49. AA. 53. Reflexive. 55. Corresponding sides of similar triangles are proportional. 58. SSS Congruence Theorem.

Chapter 10, Lesson 6 (pages 416–419)

1. $\frac{1}{2}$. 2. $\frac{1}{4}$. 10. 8 m. 11. 32 m. 12. 64 m^2. 19. $\frac{4}{5}$. 21. $\frac{16}{25}$. 25. They must be proportional. 28. 440. 30. $\frac{1}{63{,}360}$. 32. Yes. $\triangle ADE \sim \triangle ABC$ by AA. 34. No. 37. Six. 40. $\frac{1}{2}$. 44. Because all circles have the same shape. 46. $90\pi \approx 283$ ft and $120\pi \approx 377$ ft. 48. $\frac{3}{4} = 0.75$. 53. 921 ft.

Chapter 10, Review (pages 421–424)

1. $\frac{1}{2}$. 4. $\frac{5}{12}$. 9. 2.5. 10. It is 6.25 times as great. 12. 6.4. 14. 12.8. 17. Concave pentagons. 18. ABHIG $\cong$ EKJGI. 20. GFEI is a parallelogram. Both pairs of its opposite sides are equal because GFEKJ $\cong$ EIGJK. 25. It increases. 30. 2. 32. They are multiplied by 3. 34. $\frac{10}{x} = \frac{x}{5}$. Approximately 7.1 cm. 36. $\frac{\sqrt{2}}{2} \approx 0.71$. 41. In a plane, two lines parallel to a third line are parallel to each other. 46. 30 ft. 48. 70 ft. 53. $\frac{EF}{20} = \frac{6}{8}$; EF = 15. 55. $\frac{4}{3}$. 56. 80. 58. $\frac{16}{9}$.

Chapter 10, Algebra Review 1 (page 425)

1. 4 and -7. 2. 1 and $-\frac{1}{5}$. 3. $4 + \sqrt{11}$ and $4 - \sqrt{11}$. 4. 7 and -7. 5. $-2 + 2\sqrt{6}$ and $-2 - 2\sqrt{6}$. 6. $\frac{1}{3}$. 7. $\frac{4}{3}$ and -3. 8. 2 and 5. 9. -1 and -8. 10. $\frac{1 + \sqrt{26}}{5}$ and $\frac{1 - \sqrt{26}}{5}$.

Chapter 10, Algebra Review 2 (page 426)

1. $r = \frac{c}{2\pi}$. 2. $p_1 = \frac{p_2 v_2}{v_1}$. 3. $v_2 = \frac{p_1 v_1}{p_2}$. 4. $c = \sqrt{a^2 + b^2}$. 5. $a = \sqrt{c^2 - b^2}$. 6. $h = \frac{2A}{b}$. 7. $h = \frac{V^2}{2g}$. 8. $r = \sqrt{\frac{A}{4\pi}}$ or $r = \frac{1}{2}\sqrt{\frac{A}{\pi}}$. 9. $R = \frac{E}{I}$. 10. $h = \frac{3V}{\pi r^2}$. 11. $a = S(1 - r)$ or $a = S - Sr$. 12. $r = \frac{S - a}{S}$ or $r = 1 - \frac{a}{s}$. 13. $w = \frac{8M}{L^2}$. 14. $w_1 = \frac{W^2}{w_2}$. 15. $r = \frac{R}{A - 150}$. 16. $A = \frac{R + 150r}{r}$ or $A = \frac{R}{r} + 150$. 17. $d = \sqrt{\frac{1}{E}}$. 18. $l = \frac{gT^2}{4\pi^2}$. 19. $A = \frac{I_n - I_x}{d^2}$. 20. $d = \sqrt{\frac{I_n - I_x}{A}}$. 21. $R = \frac{r_1 r_2}{r_1 + r_2}$. 22. $r_1 = \frac{Rr_2}{r_2 - R}$. 23. $n = -\frac{vr + vu}{ur - vu}$ or $n = -\frac{v(r + u)}{u(r - v)}$ or $n = \frac{vr + vu}{vu - ur}$ or $n = \frac{v(r + u)}{u(v - r)}$. 24. $C = \frac{5}{9}(F - 32)$. 25. $v = c\frac{f_0^2 - f_s^2}{f_0^2 + f_s^2}$.

Chapter 11, Lesson 1 (pages 430–432)

2. The altitude to the hypotenuse. 3. AP and PC. 5. EP. 6. EP and EG. 11. Their projections on the hypotenuse. 12. AC. 15. 4. 17. $2\sqrt{10}$ (or about 6.32). 18. 4.05. 21. 16. 26. $\frac{9}{16}$. 29. False. 33. $4\sqrt{6}$. 35. $\frac{\sqrt{2}}{1}$ (or $\sqrt{2}$). 37. 113 in^2. 38. $\frac{OA}{OB} = \frac{OB}{OC}$. 41. r^2. 42. r^3. 48. 5.

Chapter 11, Lesson 2 (pages 436–440)

1. 30 ft. 2. About 95 ft. 5. About 101 ft. 8. 8 cubits. 12. 20 ft. 15. About 8.7 ft. 18. $7^2 + 24^2 = 49 + 576 = 625 = 25^2$. 21. Each number in 14-48-50 is twice the corresponding number in 7-24-25. 23. 6-8-10 and 9-12-15. 28. AB. 29. $y_2 - y_1$. 31. $d = 5x$.

33. 8.4 in by 11.2 in. 35. $\frac{27}{14}$ or about 1.9.

38. OH = $\sqrt{45}$ or $3\sqrt{5}$, OP = $\sqrt{65}$, PH = $\sqrt{20}$ or $2\sqrt{5}$. 42. The length of AP. 43. $x^2 + 20^2 = (40 - x)^2$; $x = 15$.

53. $D = R - \sqrt{R^2 - B^2}$.

Chapter 11, Lesson 3 (pages 443–447)

1. Yes. They are similar by AA. 2. $\frac{a}{1} = \frac{c}{\sqrt{2}}$.

3. $c = a\sqrt{2}$. 4. $a = \frac{c}{\sqrt{2}}$. 5. In an isosceles right triangle, the hypotenuse is $\sqrt{2}$ times the length of one leg. 11. $c = \frac{2b}{\sqrt{3}}$. 15. $8\sqrt{2}$.

16. $5\sqrt{2}$. 17. $x = 12$, $y = 6\sqrt{3}$. 19. $x = 5\sqrt{3}$, $y = 10\sqrt{3}$. 23. $\sqrt{3}$. 24. $4\sqrt{3}$.
27. $90\sqrt{2} \approx 127$ ft. 30. $75\sqrt{3} \approx 130$ ft.
31. About 23 in. 34. 21 square units.

39. $\frac{x}{2}$. 43. OA = $\frac{1}{2}$, AP = $\frac{1}{2}\sqrt{3}$.

44. $(-\frac{1}{2}, \frac{1}{2}\sqrt{3})$. 53. $\frac{1}{2}$ or 0.5. 54. $\frac{\sqrt{3}}{4}$.

Chapter 11, Lesson 4 (pages 450–453)

1. $\frac{c}{b}$. 3. $\frac{f}{d}$. 4. 0.445. 6. 57.290.

7. 5.711°. 11. $\tan A = \frac{a}{b} = \frac{a}{a} = 1$.

13. $\angle A > 45°$. 14. $\tan A > 1$.

18. $\tan 60° = \frac{BC}{AC} = \frac{b\sqrt{3}}{b} = \sqrt{3}$. 22. A right triangle because $3^2 + 4^2 = 5^2$. 24. $\frac{4}{3}$.

25. 37°. 27. $\angle A < \angle B$; $\angle A$ and $\angle B$ are complementary. 29. 16.5. 32. 20°.
35. 58°. 39. 197 ft. 43. About 22 ft.
46. 3.7°. 48. 26.565051°. 51. AB = $\sqrt{5}$, BC = $\sqrt{10}$, and AC = $\sqrt{5}$. 53. 40.8°.
55. About 4,172 mi.

Chapter 11, Lesson 5 (pages 456–460)

1. $\frac{h}{a}$ or $\frac{b}{x + y}$. 2. $\frac{h}{y}$ or $\frac{a}{b}$. 3. $\frac{y}{b}$ or $\frac{b}{x + y}$.

7. 0.996. 10. 82.935°. 11. 45.000°.

14. $\frac{12}{13}$. 15. 67°. 20. $\frac{5}{13}$. 23. 5.3.

25. 61°. 26. 36.5. 29. BH = 2.6 cm, CI = 5.0 cm, DJ = 7.1 cm, EK = 8.7 cm, and FL = 9.7 cm. 36. The sine of an acute angle of a right triangle is the ratio of the length of the opposite leg to the length of the hypotenuse. 38. In a 30°-60° right triangle, the side opposite the 30° angle is half the hypotenuse. 40. Approximately 14,500 ft, or more than 2.7 mi. 43. About 806 ft.
46. About 4°. 48. Parallel lines form equal alternate interior angles. 49. AAS.

51. 5*a*. 57. $\sin \angle 2 = \frac{d - w}{h}$.

Chapter 11, Lesson 6 (pages 463–467)

1. *h*. 3. The *x*-axis. 5. *e*. 6. *j*, $\frac{1}{4}$; *e*, −4.

11. The slope is $\tan 30° \approx 0.58$. 13. −60 m.

16. About 3.4°. 18. $\frac{5}{9}$ (or about 0.56).

23. $\frac{3}{11}$. 27. $-\frac{3}{2}$. 36. Its run is 550 m, and its rise is −10 m. 39. $(300 - \frac{x}{55})$ m above the ground. 40. 16,500 m. 42. Run = $x - x_1$; rise = $y - y_1$. 43. $y = m(x - x_1) + y_1$.
47. From the start to about 2.5 seconds.

52. $\frac{AC}{BC}$. 56. Corresponding sides of similar triangles are proportional. 58. Division.
60. l_2. 62. The altitude to the hypotenuse of a right triangle is the geometric mean between the segments of the hypotenuse.

Chapter 11, Lesson 7 (pages 471–474)

2. They are opposite each other. 4. The Law of Sines. 7. They include $\angle A$. 12. 40°.
13. 17.0. 15. 60°. 16. 32.0.

18. $\frac{\sin \angle 1}{y} = \frac{\sin \angle 4}{x}$. 21. 108°. 25. 56°.
26. 27.7 mi. 28. 34.6 mi. 32. 238,000 mi.
34. $a^2 = b^2 + c^2 - 2bc \cos A$. 35. $\cos A = \frac{1}{2}$.
40. By multiplication. 44. $c^2 = a^2 + b^2$.

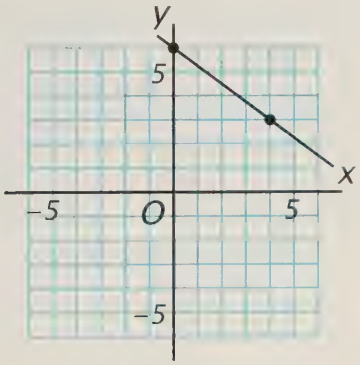
Chapter 11, Review (pages 476–480)

1. 2. 5. $-\frac{4}{3}$. 8. HF ⊥ BG. 11. $d = s\sqrt{2}$.
15. 28 ft. 16. About 24.2 ft. 19. About 195 ft. 21. About 5.7°. 26. Two.
30. 100 ft. 31. About 137.6 ft. 34. About 408 ft. 35. 0.87. 40. 7 units.
43. AF = BF. 47. About 44 ft. 53. About 380 times as far. 55. Approximately $\frac{1}{3}$ in.
57. About 600 ft. 62. $\tan \angle ABE = \frac{h - x}{d}$.

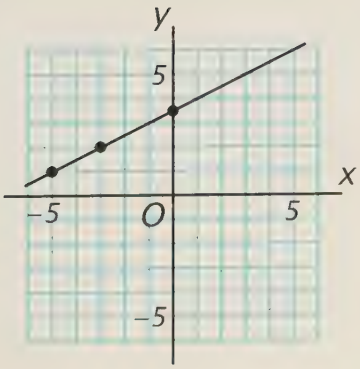
Chapter 11, Algebra Review (page 482)

1. $y = -\frac{3}{4}x + 6$. 2. $m = -\frac{3}{4}$; $b = 6$.

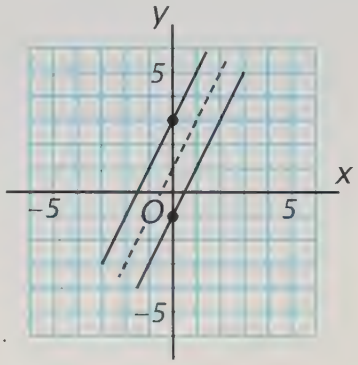
3.



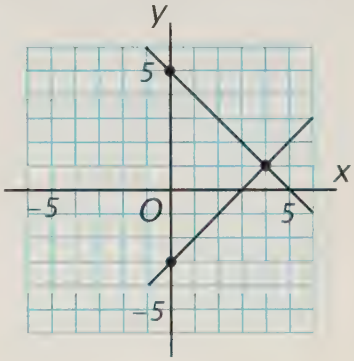
4.



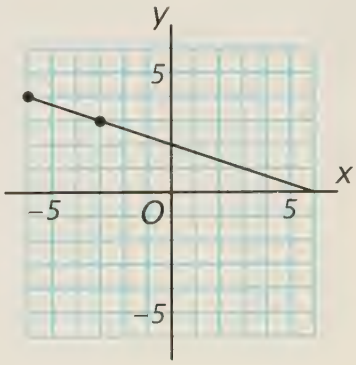
5. $y - 1 = \frac{1}{2}(x + 5)$. 6. $y = \frac{1}{2}x + \frac{7}{2}$.
7. $b = \frac{7}{2}$ (or 3.5).
8.



9. They have the same slope, 2.
10. $y = 2x + 1$. 11. $y = -x + 5$; $y = x - 3$.
12.

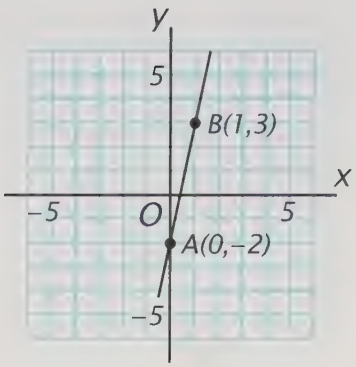


13. −1 and 1. 14. The product of their slopes is −1. 15. $-\frac{1}{3}$. 16. −6.
17.



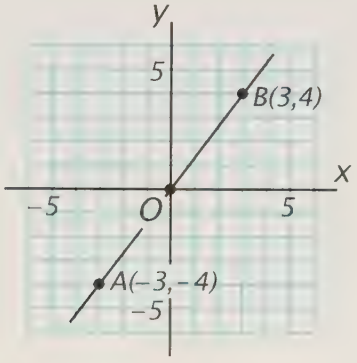
18. $y = -\frac{1}{3}x + 2$. 19. x-intercept, 6; y-intercept, 2. 20. $x + 3y = 6$.

21.



22. 5. 23. $y - 3 = 5(x - 1)$. 24. $y = 5x - 2$. 25. $5x - y = 2$. 26. $y = \frac{4}{3}x$ (or $y = \frac{4}{3}x + 0$). 27. $m = \frac{4}{3}$; $b = 0$.

28.



29. 0. 30. 10.

Chapter 12, Lesson 1 (pages 486–490)

2. Chords. 3. A diameter. 6. Five. 9. Six. 10. Concentric. 14. They are perpendicular. 17. Two points determine a line. 18. All radii of a circle are equal. 22. If a line divides a line segment into two equal parts, it bisects the segment. 26. In a plane, two points each equidistant from the endpoints of a line segment determine the perpendicular bisector of the line segment. 27. If a line through the center of a circle bisects a chord that is not a diameter, it is also perpendicular to the chord. 28. Through a point on a line, there is exactly one line perpendicular to the line. 34. An isosceles right triangle. 35. About 46 mm. 38. 1,000 seconds. 40. $\frac{10}{27}$, or about 0.37. 45. They are rhombuses because all of their sides are equal.

Chapter 12, Lesson 2 (pages 493–496)

2. In a plane, two lines perpendicular to a third line are parallel to each other. 4. $OB \perp AC$. 5. If a line through the center of a circle bisects a chord that is not a diameter, it is also perpendicular to the chord. 9. $\frac{4}{3}$. 10. $-\frac{3}{4}$. 15. AB is tangent to circle O. 16. If a line is perpendicular to a radius at its outer endpoint, it is tangent to the circle. 18. $\frac{AB}{1}$, or AB. 32. Because $\angle FDE$ is a right angle and corresponding parts of congruent triangles are equal. 34. Line AB. 36. If a line is tangent to a circle, it is perpendicular to the radius drawn to the point of contact. 41. AD = 4 cm and AF = AB = 8 cm. 42. $(8 - r)$ cm. 44. A 3-4-5 right triangle. 48. A parallelogram (also, a rectangle).

Chapter 12, Lesson 3 (pages 500–503)

1. Central angles. 2. $\overset{\frown}{PJ}$. 3. $\overset{\frown}{PCJ}$. 5. 283°. 7. 77°. 8. Two points determine a line. 10. An arc is equal in measure to its central angle. 16. The Arc Addition Postulate. 19. A semicircle. 24. No. The sum of any two sides of a triangle is greater than the third side. 27. AB = r, BC = $r\sqrt{3}$, AC = $2r$. 31. $\triangle ABC$ and $\triangle ABD$ are equilateral and congruent. 33. Line l is tangent to circle A. If a line is perpendicular to a radius at its outer endpoint, it is tangent to the circle. 41. All of the arcs have the same measure.

Chapter 12, Lesson 4 (pages 506–509)

2. 147°. 3. 24.5°. 4. 33°. 7. All radii of a circle are equal. 9. An exterior angle of a triangle is equal to the sum of the remote interior angles. 12. A central angle is equal in measure to its intercepted arc. 15. Betweenness of Rays Theorem. 17. Arc Addition Postulate. 20. An inscribed angle is equal in measure to half its intercepted arc.

22. The measure of a semicircle is 180°.
25. A 90° angle is a right angle.
28. Inscribed angles. 30. $x = 40°$, $y = 20°$, and $z = 30°$. 33. 9°. 36. They are complementary.
37. It is a right angle. An angle inscribed in a semicircle is a right angle. 43. $n°$. 44. $2n°$.
50. It is an altitude (to the hypotenuse) of the triangle. 52. BF = $\sqrt{3}$ in.

Chapter 12, Lesson 5 (pages 512–515)

2. An exterior angle of a triangle is equal to the sum of the remote interior angles. 3. An inscribed angle is equal in measure to half its intercepted arc. 5. 36. 7. 54° and 126°.
10. Subtraction. 14. 40°. 15. 75°.
18. 55°. 19. A secant angle. 20. It is less than 55°. 23. $m\overset{\frown}{AD} + m\overset{\frown}{CB} = 180°$.
27. 72°. 29. 36°. 34. 20°. 40. If a line through the center of a circle bisects a chord that is not a diameter, it is also perpendicular to the chord. 42. 17 ft. 45. Each angle is 90°. 48. About 92.5 days. 51. 6°.

Chapter 12, Lesson 6 (pages 517–521)

3. HL. 7. Inscribed angles that intercept the same arc are equal. 9. Corresponding sides of similar triangles are proportional.
12. The tangent segments to a circle from an external point are equal. 19. Right angles. An angle inscribed in a semicircle is a right angle. 24. CFOE is a square. 31. PA is tangent to circle O because it is perpendicular to radius OA at its outer endpoint.
33. PA = PB = 10. 35. 20 units.
41. (12, 10). 45. If a line through the center of a circle is perpendicular to a chord, it also bisects the chord. 46. BP.

Chapter 12, Review (pages 523–526)

2. A central angle. 7. 60°. 9. 35°.
10. 16.5. 15. 30°. 18. 60°.
25. AB = AD. The tangent segments to a circle from an external point are equal. 27. They are similar to △ABC. 29. $\frac{1}{2}$. 34. PB ⊥ AC.
36. Isosceles. (In the figure, they are also obtuse.) 40. 4 cm. 42. Approximately 3.7 cm. 44. Approximately 68°. 46. 53°.
47. 74°. 51. C is the midpoint of AB.
52. Approximately 9.7 in. 53. Approximately 75.5°. 56. $\sqrt{5}$. 58. $\frac{\sqrt{5}+1}{2}$.

Chapter 12, Algebra Review (page 528)

1. (8, 8). 2. (7, 14). 3. (11, 9). 4. (3, −4).
5. (0, −1). 6. (5, 14). 7. (14, 5).
8. (3.5, 10.5). 9. (45, 9). 10. (−2, 1).
11. (−19, −2). 12. (0, −5). 13. (51, −16).
14. (3, −11). 15. (18.5, 0.9). 16. (−1, −10).
17. (11, −4). 18. (7, 1). 19. (7, 3).
20. (9, 2). 21. (4, 0). 22. (−1, 5).
23. (8, −2). 24. (0, −8).

Chapter 13, Lesson 1 (pages 532–535)

4. Its circumcircle. 5. △ABC is inscribed in circle O. 6. They are equidistant from it.
7. Minor arcs. 13. A diameter (or a chord).
17. The lines are the perpendicular bisectors of the sides of △ABC, and the perpendicular bisectors of the sides of a triangle are concurrent. 19. No. 24. Three. 28. OA = 2OD because, in a 30°-60° right triangle, the hypotenuse is twice the shorter leg.
32. The circumcenter of the triangle with A, B, and C as its vertices. 36. It is approximately 6.4 cm. 39. They are concurrent on the hypotenuse of the triangle. 40. 7.5 cm.
42. Both ∠E and ∠ABD intercept $\overset{\frown}{AD}$. Inscribed angles that intercept the same arc are equal. 47. Approximately 31 mm.
48. 31.25 mm. 51. sin D = $\frac{a}{2r}$.

Chapter 13, Lesson 2 (pages 538–541)

1. By seeing if a pair of opposite angles are supplementary. 4. They must be concurrent.
8. Inscribed angles that intercept the same arc are equal. 10. Substitution. 14. They are isosceles because all radii of a circle are equal.
18. AB ∥ DC because they are bases of a trapezoid. 21. ∠B and ∠D are supplementary.
24. They intersect in a common point, P.
30. ABCD is not cyclic, because its opposite angles are not supplementary.
33. Vertical angles are equal. 34. Supplements of the same angle are equal. 40. 1,764.
42. The Protractor Postulate.
46. Multiplication. 54. Substitution.

Chapter 13, Lesson 3 (pages 544–547)

1. The angle bisectors of a triangle are concurrent. 3. Two. 4. They are perpendicular bisectors of the sides. 5. The vertices of the triangle. 7. Two angle bisectors. 10. Its incircle. 11. Its incenter. 13. They are perpendicular to the sides. If a line is tangent to a circle, it is perpendicular to the radius drawn to the point of contact. 15. They must be concurrent. 17. Only if the rectangle is a square. 21. They are the perpendicular bisectors of the sides. 25. 2π and 4π units. 28. EF seems to be tangent to the incircle. 32. Exterior angles. 38. It proves that AP bisects $\angle DAE$. 40. FDEO is a square. 42. $2\pi r$ units. 46. $\frac{\pi}{4}$. 49. $3 - r$. 51. $7 - 2r$. 55. π square units. 59. The area of a triangle is half the product of its base and altitude. 61. $\frac{2\pi r}{p}$.

Chapter 13, Lesson 4 (pages 550–553)

4. The fact that the altitudes of a triangle form right angles with the lines of its sides. 5. Two points determine a line. 6. The Ruler Postulate. 8. A midsegment of a triangle is parallel to the third side. 11. A line segment that connects a vertex of a triangle to the midpoint of the opposite side is a median. 14. Through a point not on a line, there is exactly one line parallel to the line. 18. In a plane, a line perpendicular to one of two parallel lines is also perpendicular to the other. 23. GD and EH bisect each other. The diagonals of a parallelogram bisect each other. 28. $\triangle ABC$ is obtuse. 29. No. The altitude *segments* do not intersect. 32. GE, AF, and CD. 33. At point B. 41. Corresponding sides of similar triangles are proportional. 42. Substitution. 49. PD and EC are altitudes of $\triangle APE$.

Chapter 13, Lesson 5 (pages 557–560)

1. CF. 5. No. GF is not a cevian, because neither of its endpoints is a vertex of the triangle. 6. 3.2. 8. Not concurrent. 11. 2. 15. That this product is 8. 18. That this sum is 2. 21. 3. 24. AX = 4; XB = 6. 28. $\triangle AXZ \sim \triangle CPZ$ and $\triangle BXY \sim \triangle CPY$. 32. Division (and substitution). 34. If a line parallel to one side of a triangle intersects the other two sides in different points, it divides the sides in the same ratio. 41. $\frac{AX}{BX} \cdot \frac{BY}{CY} \cdot \frac{CZ}{AZ} = 1$ (by Ceva's Theorem). 45. In a plane, a line perpendicular to one of two parallel lines is also perpendicular to the other. 52. $\triangle AXC \sim \triangle AZB$.

Chapter 13, Lesson 6 (pages 562–565)

1. AB and DC. 2. $-\frac{3}{2}$. 4. $\sqrt{13}$ units. 7. M(9, 0); L(3, 6). 8. (8, 4). 11. CN = NB = $\sqrt{72}$ or $6\sqrt{2}$. 16. mNT = 1 and mCB = −1. 18. (9, 3). 24. At V. 27. (6, 6). 36. 48 units. 38. 150°. 45. No. AC and CB are not perpendicular, because the product of their slopes is not −1. 46. G(3, 14); I(17, 14). 53. AB = DC = 6; AD = BC = $\sqrt{10}$. 57. mXZ = 9; mWY = $-\frac{1}{9}$.

Chapter 13, Review (pages 566–570)

2. $\triangle ABC$ is inscribed in the circle. 6. $\triangle ABC$ is a right triangle because $\angle ACB$ is inscribed in a semicircle. 12. F. 14. IJKL. 18. Midsegments. 24. AX = 4, YB = 3, and ZC = 2. 28. Cyclic. 34. $\frac{3}{4}$ in. 36. $\angle AED + \angle AGD = 180°$. $\angle AED$ and $\angle AGD$ are supplementary because they are opposite angles of a cyclic quadrilateral. 38. $\angle EGD = \angle EAD$ because they are inscribed angles that intercept the same arc. 39. $\angle EGD = 60°$ because $\angle EGD = \angle EAD$. 42. $\angle BHC = 120°$ because it is supplementary to $\angle BFC$. 48. 125°. 53. They appear to bisect its angles. 55. DB and FB. 61. ABCD is concave.

Chapter 14, Lesson 1 (pages 574–578)

3. The radii of the polygons. 5. Yes. If a triangle is equilateral, it is also equiangular. 8. No. 10. Yes. (*All* regular polygons are cyclic.) 13. 60°. 14. Equilateral.

18. Three squares. 20. They are factors of it. 21. Equilateral triangles and regular pentagons. 27. Regular polygons having an even number of sides. 31. If it isn't regular, it must not be equiangular. 33. Isosceles right triangles. 39. 22.5°. 41. 135°. 45. Apothems. 47. 30°-60° right triangles. 52. They are equal because the sides of the pentagon are chords of the circle, and equal arcs in a circle have equal chords. 54. 72°.

Chapter 14, Lesson 2 (pages 581–584)

2. The number of sides. 4. 3.0902. 10. Radii. 14. PB bisects $\angle ABC$. 16. 9 sin 20°. 17. 3.078. 21. The radius of the hexagon appears to be longer. 22. 108°. 26. 5 sin 36° $\approx$ 2.94. 28. Approximately 2.13 angstroms. 31. 74 mm. 37. $r\sqrt{3}$. 39. $p \approx 5.196r$. 41. 2. 44. By the Law of Sines. 45. Approximately 1.9. 48. Approximately 2.2. 49. 18°. 52. 3.0901699. . . .

Chapter 14, Lesson 3 (pages 587–590)

3. 0. 5. 1.30. 9. The perimeter of the nonagon. 14. $4a^2$. 16. $r = a\sqrt{2}$. 18. $2r^2$. 22. 628 units. 28. 31,359 square units. 31. 30°-60° right triangles. 32. $\frac{r}{2}$. 34. $\frac{\sqrt{3}}{4}r^2$. 42. They are isosceles. 43. That they are 15°. 47. All are $(2 \sin 15°)r \approx 0.518r$. 49. $\frac{1}{4}r^2$. 54. 7.1 cm. 57. 38 cm.

Chapter 14, Lesson 4 (pages 593–597)

2. π. 5. The circumference of the hat. 7. Approximately 1,500,780 mi. 10. πa. 16. 704 ft/minute. 17. 14π ft. 20. 10 units. 21. 8 units. 24. $AC = \frac{1}{2}AB$. If a line through the center of a circle is perpendicular to a chord, it also bisects the chord. 26. 3.1415927. 28. $2\sqrt{2}$. 29. 2.83. 35. Approximately 24,880 mi. 36. Approximately 1,040 mi. 37. 1,040 mi/hour. 42. Approximately 19 mi/second. 45. $r^2 = \frac{5}{4}x^2$. 46. $r^2 = \frac{1}{2}y^2$. 49. $2a + 2b + 2c$. 52. 825 ft. 53. $\pi \approx 3.14$ in.

Chapter 14, Lesson 5 (pages 600–604)

1. 300 mi. 2. 940 mi. 3. 71,000 mi^2. 4. 15 mi. 5. 90 mi. 9. x^2. 12. 100 circular inches. 14. 3,600π cm^2. 15. $60t$ cm. 19. 36π square units. 23. π^2. 25. 6 units. 27. 3.1425. 31. $3\pi x^2$. 33. $(4 - \pi)x^2$. 35. $(5\pi - 8)x^2$. 36. $(3\sqrt{3} - \pi)x^2$. 42. 72.6. 43. 290. 47. $a + b$. 48. $\pi a^2 + \pi ab$ or $\pi a(a + b)$.

Chapter 14, Lesson 6 (pages 606–611)

1. A sector. 2. 45°. 3. $\frac{1}{4}\pi r$. 4. $\frac{1}{8}\pi r^2$. 5. The length of $\overset{\frown}{AB}$ in meters. 8. 15,700 m^2. 9. About 6,220 mi. 12. 2,009 ft^2. 13. 887 ft^2. 16. πr^2. 18. 60°. 19. $\frac{1}{3}\pi x$. 22. x. 23. About 9 in. 25. About 0.0007 second. 27. 360°. 28. $\frac{1}{2\pi}$. 30. 57.3°. 31. 46 in. 32. 469 in^2. 34. About 53°. 37. About 93 square units. 39. About 45 square units. 41. 63.66 m. 43. 103.77 m. 45. About 188 m. 47. $2\sqrt{2}$. 49. 4π. 50. 8. 51. $4\pi - 8$. 53. π. 58. 1,256.64 in^2. 62. 34.64 in. 64. 246 in^2. 66. $2a + b$. 67. $2a + 2b$.

Chapter 14, Review (pages 612–616)

2. It must be convex, equilateral, and equiangular. 4. 120°. 6. 48°. 9. 3.12. 10. 3.05. 12. $6.24r$. 15. About 420 turns. 18. $2\pi^2$. 20. 4π. 21. $\frac{\pi}{8}a^2$, $\frac{\pi}{8}b^2$, and $\frac{\pi}{8}c^2$. 24. $\frac{1}{2}\pi r^2$. 28. $\sqrt{3} \approx 1.732$. 29. $\frac{2}{3}\pi \approx 2.094$. 31. $\frac{4}{3}\pi - \sqrt{3} \approx 2.457$. 33. 45°. 34. 0.79. 35. $\sqrt{2} - 1$. 37. 0.98. 39. About 1,037 mi. 40. About 3,034 mi. 42. About 794 mi. 45. $\sqrt{2}$ (or about 1.414). 47. 8.78. 52. $\pi R^2 - \pi r^2$ or $\pi(R^2 - r^2)$. 55. 4π. 58. $(360 - 2x)°$. 59. $\frac{180 - x}{90}\pi$. 63. About 83 ft.

Chapter 15, Lesson 1 (pages 620–624)

1. The second. 7. It appears to be oblique to the ground. 8. A vertical plane. 10. That they lie in a horizontal plane. 11. No. AB is perpendicular to the floor if it is perpendicular to *every* line in the floor that passes through point B. 13. Parallelograms. 15. EFGH. 21. They intersect in line l (or they both contain it). 23. One plane must contain a line that is perpendicular to the other plane. 25. CF is perpendicular to plane EFGH. 27. If a plane intersects two parallel planes, it intersects them in parallel lines. 32. Three noncollinear points determine a plane. 33. If two points lie in a plane, the line that contains them lies in the plane. 34. They lie in it. 40. No. Two horizontal lines can be skew. They can also intersect. 42. About 91 cm. 43. About 62°. 45. SAS. 47. A quadrilateral is a parallelogram if its diagonals bisect each other. 55. Nine.

Chapter 15, Lesson 2 (pages 630–633)

1. Six. 2. Eight. 3. 12. 4. F. 7. CG, DH, EH, and FG. 10. 0.025 cm. 12. 0.225 cm^2. 14. 185 units. 15. 697 units. 18. $8x + 10 = 106$. 20. A regular hexagon. 21. $\triangle HEM$ and $\triangle HEN$ are congruent right triangles. 25. $\angle EAC$ is a right angle. 28. An isosceles trapezoid. 30. 6 cm. 32. 45°. 33. 60°. 35. About 35°. 39. $3\sqrt{2}$ cm. 44. $\frac{e}{2}\sqrt{3}$. 46. 54.74°. 49. One. The structure appears to contain $27 + 1 = 28$ cubes.

Chapter 15, Lesson 3 (pages 635–638)

1. L, I, and E. 3. Prisms. 5. Two. 6. Three. 10. The bases. 12. n. 13. $n + 2$. 18. 1,624 ft^2. 19. A parallelogram. 20. BAB'C. 23. EDE'F. 26. They appear to be parallel and equal. 29. Pentagons. 33. 1,458 ft^2. 35. They are rectangles. 38. Yes. The bases of *every* prism are congruent. 44. AX > AE. 46. A right triangular prism. 50. $2n$.

Chapter 15, Lesson 4 (pages 642–646)

2. "x cubed." 4. Its volume. 5. 27. 7. 36. 8. 1,296. 12. About 1.6 ft^3. 14. 1,518,750 ft^3. 16. 1 m^3. 17. 6 m^2. 20. 12 m^2. 27. A right hexagonal prism and an oblique hexagonal prism. 31. 108 cubic units. 33. 288 cubic units. 34. 79 cm^3. 36. 55 cm^3. 38. 0.004 in. 40. 187.156 in^2. 41. $(a + b)^3$. 43. Two of them. 44. a^2b. 49. $ab + ac + bc + c\sqrt{a^2 + b^2}$. 51. 3 cm. 54. 236.5 ft^2. 55. 4,730 ft^3. 57. 9 ft^3.

Chapter 15, Lesson 5 (pages 649–653)

1. A pyramid. 2. Its apex. 4. A cross section. 5. A lateral face. 10. 128 cubic units. 12. 70 cubic units. 13. They are equal. 19. 576 yd^3. 22. $V = \frac{1}{3}Bh$. 23. 55.125 square units. 25. 385.875 cubic units. 28. $\frac{1}{6}x^3$. 30. $\frac{1}{2}x$. 33. 231,000 ft^2. 34. 612 ft. 35. 231,000 ft^2. 37. 92,000,000 ft^3. 38. $n + 1$. 43. It is a pyramid. 45. Isosceles right triangles. 46. $\frac{3}{4}s^2$. 47. $\frac{1}{6}s^3$. 48. $\frac{1}{3}a^2(x + h)$. 51. $OM = \frac{1}{2}a$ and $PN = \frac{1}{2}b$. 52. Corresponding sides of similar triangles are proportional. 53. $\frac{x + h}{x} = \frac{a}{b}$.

Chapter 15, Lesson 6 (pages 656–661)

6. Product. 10. Its axis. 11. The altitude. 12. The lateral surface. 13. The bases. 15. 45π in^3. 19. 7 in^2. 22. 4,800 m^3. 25. They have equal volumes. 26. Cavalieri's Principle. 27. 35π cm^3. 28. 110 cm^3. 31. 65%. 34. That they have the same length. 38. The diameter of its base. 41. 352 in^3. 42. πh^2. 44. It is multiplied by four. 48. 285 in^3. 52. $2\pi rh$. 54. A sector of a circle. 55. $2\pi r$. 57. πrl.

Chapter 15, Lesson 7 (pages 665–669)

1. They are equal. 2. It is four times as large. 3. About 12.6 km^2. 4. About 4.2 km^3. 7. $\frac{1}{3}\pi x^3$. 11. All radii of a sphere are equal. 12. If a line is perpendicular to a plane, it is perpendicular to every line in the plane that passes through the point of intersection. 18. It decreases. 19. 212 cm^3. 21. 0.7 gram/cm^3. 28. $\frac{2}{3}\pi r^3$. 30. $\frac{3}{2}\pi r^2$. 33. Either leg of a right triangle is the geometric mean between the hypotenuse and its projection on the hypotenuse. 35. 3,895 mi. 38. About 15 hours. 39. About 13 hours. 44. 325,000,000 mi^3. 47. The second expression.

Chapter 15, Lesson 8 (pages 672–676)

2. Two cubes are always similar. 6. It is multiplied by 9. 7. It is multiplied by 27. 10. It is multiplied by n^2. 11. It is multiplied by n^3. 13. 10.24. 14. 32.768. 17. That Gulliver's body was similar to their bodies. 20. About 3.6 times. 23. About 22 mm. 25. About 1.8 times. 30. $24x^2$. 32. $8x^3$. 35. The square. 38. $x = \sqrt[3]{2}$; $y = \sqrt[3]{4}$. 39. PA, 1; PB, 2; PC, 4; PD, 8. 40. It is twice as large. 45. Eight. 51. About 47 pounds. 52. About 93 pounds.

Chapter 15, Lesson 9 (pages 678–682)

1. Eight. 3. Quadrilateral. 6. Yes. 9. The area of the larger triangle is four times the area of the smaller one. 10. The surface area of the larger tetrahedron is $3^2 = 9$ times the surface area of the smaller one. 14. 12. 16. Three. 18. 20. 20. An icosahedron. 26. 12. 29. That they are congruent. 30. 180°. 34. 270°. 38. $\frac{e}{2}\sqrt{2}$. 41. $\frac{1}{6}e^3$. 44. $\angle ABP = 54°$ and $\angle BAP = 36°$. 45. 1.618. 46. About 1.8. 52. 0.714 unit. 53. 10.5 square units. 58. 2.8 cubic units.

Chapter 15, Review (pages 684–688)

1. 4 and 8. 3. They appear to be perpendicular. 10. 16 ft. 13. A prism and a pyramid. 15. 80,000 ft^3. 18. It is four times as large. 21. 250,000. 23. 1,250,000,000. 25. $e\sqrt{2}$. 27. A regular tetrahedron. 29. $\frac{1}{6}e^3$. 30. $\frac{1}{3}e^3$. 36. $2\pi x^3$. 37. $\frac{2}{3}$. 40. $6\pi x^2$. 45. 40 square units. 50. The area of the horizontal cross section halfway up the barrel. 51. $\pi r^2 h$. 58. Four. 59. Six.

Chapter 16, Lesson 1 (pages 692–695)

1. At the center of the sphere. 2. A radius of the sphere. 3. Two hemispheres. 7. Euclidean geometry. 10. Both geometries. 16. 186°. 18. $\angle A = 72°$, $\angle B = \angle C = 60°$; $\angle A + \angle B + \angle C = 192°$. 22. Yes. 25. Arcs of great circles. 30. Both curves cannot correspond to lines. 31. One. 32. No. 40. No. The sum of the angles of $\triangle ABC$ is greater than 180°. 45. It has three right angles rather than one. 46. It is four times as long.

Chapter 16, Lesson 2 (pages 698–700)

1. Its legs. 2. The base angles. 3. The summit. 5. WY and MO. 6. $\angle Y$ and $\angle O$. 7. Two points determine a line. 9. All right angles are equal. 14. The Ruler Postulate. 16. It is a birectangular quadrilateral whose legs are equal. 17. The summit angles of a Saccheri quadrilateral are equal. 20. An exterior angle of a triangle is greater than either remote interior angle. 23. A Saccheri quadrilateral. 25. The fact that $\angle D > \angle C$. 26. If the legs of a birectangular quadrilateral are unequal, the summit angles opposite them are unequal in the same order. 29. CD. 31. DM = CM because $\triangle ADM \cong \triangle BCM$ (SAS). 33. In a plane, two points each equidistant from the endpoints of a line segment determine the perpendicular bisector of the line segment. 38. DA. 40. The line segment connecting the midpoints of the base and summit of a Saccheri quadrilateral is perpendicular to both of them.

42. In a plane, two lines perpendicular to a third line are parallel. 44. The opposite angles of a parallelogram are equal.

Chapter 16, Lesson 3 (pages 704–706)

1. The summit angles of a Saccheri quadrilateral are acute in Lobachevskian geometry. 3. Substitution. 4. A line segment has exactly one midpoint. 6. The line segment connecting the midpoints of the base and summit of a Saccheri quadrilateral is perpendicular to both of them. 7. A quadrilateral that has two sides perpendicular to a third side is birectangular. 9. The Addition Theorem of Inequality. 14. A birectangular quadrilateral whose legs are equal is a Saccheri quadrilateral. 15. The summit of a Saccheri quadrilateral is longer than its base in Lobachevskian geometry. 20. Through a point not on a line, there is more than one line parallel to the line. 28. Birectangular. 34. In Riemannian geometry, the summit of a Saccheri quadrilateral is shorter than its base. 37. They are all half as long. 42. They are congruent squares.

Chapter 16, Lesson 4 (pages 709–712)

2. $\angle D$ and $\angle E$. 4. A Saccheri quadrilateral. 5. The summit angles of a Saccheri quadrilateral are acute in Lobachevskian geometry. 8. Substitution. 10. The sum of the angles of a triangle is less than 180° in Lobachevskian geometry. 14. Corresponding angles of similar triangles are equal. 17. The angles in a linear pair are supplementary. 21. No. The sums of the angles of the two triangles may not be equal. 26. $\angle A + \angle B + \angle C > 180°$. 27. Riemannian geometry. 31. $\angle 1 = \angle A + \angle B$. 35. $\triangle ADE \sim \triangle ABC$ (AA). 41. $\angle F$ is a right angle. 42. CFDE is a birectangular quadrilateral (in two different ways). 43. $\angle FDE$ is acute. 46. $DE > 2BC$.

Chapter 16, Review (pages 714–717)

1. A great circle. 2. False. 4. True (because antipodal points are considered to be identical). 7. AD. 12. 360°. 16. $\angle ABC = 90°$; $\angle BCD = 120°$; $\angle CDA = 90°$; $\angle DAB = 120°$. 17. The quadrilateral is a parallelogram. 19. It is a rhombus. 23. No. 26. Saccheri quadrilaterals. 27. Acute. 32. Substitution. 34. A midsegment of a triangle is less than half as long as the third side. 37. Two points determine a line. 40. The sum of the angles of a triangle is less than 180°.

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About the Author:

HAROLD R. JACOBS is teacher of mathematics and science, writer, and well-respected speaker. He received his B.A. from U.C.L.A. and his M.A.L.S from Wesleyan University. His other publications include *Mathematics: A Human Endeavor*, *Elementary Algebra* and articles for *The Mathematics Teacher* and the *Encyclopedia Britannica*. Mr. Jacobs has received the Most Outstanding High School Mathematics Teacher in Los Angeles award, the Presidential Award for Excellence in Mathematics Teaching, and was featured in the book *101 Careers In Mathematics* published by the Mathematical Association of America.

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