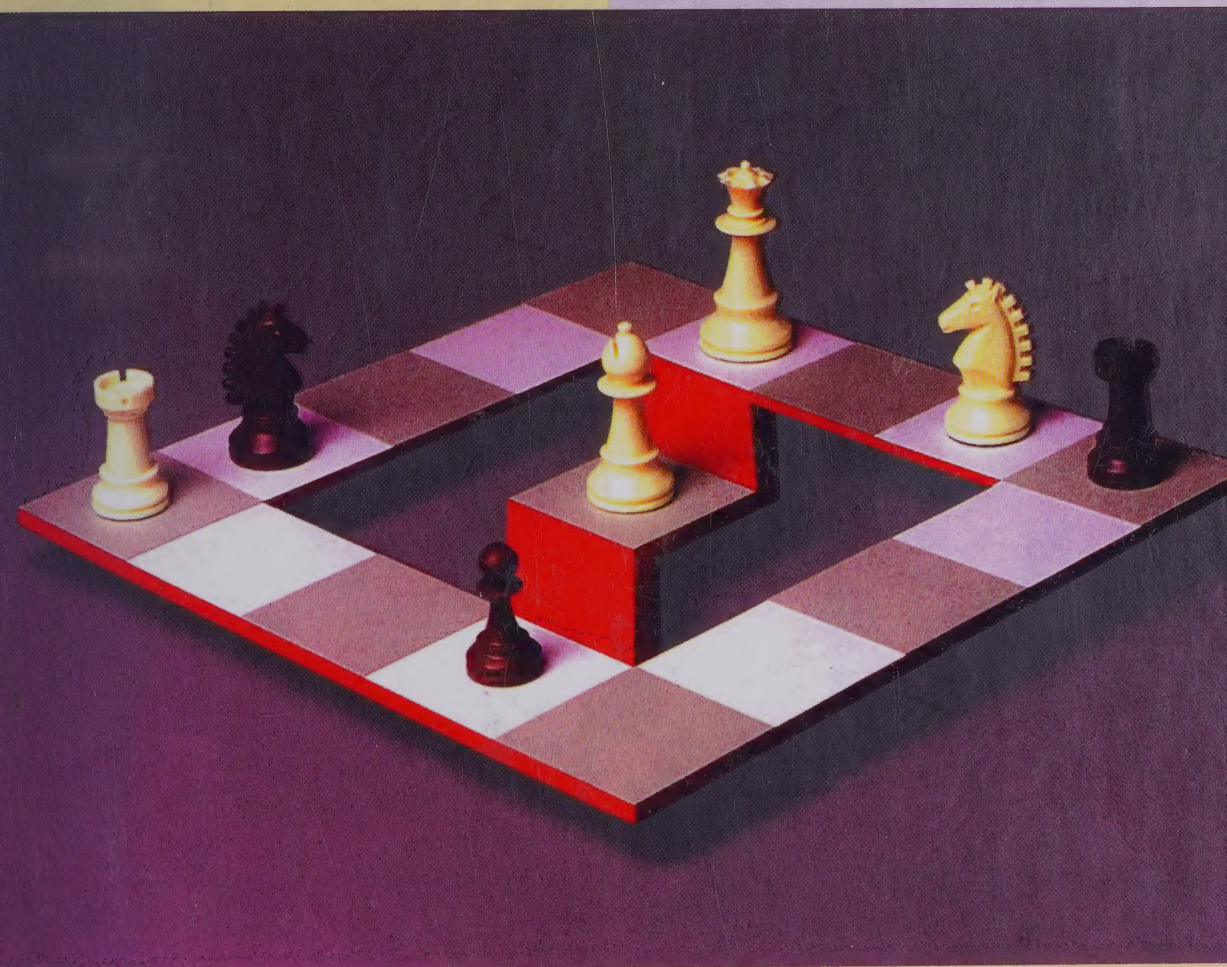


GEOMETRY

Seeing, Doing, Understanding

Third Edition



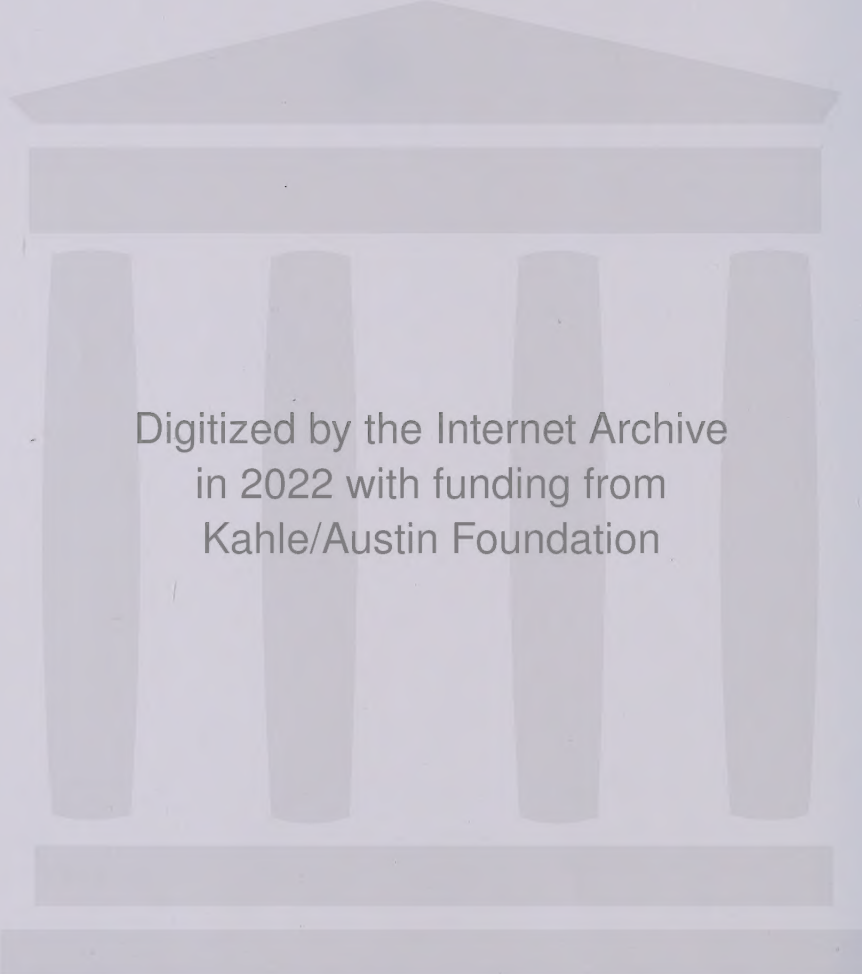
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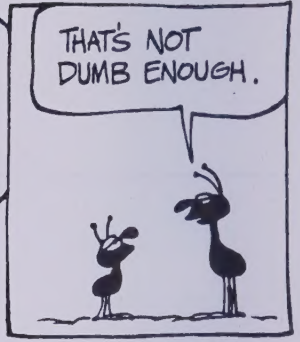
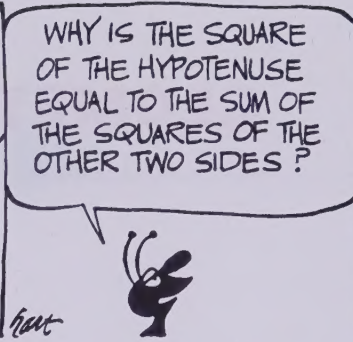
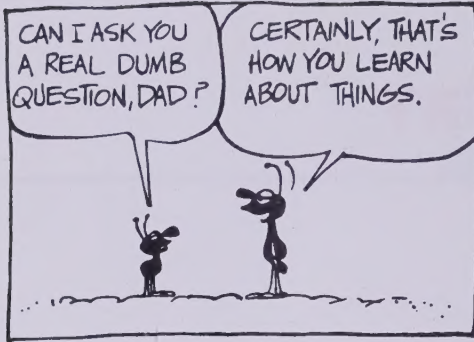
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Harold R. Jacobs



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Seeing, Doing, Understanding

Third Edition

Harold R. Jacobs

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EXECUTIVE EDITOR: Craig Bleyer
SENIOR DEVELOPMENT EDITOR: Mary Johenk Maechling
CONTRIBUTING EDITOR: Dr. Peter Renz
MARKETING MANAGER: Cindi WeissGoldner
PROJECT EDITOR: Jane O'Neill
COVER AND TEXT DESIGNER: Diana Blume
ILLUSTRATION COORDINATOR: Shawn Churchman
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Forewords

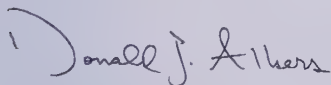
This is one of the great geometry books of all time. It is the book that I would like to have had for the geometry classes that I taught at Menlo School and College. It is the finest example of instructional artistry that I have encountered.

When I taught statistics courses, I used the counting and probability chapters from *Mathematics: A Human Endeavor* by Jacobs. His approach engaged my students and enabled them to discover and master this subject. In the present book, Jacobs does the same thing for geometry.

Harold Jacobs—author and master teacher—loves geometry, and this book shows it. Its thousands of photographs, diagrams, and figures draw the reader into the material. Jacobs has a genius for finding illustrations that reveal a subject and make it come alive. A look at his sources shows the broad search that he made to assemble this visual feast.

Teachers know that in most classes the goal for students is to complete the homework exercises with as little pain as possible. When students get their assignment, they first go to the exercises. If the teacher worked an example in class, the student follows that. If not, then the student scans the text in search of an example that is similar to the exercise. So many textbooks are little more than a bunch of examples, followed by exercises that can be solved by using the methods of the examples. And the student learns by creative copying.

As you page through this book, you will be struck by how few words Jacobs uses in introducing each lesson. Jacobs knows that students will probably not read much introductory material and want to get to the exercises as quickly as possible—solutions are still the goal. But, for Jacobs, the exercises are the beating heart of the book. You will first notice that they are beautifully illustrated with real-world material. As you read, you will discover that they are engaging, carefully sequenced, and structured so that students discover ideas for themselves. Students come away from his exercise sets empowered, because the ideas have become “their” ideas. Students using this book, working alone or in groups, will learn geometry by doing it. The result is that they will enjoy learning a body of integrated concepts that has become their intellectual possession.



Donald J. Albers
The Mathematical Association of America

Twenty-six hundred years ago Thales of Miletus launched the idea that Nature itself is subject to the laws of logic. Geometry, which had previously existed as a practical science, was found during the next few centuries to be tightly bound by logic. Some complex geometric facts were shown to be consequences of far simpler facts, and soon similar analyses of more general ideas about the world became standard for the Greek philosophers. The principle of stitching a few fundamental ideas together by logical analysis is at the heart of all modern science.

About 300 B.C.E., Euclid compiled his famous *Elements*, containing much of the geometry known at the time. Although the *Elements* does not fully meet today's standards of rigor, it quite rightly became and remained the principal textbook for teaching geometry for more than 2,000 years. In the past 100 years, however, the idea of *proof* has slowly receded (from American textbooks, at least), and now there is often little distinction between statements that have been proved formally and those that appear true on the basis of a drawing or a few measurements. This is an immense loss for American education, especially for science education. Some have tried to introduce formal proofs in algebra in place of those in geometry, but the results in algebra do not have the appeal of theorems such as that of Pythagoras. Geometry offers surprising theorems that can be proved by ingenious reasoning.

This book restores the idea of *proof* to its rightful place in geometry. Whoever studies this text will know what a proof is, what has been proved, and what has not.

Andrew M. Gleason

Andrew M. Gleason
Harvard University



Photograph by Roy Bishop

A Letter to the Student

The most useful question in the history of science is, Why? Because mathematics is the language of science, the same question is important in mathematics as well.

The oldest standing examples of applied geometry are the pyramids of Egypt, built about 2600 B.C. The bases of these structures are almost perfect squares, and their faces are precisely constructed triangles. There is evidence that the Egyptians even knew how to compute the *volume* of a pyramid but no evidence that they ever tried to understand or explain their methods by asking, “Why?” Their mathematics was based on intuition and experience; even for other ancient civilizations from Babylon to Rome, intuition and experience were as far as geometry went. The subject consisted of disconnected rules, and the fact that these rules “worked” in measuring land and constructing buildings seemed to be enough.

Your understanding of geometry has been like that of these people of long ago. You have a good intuitive knowledge of the subject simply by growing up in the world. As a small child, you may have played with blocks in the shape of *cubes*. On the playground, you have played with balls in the shape of *spheres*. In school, you use paper ruled with *parallel* and *perpendicular lines*. You drink from cans in the shape of *cylinders* and eat ice cream from containers in the shape of *cones*. Geometry is everywhere, and so it is natural for you to take it for granted without ever asking, “Why?”

About 600 B.C., the Greeks made a discovery that led to a new way of looking at mathematics. Thales, one of the seven wise men of antiquity, saw how geometric ideas that had been discovered intuitively could be related to one another logically. If some were true, then others must follow without question. He raised the question Why? about a mathematical idea and then successfully answered it. For example, the four edges of the Great Pyramid that meet at its top are equal in length. Thales explained why it follows from this fact that the angles that these edges form with the sides of the base also must be equal.

After Thales came other Greek mathematicians, including Pythagoras, Hippocrates, and, eventually, Euclid. Euclid, about whom you will learn more in this book, presented geometry as a logical system of connected ideas so well that his book titled the *Elements* became the most widely used textbook ever written. Euclid's approach to geometry, centered on the question *why*, was so successful that it even inspired writers in other fields to organize their ideas in the same way. One of the most influential books of modern science, the *Principia* by Sir Isaac Newton, was modeled after Euclid's *Elements*.

The word *geometry* derives from the Greek words for "earth" and "measure" and in some languages is still used today to mean "earth measurement" or "surveying." Remarkably, although Euclid's *Elements* is the most famous geometry book of all time, the Greek word for geometry never appears in it. The *Elements* was not a book about measuring the earth; rather, by bringing geometry together as a coherent whole, it made geometry understandable *and* provided a model for other sciences.

As you pursue your study of geometry, you will encounter not only many familiar ideas but also many ideas new to you that are both beautiful and surprising. As the title of this book suggests, it is about seeing, doing, and understanding. As you read the lessons, you will *see* the main ideas and some of their applications. With the exercises comes your chance to *do*, *understand*, and, consequently, know *why*. I hope that in using this book you will discover for yourself the wonder and excitement of geometry so that you will find your study of it to be an enjoyable and rewarding endeavor.

Harold Jacobs

Acknowledgments

Since beginning work on the first edition of *Geometry* some 30 years ago, I have benefited from the comments and suggestions of many people. Many teachers have been generous in sharing their ideas, both at conferences and through correspondence, and students have contributed as well.

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GEOMETRY
